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Mass Customization of Reciprocal Frame Structures

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Abstract

A reciprocating system uses relatively short members to build a structure with a span longer than that of its individual parts where each beam supports and is supported by other beams in the structure (Pawlyn [8]). For reasons regarding recyclability, dis-assembly of parts and re-assembly elsewhere, wooden reciprocal frame structures have been the core of extensive research in recent years as advances in computational design tools allow for the application of their principles to complex shapes.

While elongated bars that are similar to each other are populated over a surface on a conventional linear reciprocal frame structure, the members can also be different from each other as long as there is a tessellation pattern that is repeated (Kohlhammer and Kotnik [5]). The objective of this paper is to explore this option by creating a digital workflow that automates the fabrication of mass-customized planar reciprocal frame structures. Local material differentiation of the framework can be introduced by reading the results of an overall structural analysis of the assembly and a multiple load-case study of each member. The boards can have diverse tectonic features according to their position and function within the structure and the order in which they have to be assembled.

Designed using optimization algorithms, the planar reciprocal frame structures featured in this paper incorporate notching techniques. The three-dimensional nature of the intersections between members is processed computationally taking into consideration the constraints of the fabrication method where the elements have been manufactured by means of a six-axis robot.

Keywords: reciprocal frame structures, mass customization, computational design, digital optimization.

1. Introduction

1.1 Fundamentals of planar reciprocal frame structures

A reciprocating system uses relatively short members to build a structure with a span longer than that of its individual parts where each beam supports and is supported by other beams in the structure (Pawlyn [8]). The origin and development of reciprocal frame (referred to as RF in this paper) structures occurred both in Europe and in the Orient making use of linear elongated elements. In the West, the goal of covering large spans with the available short elements made these wooden structures ubiquitous. In the East, large structures called ‘mandala roofs’ were created as a result of scaling up the system used in traditional interwoven baskets made of bamboo (Baverel and Pugnale [2]). The simplest assembly of mutually supporting beams placed in a closed circuit consists of three members, which form a unit that resembles a fan-like structure (Popovic Larsen [9]) (Figure 1).

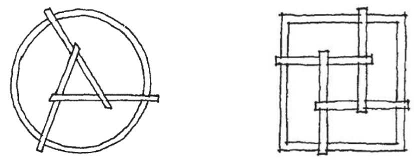

Figure 1: Three and four-member RF unit (Popovic Larsen [9])

A planar RF structure is understood as the type that is composed of 'thick' linear elements. Each connection between a pair of members features the same three parameters present in linear ones: eccentricity *e*, engagement length *x*, and the condition of whether member 1 sits on top or lies under member 2 (Parigi and Kirkegaard [7]). However, while the value of the eccentricity between members in a linear RF structure using bars determines the diameter of these elements, in planar reciprocal frame structures, the sizing of the members - their depth - is an external parameter. In both linear and planar RF structures, the values of the eccentricity between bars and the length of the elements control the convexity or concavity of the underlying geometry. This can also be modified by notching the bars or bending the elongated members. Together, the eccentricity and the depth of the members define the depth of the notch in the case of planar RFs (Figure 2). An opportunity to introduce heterogeneity, local differentiation and integration to customize the structure has been observed in the larger surface available on the members of a planar RF.

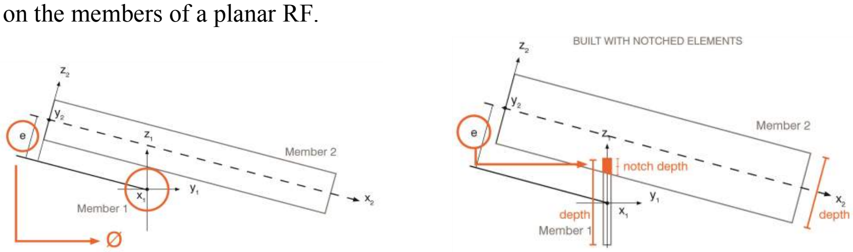

Figure 2: Parameters in Members 1 and 2 for linear (left) and planar (right) RF.

