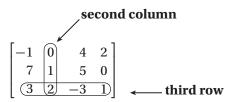**GOAL** Organize data in a matrix; add and subtract matrices; perform scalar multiplication.

A $\boxed{\text{matrix}}$ is a rectangular arrangement of numbers in rows and columns. The plural of matrix is *matrices*. The $\boxed{\text{dimensions}}$ of a matrix are the numbers of rows and columns. The matrix below has three rows and four columns, so it is called a 3×4 (read "3 by 4") matrix. Each number in a matrix is called an $\boxed{\text{entry}}$. The entry in the third row and second column of the matrix below is 2.

second column

$$\begin{bmatrix} -1 & 0 & 4 & 2 \\ 7 & 1 & 5 & 0 \\ 3 & 2 & -3 & 1 \end{bmatrix}$$ ⟵ **third row**

EXAMPLE 1 **Writing Matrices**

Write and label matrices to organize the following data about concession sales at a high school play on two different nights.

Friday
Before the show, the concessions committee sold 20 beverages, 6 cookies, and 12 candy items. During intermission they sold 26 beverages, 11 cookies, and 19 candy items.

Saturday
Before the show, the concessions committee sold 14 beverages, 3 cookies, and 9 candy items. During intermission they sold 31 beverages, 13 cookies, and 15 candy items.

SOLUTION

One way to organize the data is to use 2×3 matrices, as shown.

Friday

	Beverages	Cookies	Candy
Before show	20	6	12
Intermission	26	11	19

Saturday

	Beverages	Cookies	Candy
Before show	14	3	9
Intermission	31	13	15

You can also organize the data using 3×2 matrices, where the row labels are the types of concessions (beverages, cookies, and candy), and the column labels are the times of sales (before show and intermission).

Friday

	Before show	Intermission
Beverages	20	26
Cookies	6	11
Candy	12	19

Saturday

	Before show	Intermission
Beverages	14	31
Cookies	3	13
Candy	9	15

Adding and Subtracting Matrices To add or subtract matrices, add or subtract their corresponding entries. You can add or subtract matrices only if they have the same dimensions.

EXAMPLE 2 **Adding and Subtracting Matrices**

Perform the indicated operation, if possible. If it is not possible, explain why not.

a. $\begin{bmatrix} 7 & 6 \\ 4 & -3 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} -4 & 4 \\ -2 & 1 \\ 6 & 9 \end{bmatrix}$ **b.** $\begin{bmatrix} 1.2 & -1.1 \\ 1.7 & 2.3 \end{bmatrix} - \begin{bmatrix} 0.7 & 1.4 \\ -9.0 & 1.6 \end{bmatrix}$ **c.** $\begin{bmatrix} 3 & 6 & 5 \end{bmatrix} + \begin{bmatrix} -4 \\ 5 \end{bmatrix}$

SOLUTION

a. $\begin{bmatrix} 7 & 6 \\ 4 & -3 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} -4 & 4 \\ -2 & 1 \\ 6 & 9 \end{bmatrix} = \begin{bmatrix} 7 + (-4) & 6 + 4 \\ 4 + (-2) & -3 + 1 \\ 2 + 6 & 8 + 9 \end{bmatrix} = \begin{bmatrix} 3 & 10 \\ 2 & -2 \\ 8 & 17 \end{bmatrix}$

b. $\begin{bmatrix} 1.2 & -1.1 \\ 1.7 & 2.3 \end{bmatrix} - \begin{bmatrix} 0.7 & 1.4 \\ -9.0 & 1.6 \end{bmatrix} = \begin{bmatrix} 1.2 - 0.7 & -1.1 - 1.4 \\ 1.7 - (-9.0) & 2.3 - 1.6 \end{bmatrix} = \begin{bmatrix} 0.5 & -2.5 \\ 10.7 & 0.7 \end{bmatrix}$

c. Because they have different dimensions, it is not possible to add the matrices.

EXAMPLE 3 **Using Matrix Subtraction**

The revenue and expenses of the junior and senior classes for three months are shown in the matrices below. Write and label a matrix that shows the monthly profit for each class.

