LESSON 11

Geometric Probability

Key Concept

Students already have experience finding lengths, areas, and volumes. In this lesson, they use these geometric measures to solve probability problems. Some of the problems are purely geometric (such as finding the probability that a randomly chosen point inside a figure lies within a shaded portion of the figure). Other problems model real-world phenomena (such as finding the probability that a bus is waiting at a bus stop).

Key Question: Example 1

Why is the line segment divided into 12 equal units?

The line segment represents 6 minutes and the interval of time that the bus waits is $\frac{1}{2}$ minute. By dividing the line segment into 12 equal units, you can use 1 unit to represent the interval of time the bus waits.

Key Question: Example 2

How can you use estimation to show that the answer is reasonable?

The area of the square is 100 cm^2 and the area of the circle is about $3 \cdot 5^2 = 75$ cm^2. Therefore, the area of the shaded region is about 25 cm^2 and it is approximately $\frac{1}{4}$ of the square. The probability that a randomly chosen point in the square lies in the shaded region should be close to $\frac{1}{4}$. Since 21.5% is close to 25%, the answer is reasonable.

Differentiated Instruction

Example 2 There are several steps involved in solving this problem. Some students may get lost in the middle of the calculations and lose sight of the big picture. To help these students, begin by discussing the overall strategy for solving the problem and then outline the steps that will be used. One such outline is shown below. As students work through the problem, they can use this roadmap to check off the steps as they complete them.

Steps for Solving Example 2

1. Find the area of the square.
2. Find the area of the shaded region.
   a. Find the area of the circle.
   b. Subtract the area of the circle from the area of the square.
3. Find the ratio of the area of the shaded region to the area of the square.

Avoiding Common Errors

Example 2 Students may think that they are supposed to find the ratio of the areas of the shaded and unshaded regions in the figure. Point out that the figure is a square and that the area of the shaded region is to be compared to the area of the square. Similarly, students are to compare the area of the shaded triangle to the area of the rectangle in Check Exercise 2, and they are to compare the area of the shaded triangle to the area of the large triangle in Check Exercise 3.

Key Question: Example 3

What do the variables in the volume formulas represent?

In the formula for the volume of a rectangular prism, *l* is the length, *w* is the width, and *h* is the height. In the formula for the volume of a cylinder, *r* is the radius and *h* is the height.

Avoiding Common Errors

Example 3 Students may assume that the cylinder and the rectangular prism have the same height and incorrectly substitute 3 for *h* in the formula for the volume of the cylinder. Point out that the cylinder is shorter than the prism.

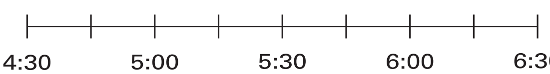
Closing the Lesson

Have students answer the following question: What is the general method for finding a geometric probability?

Find the ratio of the length, area, or volume of a portion of a figure to the length, area, or volume of the entire figure.

Teaching Strategy

Exercise 1 You may wish to provide scaffolding to help students get started with Exercise 1. In this case, show students the following line segment. Then have them mark an interval on the segment that could represent the time Mia spends on math homework.



Homework Help

Example 1: Exs. 1, 2
Example 2: Exs. 3–7
Example 3: Exs. 8–11
Enrichment: Ex 12

Homework Check

To quickly check student understanding of key concepts, go over the following exercises: 1, 3, 7, 11.

ANSWERS

Check Answers

1. about 76.7%
2. 50%
3. about 11.1%
4. about 19.9%

Exercise Answers

1. 37.5%
2. about 11.1%
3. 50%
4. 25%
5. 80%
6. about 45.8%
7. about 11.1%
8. about 32.7%
9. about 11.1%
10. about 3.7%
11. about 2.2%
12. Check students' drawings. The irregular shape should be entirely contained within a square. The area of the square can be calculated by measuring one of its sides. To find the area of the irregular shape, use the formula for geometric probability: $P(\text{point lies in shape}) = \frac{\text{Area of shape}}{\text{Area of square}}$, or $P(\text{point lies in shape}) \bullet (\text{Area of square}) =$ Area of shape. The geometric probability P(point lies in shape) can be estimated by finding the ratio of the number of darts that land in the shape to the total number of darts thrown. The area of the square is known. Thus, the product $P(\text{point lies in shape}) \bullet (\text{Area of square})$ gives an estimate of the area of the shape.