

Matrices

Key Concept

Matrices have a wide range of uses in mathematics. They can represent transformations and they are a convenient tool for solving systems of equations. In this lesson, the emphasis is on the use of matrices for organizing data. Along the way, students learn three basic matrix operations: addition, subtraction, and scalar multiplication.

Key Question: Example 1

Why does each matrix have 6 entries?

On each day, 3 different items are sold and each item is sold at 2 different times, so there are 6 items of data for each day.

Teaching Strategy

Example 1 When you write the dimensions of a matrix, the number of rows comes before the number of columns. For example, a 4×5 matrix has 4 rows and 5 columns. A simple mnemonic, such as *RC Cola*, can help students remember that rows come before columns.

Key Question: Example 2

Is matrix addition commutative? That is, does the order in which you add the matrices matter? Why or why not?

Matrix addition is commutative because the entries are real numbers and addition of real numbers is commutative.

Key Question: Example 3

How can you use the profit matrix to find the overall profit for each class during the three-month period?

Add the values in each column. The overall profit for the junior class is \$200 and the overall profit for the senior class is $-\$55$ (a loss of \$55).

Differentiated Instruction

Example 3 Introduce the algebra of matrices to advanced learners. In this example, represent the revenue matrix by R , the expenses matrix by E , and the profit matrix by P . Tell students that $R - E = P$. Then ask students if they think the typical algebraic properties of equality apply. For example, have students investigate whether it is possible to add E to both sides of the equation to write $R = P + E$.

Key Question: Example 4

In part b, how does the order of operations apply to the matrices?

Scalar multiplication should be performed before addition and subtraction, so perform the scalar multiplications first and then subtract the resulting matrices.

Key Question: Example 5

Is it possible to use scalar multiplication to find a matrix that shows the recipe for a double batch of waffles and single batch of pancakes? Why or why not?

No; when you use scalar multiplication, you must multiply every entry in the matrix by the scalar.

Closing the Lesson

Have students answer the following question: What is the connection between scalar multiplication by a whole number and matrix addition?

Scalar multiplication by a whole number n is the same as repeated addition of the matrix with a total of n addends.

Vocabulary

Exercises 26–31 Point out to students that many concepts that apply to real numbers have counterparts in the world of matrices. These exercises introduce *equal matrices*. As with real numbers, once equality has been defined, it makes sense to think about equations and their solutions. You may want to extend these exercises by having students write their own matrix equations. Have students exchange their work with a partner and ask them to solve each other's equations.

Homework Help

Example 1: Exs. 1–3, 19

Example 2: Exs. 4–7

Example 3: Exs. 8, 9, 22

Example 4: Exs. 10–13, 15–18

Example 5: Exs. 14, 20, 21

Enrichment: Exs. 23–31

Homework Check

To quickly check student understanding of key concepts, go over the following exercises: 1, 6, 8, 14, 15.

ANSWERS

Check Answers

1. Sample answer:

	Siblings	
	Sisters	Brothers
Al	2	2
Moe	3	1
Don	1	0
Janet	0	0

2. $\begin{bmatrix} 7 & 8 \\ 4 & 9 \end{bmatrix}$

3. $\begin{bmatrix} 2 & 0 \\ 8 & 6 \\ 0 & 7 \end{bmatrix}$

4. Not possible; the matrices do not have the same dimensions.

5. Sample answer:

	Monday		
	History	Math	English
Rob	40	30	45
Jill	30	25	50

	Tuesday		
	History	Math	English
Rob	20	35	60
Jill	40	30	40

6. Sample answer: **Total Homework Time**

	History	Math	English
Rob	60	65	105
Jill	70	55	90

7. $\begin{bmatrix} -6 & -12 & 9 \\ -3 & 0 & -21 \end{bmatrix}$

8. $\begin{bmatrix} 18 & 36 \\ 9 & 0 \end{bmatrix}$

9. Not possible; the matrices do not have the same dimensions.

10. **Total Money Received**

	Comedy	Drama
Saturday	84	129.50
Sunday	56	70

Exercise Answers

1. Sample answer:

	Book Collection		
	History	Science fiction	Mystery
Hardbacks	2	9	6
Paperbacks	1	12	8

2. Sample answer:

	Car Reservations	
	Budget	Luxury
Sedan	16	14
Convertible	5	12
Van	8	13
Sport Utility	2	22

3. Sample answer:

	Sports Club Locations	
	East	West
Treadmill	12	10
Stationary bike	6	8
Stair machine	4	3

4. Not possible; the matrices do not have the same dimensions.

5. $\begin{bmatrix} 1.9 & 2.2 \\ -1.1 & 3.5 \end{bmatrix}$

6. $\begin{bmatrix} 11 & -19 \\ 12 & 13 \\ 2 & 3 \end{bmatrix}$

7. $\begin{bmatrix} -3 & 7 & 2 \\ -1 & -2 & -4 \\ -1 & 6 & 6 \end{bmatrix}$

8.