Revenue

	Juniors	Seniors
January	0	200
February	190	0
March	310	0

Expenses

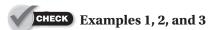|	Juniors	Seniors
January	40	55
February	40	200
March	200	0

SOLUTION

Profit is the difference between revenue and expenses. To find the *profit matrix*, you can subtract the *expenses matrix* from the *revenue matrix*. A negative entry in the profit matrix indicates a loss.

$\begin{bmatrix} 0 & 200 \\ 190 & 0 \\ 310 & 0 \end{bmatrix} - \begin{bmatrix} 40 & 55 \\ 40 & 200 \\ 200 & 0 \end{bmatrix} = \begin{bmatrix} 0 - 40 & 200 - 55 \\ 190 - 40 & 0 - 200 \\ 310 - 220 & 0 - 0 \end{bmatrix} = \begin{bmatrix} -40 & 145 \\ 150 & -200 \\ 90 & 0 \end{bmatrix}$

Label the resulting matrix to identify the monthly profit for each class.

Profit

	Juniors	Seniors
January	-40	145
February	150	-200
March	90	0

1. Al has 2 sisters and 2 brothers. Moe has 3 sisters and 1 brother. Don has only 1 sister. Janet has no siblings. Write and label a matrix to organize the data.

Perform the indicated operation, if possible. If it is not possible, explain why not.

2. $\begin{bmatrix} 9 & 5 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ -1 & 3 \end{bmatrix}$ 3. $\begin{bmatrix} 0 & 0 \\ 12 & 6 \\ -4 & 7 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 4 & 0 \\ -4 & 0 \end{bmatrix}$ 4. $\begin{bmatrix} 6 & 8 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

In Exercises 5 and 6, use the following information.

Rob and Jill do their homework together each night. On Monday night, Rob spends 40 minutes on history, 30 minutes on math, and 45 minutes on English. Jill spends 30 minutes on history, 25 minutes on math, and 50 minutes on English. On Tuesday night, Rob spends 20 minutes on history, 35 minutes on math, and 60 minutes on English. Jill spends 40 minutes on history, 30 minutes on math, and 40 minutes on English.

5. Write and label matrices to organize the data.

6. Write and label a matrix that shows the total time for the two nights that each student spent on each subject.

Scalar Multiplication When a matrix is multiplied by a real number, that number is called a **scalar**. To multiply a matrix by a scalar, multiply each entry in the matrix by the scalar. This process is called **scalar multiplication**.

EXAMPLE 4 **Multiplying a Matrix by a Scalar**

Perform the indicated operation(s), if possible. If it is not possible, explain why not.

a. $5\begin{bmatrix} -5 & 2 \\ 0 & 1 \end{bmatrix}$ b. $3\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} - 2\begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$

SOLUTION

a. $5\begin{bmatrix} -5 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5(-5) & 5(2) \\ 5(0) & 5(1) \end{bmatrix} = \begin{bmatrix} -25 & 10 \\ 0 & 5 \end{bmatrix}$

b. $3\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} - 2\begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 3(1) \\ 3(3) \\ 3(-2) \end{bmatrix} - \begin{bmatrix} 2(6) \\ 2(4) \\ 2(0) \end{bmatrix}$ *Multiply matrices by scalars.*

$= \begin{bmatrix} 3 \\ 9 \\ -6 \end{bmatrix} - \begin{bmatrix} 12 \\ 8 \\ 0 \end{bmatrix}$ *Simplify.*

$= \begin{bmatrix} -9 \\ 1 \\ -6 \end{bmatrix}$ *Then subtract.*

EXAMPLE 5 Using Scalar Multiplication

The matrix below shows recipes for a batch of waffles and a batch of pancakes. Use scalar multiplication to write and label a matrix that shows recipes for double batches of waffles and pancakes.

Single Batches

	Waffles	Pancakes
Mix (cups)	2	1
Milk (cups)	$1\frac{1}{3}$	$\frac{1}{2}$
Oil (tablespoons)	2	0
Eggs	1	1

SOLUTION

To find the recipes for double batches, multiply the matrix above by 2.

$$2\begin{bmatrix} 2 & 1 \\ 1\frac{1}{3} & \frac{1}{2} \\ 2 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2(2) & 2(1) \\ 2\left(1\frac{1}{3}\right) & 2\left(\frac{1}{2}\right) \\ 2(2) & 2(0) \\ 2(1) & 2(1) \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2\frac{2}{3} & 1 \\ 4 & 0 \\ 2 & 2 \end{bmatrix}$$

Label the resulting matrix.

