LESSON 16

Measures of Central Tendency

GOAL Find the mean, median, and mode(s) of data and choose which measure best describes a set of data.

A **measure of central tendency** indicates where the center of a data set is by identifying a typical value for the data set. The *mean*, the *median*, and the *mode* are three commonly used measures of central tendency.

Mean, Median, and Mode

The **mean**, or *average*, of n numbers is the sum of the numbers divided by n. The mean is represented by $\bar{x}$, which is read as "x-bar."

For a set of data $x_1, x_2, \ldots, x_n$, the mean is $\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$.

The **median** of n numbers is the middle number when the numbers are written in numerical order. If n is even, the median is the average of the two middle numbers.

The **mode** of n numbers is the number or numbers that occur most frequently. There can be no mode, one mode, or many modes.

EXAMPLE 1 Finding Mean, Median, and Mode

The lengths (in minutes) of 13 movies are given below. Find the mean, median, and mode(s) of the data.

90, 102, 120, 180, 90, 85, 90, 137, 120, 145, 97, 93, 120

SOLUTION

Mean: To find the mean, add the 13 numbers and divide by 13.

$$\bar{x} = \frac{90 + 102 + 120 + 180 + 90 + 85 + 90 + 137 + 120 + 145 + 97 + 93 + 120}{13}$$
$$= \frac{1469}{13}$$
$$= 113 \text{ minutes}$$

Median: To find the median, first write the data in numerical order.

$$\underbrace{85, 90, 90, 90, 93, 97,}_{\text{6 values}}\ \underbrace{\mathbf{102}}_{\text{median}},\ \underbrace{120, 120, 120, 137, 145, 180}_{\text{6 values}}$$

The median is the middle value: 102 minutes.

Modes: The modes are the values that occur most frequently: 90 minutes and 120 minutes.

CHECK **Example 1**

Find the mean, median, and mode(s) of the data.

1. 62, 23, 27, 56, 52, 34, 42, 40, 70, 45, 77

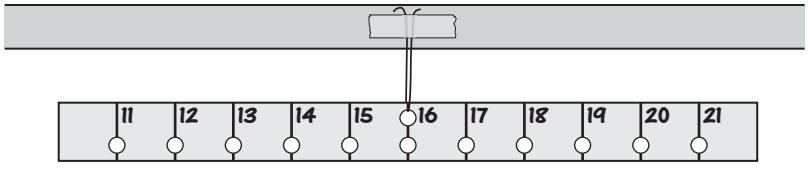
2. 38, 40, 29, 34, 8, 40, 47, 26, 43, 49, 14, 40

3. 32, 72, 81, 95, 98, 58, 77, 75, 87, 93, 45, 89, 93, 57, 82, 97, 52, 75

Activity

Modeling the Mean

1. On a 12 inch by 1 inch strip of cardboard, mark off segments every inch, and label them as shown. Punch holes where indicated. Use string and tape to hang the strip from the edge of a desk, as shown. Make sure the strip is balanced.



2. For each number listed below, hang one paper clip from the corresponding hole. Then balance the strip of cardboard by hanging one paper clip for each blank. Which number balances the strip?

   11, 13, 19, ___?___

3. Find the mean of the numbers for which paper clips are hung.

4. Repeat Steps 2 and 3 for the following sets of numbers. (Although the given numbers are in numerical order, the missing numbers are not necessarily greater than or equal to the given numbers.)

   a. 13, 13, 20, ___?___

   b. 15, 16, 17, ___?___, ___?___

   c. 12, 15, 20, ___?___, ___?___

5. How does the mean of each set of numbers compare with the number at the center of the strip of cardboard?

6. Will the mean change if you hang a paper clip or paper clips from the hole corresponding to the center number? Why or why not?

7. Under what conditions will the strip of cardboard balance around the median of a set of data?

8. Under what conditions will the strip of cardboard balance around the mode of a set of data?

Appropriate Measures of Central Tendency For a set of data, the three measures of central tendency may be nearly identical or may vary widely. Determining which of the three measures is most useful depends on the context in which the data are being used.

EXAMPLE 2 Choosing Mean, Median, or Mode

Tell whether the *mean, median,* or *mode(s)* best represents the data. Explain your reasoning.

a. Waiting times (in minutes) for patients at a doctor's office:

24, 19, 39, 19, 22, 26, 58, 16, 64, 27

b. Money (in dollars) earned baby-sitting:

30, 10, 15, 25, 10, 35, 20, 25, 30, 10

c. Shoes sizes of females at a bowling alley:

$5\frac{1}{2}$, 9, 6, $7\frac{1}{2}$, 6, 8, 6, $8\frac{1}{2}$, 6, $9\frac{1}{2}$, 6, 7, $6\frac{1}{2}$, 9

SOLUTION

a. The mean is 31.4 minutes.

The median is 25 minutes.

The mode is 19 minutes.

The mean is longer than most of the waiting times. The mode is shorter than most of the waiting times. Therefore, the median best represents the data.

b. The mean is \$21.

The median is \$22.50.

The mode is \$10.

The mean and median both represent the data well, because both are greater than half of the data and less than the other half.

c. The mean is about 7.18.

The median is 6.75.

The mode is 6.

The mean and median do not make sense because shoes come in only half and whole sizes. The mode gives information about the most common shoe size, so the mode best represents the data.

CHECK Example 2

Tell whether the *mean, median,* or *mode(s)* best represents the data. Explain your reasoning.

4. 21, 25, 10, 18, 9, 7, 3, 8, 14, 30, 12, 29, 11, 6, 35, 30

5. 3.65, 4, 3.25, 4.15, 3, 3.05, 3.25, 3.85

6. 6, 6, 5, 6, 6, 20, 6, 6, 6, 6

7. For a set of data containing all whole numbers, are the mean, median, and mode(s) necessarily whole numbers? Explain.

EXERCISES

Find the mean, median, and mode(s) of the data.