	Temperature Difference			
	January	April	July	October
Chicago	15.1	18.9	18.7	19.1
New York City	12.3	17.5	16.7	15.7
Los Angeles	18	14.6	12.4	15.1

9. **Total Free Throws Made and Missed**

	Free throws made	Free throws missed
Game 1	12	7
Game 2	9	13
Game 3	9	5

10. $\begin{bmatrix} \frac{10}{3} & -\frac{5}{3} \\ -1 & 4 \\ 3 & -\frac{25}{3} \end{bmatrix}$

11. $\begin{bmatrix} \frac{1}{4} & -3 & -\frac{3}{8} \\ 2 & 5 & -\frac{1}{8} \end{bmatrix}$

12. $\begin{bmatrix} -35 & -28 & 0 & 14 & -7 \\ -21 & 7 & -4.2 & -14 & -105 \end{bmatrix}$

13. $\begin{bmatrix} -0.75 & 6 & 15 & 0.75 \\ -3 & 2.25 & -3.75 & -30 \end{bmatrix}$

14. Sample answer:

	Total Coupon Value		
	Football players	Cheerleaders	Band
East	20.25	6	47.25
Central	31.50	13.50	84

15. $\begin{bmatrix} 10 & 27 \\ 26 & 36 \end{bmatrix}$

16. $\begin{bmatrix} 2 & -7 \\ 0 & -6 \\ 6 & -1 \end{bmatrix}$

17. Not possible; the matrices do not have the same dimensions.

18. $\begin{bmatrix} 7 & 12 & -3 \\ -13 & 19 & -12 \end{bmatrix}$

19. Sample answer:

	Child Tickets	
	Saturday	Sunday
2:55 P.M. show	114	106
5:05 P.M. show	91	122
7:10 P.M. show	126	88

	Adult Tickets	
	Saturday	Sunday
2:55 P.M. show	205	214
5:05 P.M. show	226	198
7:10 P.M. show	187	211

20. Sample answer: Revenue for Child Tickets

	Saturday	Sunday
2:55 P.M. show	570	530
5:05 P.M. show	455	610
7:10 P.M. show	630	440

21. Sample answer: Revenue for Adult Tickets

	Saturday	Sunday
2:55 P.M. show	1640	1712
5:05 P.M. show	1808	1584
7:10 P.M. show	1496	1688

22. Sample answer:

	Total Revenue	
	Saturday	Sunday
2:55 P.M. show	2210	2242
5:05 P.M. show	2263	2194
7:10 P.M. show	2126	2128

23. Sample answer:

	Average Weekly Wages		
	Illinois	Maryland	New Jersey
Body Repairer	827.60	857.20	754.40
Glass Installer	568.40	648.40	630.80
Service Technician	780.80	809.20	749.20

24. Sample answer:

	Average Hourly Wages		
	Illinois	Maryland	New Jersey
Body Repairer	21.10	21.86	19.24
Glass Installer	14.49	16.53	16.09
Service Technician	19.91	20.63	19.10

25. Sample answer: Use matrix addition as shown.

$$\begin{bmatrix} 20.69 & 21.43 & 18.86 \\ 14.21 & 16.21 & 15.77 \\ 19.52 & 20.23 & 18.73 \end{bmatrix} + \begin{bmatrix} 0 & 1.25 & 0 \\ 0 & 1.25 & 0 \\ 0 & 1.25 & 0 \end{bmatrix} = \begin{bmatrix} 20.69 & 22.68 & 18.86 \\ 14.21 & 17.46 & 15.77 \\ 19.52 & 21.48 & 18.73 \end{bmatrix}$$

26. $-4; 3$

27. $2; 3$

28. $5; 0$

29. $4; -15$

30. $\frac{1}{2}; 1$

31. $3; 2$