Double Batches

	Waffles	Pancakes
Mix (cups)	4	2
Milk (cups)	$2\frac{2}{3}$	1
Oil (tablespoons)	4	0
Eggs	2	2

CHECK **Examples 4 and 5**

Perform the indicated operation(s), if possible. If it is not possible, explain why not.

7. $-3\begin{bmatrix} 2 & 4 & -3 \\ 1 & 0 & 7 \end{bmatrix}$ **8.** $4\begin{bmatrix} 2 & 4 \\ 1 & 0 \end{bmatrix} + 5\begin{bmatrix} 2 & 4 \\ 1 & 0 \end{bmatrix}$ **9.** $4\begin{bmatrix} 3 & 4 \\ 5 & 5 \end{bmatrix} - [1 \ 2 \ 3 \ 4]$

10. The matrix below shows the numbers of movies rented in two categories on each of two days. Each movie rental costs $3.50. Use scalar multiplication to write and label a matrix that shows the total money received in rental costs in each category for the two days.

Movies Rented

	Comedy	Drama
Saturday	24	37
Sunday	16	20

In Exercises 1–3, write and label a matrix to organize the data.

1. Your book collection:

   Hardbacks: 2 history, 9 science fiction, 6 mystery

   Paperbacks: 1 history, 12 science fiction, 8 mystery

2. Car rental reservations:

   Sedan: 16 budget, 14 luxury

   Convertible: 5 budget, 12 luxury

   Van: 8 budget, 13 luxury

   Sport utility: 2 budget, 22 luxury

3. A sports club has 2 locations in your city. At the east location there are 12 treadmills, 6 stationary bicycles, and 4 stair machines. At the west location there are 10 treadmills, 8 stationary bicycles, and 3 stair machines.

In Exercises 4–7, perform the indicated operation, if possible. If it is not possible, explain why not.

4. $\begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -7 & 1 \\ 4 & 0 \end{bmatrix}$

5. $\begin{bmatrix} 1.4 & 1.2 \\ -1.1 & 2.3 \end{bmatrix} + \begin{bmatrix} 0.5 & 1.0 \\ 0 & 1.2 \end{bmatrix}$

6. $\begin{bmatrix} 9 & -8 \\ 3 & 7 \\ 2 & -9 \end{bmatrix} - \begin{bmatrix} -2 & 11 \\ -9 & -6 \\ 0 & -12 \end{bmatrix}$

7. $\begin{bmatrix} 6 & 5 & 9 \\ 4 & 3 & 1 \\ 0 & 4 & 7 \end{bmatrix} - \begin{bmatrix} 9 & -2 & 7 \\ 5 & 5 & 5 \\ 1 & -2 & 1 \end{bmatrix}$

8. The matrices below show the average maximum and minimum temperatures (in degrees Fahrenheit) in Chicago, New York City, and Los Angeles. Use matrix subtraction to write and label a matrix that shows the differences in maximum and minimum temperatures.

Maximum Temperatures

	January	April	July	October
Chicago	30.0	59.0	84.4	63.7
New York City	37.6	61.2	85.1	65.3
Los Angeles	65.7	67.3	75.2	74.3

Minimum Temperatures

	January	April	July	October
Chicago	14.9	40.1	65.7	44.6
New York City	25.3	43.7	68.4	49.6
Los Angeles	47.7	52.7	62.8	59.2

9. The matrices below show the number of free throws made and free throws missed in each of three games by two students on the same basketball team, Tom and Mark. Use matrix addition to write and label a matrix that shows the total number of free throws made and free throws missed by Tom and Mark in the three games.

Tom

	Free throws made	Free throws missed
Game 1	5	5
Game 2	6	8
Game 3	4	5

Mark

	Free throws made	Free throws missed
Game 1	7	2
Game 2	3	5
Game 3	5	0

In Exercises 10–13, perform the indicated operation.