1. 3, 15, 12, 13, 5, 10, 12

2. 6, 16, 10, 16, 15, 9, 12

3. 25, 35, 29, 26, 31, 22, 21

4. 20, 39, 29, 25, 19, 25, 39

5. 15, 11, 19, 15, 14, 14, 13, 17

6. 15, 7, 1, 27, 13, 14, 17, 21, 3, 11

7. 75, 78, 99, 98, 54, 78, 85, 61

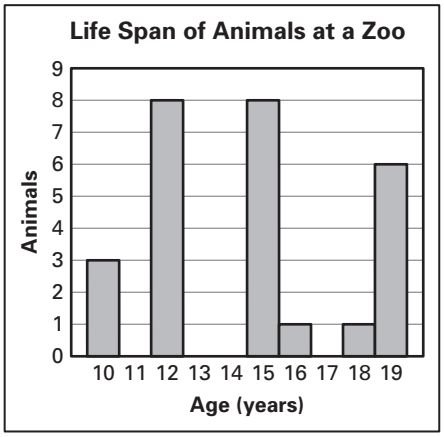
8. 49, 42, 62, 57, 71, 57, 72, 101, 74

9. 9.2, 6.4, 5.6, 8.4, 5.4, 7.5, 5.8

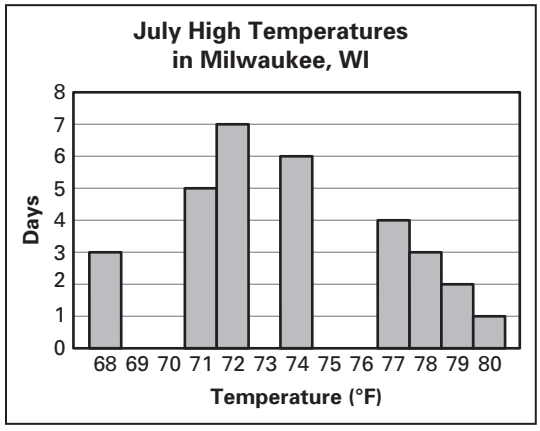
10. 7, 9.1, 7.8, 7.5, 7.8, 8.8, 8.5, 5.9

Find the mean, median, and mode(s) of the data displayed in the histogram.

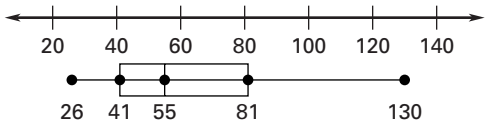
11.



12.



13. Tell whether you can find the *mean*, *median*, or *mode*(*s*) of the data from the box-and-whisker plot. Explain your reasoning.



Tell whether the *mean, median,* or *mode(s)* best represents the data. Explain your reasoning.

14. The length (in days) of 20 recent trials at a courthouse:

1, 14, 12, 1, 1, 4, 5, 2, 3, 1, 3, 3, 7, 2, 1, 1, 2, 3, 3, 2

15. Commuting times (in minutes) of 14 employees:

20, 55, 10, 45, 20, 60, 20, 5, 30, 15, 55, 35, 10, 40

16. Sizes (in fluid ounces) of 12 milk containers bought at a convenience store:

128, 16, 128, 32, 32, 128, 16, 64, 32, 64, 128, 32

Tell whether the *mean, median,* or *mode(s)* best represents the situation. Explain your reasoning.

17. You ask each of 100 schoolmates for the call number (for example, 98.9 FM) of his or her favorite radio station.

18. You collect data on the salaries of 100 workers at a company: 90 factory workers, 8 managers, the vice president, and the president.

19. In her first five basketball games, Juanita makes 7, 5, 8, 3, and 8 free throws. In her first six games, the mean number of free throws that Juanita makes is 6. How many free throws does Juanita make in the 6th game?

20. Your scores on four history tests are 85, 95, 88, and 94. There is one test left and you need a mean of at least 90 on the tests to get an A in the class.

a. What is the minimum score that you need on the last test in order to get an A?

b. After the exam, your teacher tells you that the median score of all your tests is 88. Can you conclude that you will get an A in the class? Explain.

c. Your teacher also tells you that the mode of your scores is 88. Can you conclude that you will get an A in the class? Explain.

21. The singers in a group sold calendars to raise money. The amounts raised are given below.

$35, $95, $75, $35, $50, $65, $50

a. Find the mean, median, and mode(s) of the data.

b. In addition to the amounts raised by the singers, the director of the choral group raised $50. Predict how adding this amount will affect the mean, median, and mode(s). Then check your prediction.

22. For 9 days, Kyle recorded the daily high temperature in Roseville and Summerton. His data are shown in the table.

Daily High Temperatures (°F)									
Roseville	75	80	85	78	88	92	90	75	75
Summerton	84	72	83	64	84	89	68	65	84

a. Which data set has a greater median?

b. Which data set has a greater mean?

c. Which data set has a greater mode?

d. Which measure of central tendency should Kyle use to compare the data sets if he wants to convince his sister that Roseville is warmer than Summerton?

In Exercises 23 and 24, use the table, which shows the number of wins per season from 2002 to 2008 for two teams in the National Football League.

	2002	2003	2004	2005	2006	2007	2008
Miami Dolphins	9	10	4	9	6	1	11
New Orleans Saints	9	8	8	3	10	7	8

23. Which measure of central tendency would you use to compare the data sets if you wanted to convince someone that the Miami Dolphins was the more successful team during this period? Explain.

24. Which measure of central tendency would you use to compare the data sets if you wanted to convince someone that the New Orleans Saints was the more successful team during this period? Explain.