10. $-\dfrac{1}{3}\begin{bmatrix} -10 & 5 \\ 3 & -12 \\ -9 & 25 \end{bmatrix}$

11. $\dfrac{1}{2}\begin{bmatrix} \frac{1}{2} & -6 & -\frac{3}{4} \\ 4 & 10 & -\frac{1}{4} \end{bmatrix}$

12. $-7\begin{bmatrix} 5 & 4 & 0 & -2 & 1 \\ 3 & -1 & 0.6 & 2 & 15 \end{bmatrix}$

13. $1.5\begin{bmatrix} -0.5 & 4 & 10 & 0.5 \\ -2 & 1.5 & -2.5 & -20 \end{bmatrix}$

14. You manage the concession stand during football games at your school. You give a $.75 coupon to each player, cheerleader, and band member from each school. East High School has 27 football players, 8 cheerleaders, and 63 band members. Central High School has 42 football players, 18 cheerleaders, and 112 band members. Use scalar multiplication to write and label a matrix that shows the total coupon value (in dollars) given to each school.

Perform the indicated operations, if possible. If it is not possible, explain why not.

15. $3\begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix} + \begin{bmatrix} -2 & 9 \\ 5 & 9 \end{bmatrix}$

16. $-1\begin{bmatrix} -4 & 5 \\ -2 & 6 \\ -7 & 0 \end{bmatrix} - \dfrac{1}{4}\begin{bmatrix} 8 & 8 \\ 8 & 0 \\ 4 & 4 \end{bmatrix}$

17. $-2\begin{bmatrix} 6 & 8 \end{bmatrix} + 5\begin{bmatrix} 1 & 2 & 4 \end{bmatrix}$

18. $5\begin{bmatrix} 3 & 4 & 1 \\ -9 & 3 & 0 \end{bmatrix} - 4\begin{bmatrix} 2 & 2 & 2 \\ -8 & -1 & 3 \end{bmatrix}$

In Exercises 19–22, use the table below, which shows the ticket sales for a movie at a local theater.

	Saturday		Sunday	
	Child	**Adult**	**Child**	**Adult**
2:55 P.M. show	114	205	106	214
5:05 P.M. show	91	226	122	198
7:10 P.M. show	126	187	88	211

19. Write and label a matrix to organize the data about child tickets and another matrix to organize the data about adult tickets.

20. The price of one child ticket is $5. Use scalar multiplication to write and label a matrix that shows the total revenue in child ticket sales for each show.

21. The price of one adult ticket is $8. Use scalar multiplication to write and label a matrix that shows the total revenue in adult ticket sales for each show.

22. Use matrix addition to write and label a matrix that shows the total revenue (including both child and adult tickets) for each show.

In Exercises 23–25, use the following information.

The table shows data from the United States Department of Labor for 2008.

Automotive Industry: Average Hourly Wages			
Occupation	Illinois	Maryland	New Jersey
Body Repairer	$20.69	$21.43	$18.86
Glass Installer	$14.21	$16.21	$15.77
Service Technician	$19.52	$20.23	$18.73

23. Use scalar multiplication to write and label a matrix that shows the average weekly wage for each occupation in each state. Assume there are 40 hours in a work week.

24. Suppose wages in the automotive industry increase by 2% from 2008 to 2009. Write and label a matrix that shows the average hourly wage for each occupation in each state in 2009.

25. Suppose the average hourly wages for these occupations increase by $1.25 per hour in Maryland while the wages remain constant in the other states. Describe how you could use one or more matrix operations to represent this situation.

In Exercises 26–31, use the following information to solve the matrix equation for x and y.

Two matrices are *equal* if their dimensions are the same and the entries in corresponding positions are equal.

26. $\begin{bmatrix} 4 & 2x \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ -y & 0 \end{bmatrix}$

27. $\begin{bmatrix} -1 & 3 \\ 2y+1 & 12 \end{bmatrix} = \begin{bmatrix} x-3 & 3 \\ 7 & 12 \end{bmatrix}$

28. $\begin{bmatrix} 9 & 4 \\ 2x & -2 \end{bmatrix} + \begin{bmatrix} 1 & 2y \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 4 \\ 13 & -1 \end{bmatrix}$

29. $3\begin{bmatrix} x \\ 4 \end{bmatrix} + \begin{bmatrix} -5 \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$

30. $\begin{bmatrix} -3 & -2x \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 2y & 1 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ 2 & 10 \end{bmatrix}$

31. $2x\begin{bmatrix} -1 & 0 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 9y & -24 \end{bmatrix}$