2. Research background

Research on reciprocity has extensively explored the generation and assembly of RF with linear elements, while few but quite interesting texts have been written on the second type composed of planar members. Representatives of both types are summarized in the next section.

2.1. Structure and design

In planar RF structures, the members provide depth to explore new design possibilities. This is the case of Balmond, Souza and de Moura's Serpentine Gallery Pavilion, which features 427 unique members, and wherein one direction of the interlocked beams is made predominant over the other by kinking its beams at the top (Balmond [1]). Of particular interest to this type of structure are also the studies by Douthe and Baverel on double layer reciprocal structures, for which they built a dome using ladders [4] (Figure 3). This image shows the potential of removing material from the surface of the planar elements. Another example is the Coca-Cola Beatbox Pavilion, a RF structure designed by Khan and Ohrstedt and built with panels where three interacting lightweight ETFE cushions are braced against each other to make an overall stable system.

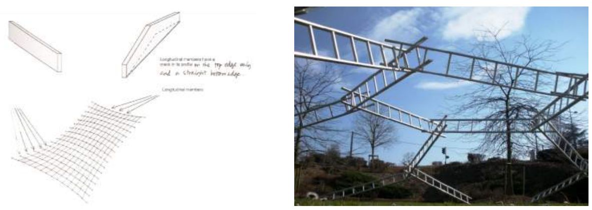

Figure 3: Serpentine Gallery Pavilion (Balmond [1]) and ladder dome (Douthe and Baverel [4])

2.2. Connection methods

Connections in RF assemblies ensure the transmission of the loads, determine how easy it is to assemble the structure and govern the deformation of the overall structure. There are several built examples using notching techniques which is the most common type employed. In the planar reciprocal frame assembly of the aforementioned Serpentine Gallery Pavilion, a simple system of connections was created by having all the elements the same length and fixing the dimensions of the mortise and tenon solution [1]. Thönissen developed two-dimensional contour sections and notches and milled them into three dimensional cross-lap joints using a CNC machine [12] (Figure 4).

Figure 4: Mortise and tenon of the Serpentine Gallery Pavilion (Balmond [1]) and notching system designed by Thönissen [12]

2.3. Morphogenesis

Considering that linear reciprocal structures are low-cost and relatively simple to fabricate, complex free-form shapes with standard elements and simple jointing techniques can be built using these systems. The most used method to generate RFs computationally is the top-down approach, where optimization strategies can be introduced. In this process, design algorithms use a reference surface geometry and 'force' the assembly of bars to become a reciprocal system. This is the case of Dermoid, a project that consists of a wooden reciprocal frame with no fasteners that is laid out in a hexagonal grid and distributed across a doubly curved surface (Burry [3]). Piker's component for Grasshopper generates free-form RF structures from triangular, quadrangular or hexagonal meshes where tangency between rods is achieved through dynamic relaxation (Piker [9]) (Figure 5).

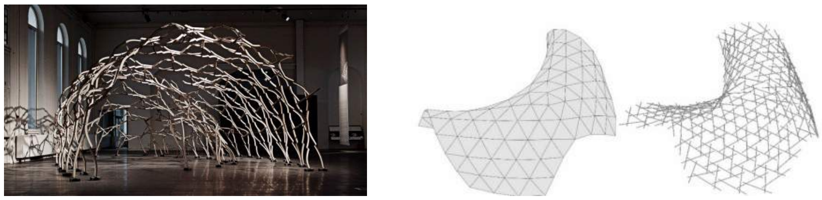

Figure 5: Dermoid by Burry [3] and RF on a free-form shape by Piker [9]

2.4. Rapid prototyping

Advances in digital fabrication technologies allow for the manufacturing of each unique member in a structure in an integrated process. These tools have been used in the following free-form RF linear structures. In Lamella Flock (Tamke *et al*. [11]) the beams have a precisely cut tenon at the ends and house the connection point in the middle. Digital fabrication allowed for self-registration and all the elements were labeled using specialized screws that don't require drilling. In the case of the self-supporting framework built at the University of Kassel, the data generated enabled the cutting to the right length of 180 beams using a CNC machine (Günther and Proll [5]) (Figure 6).

Figure 6: Lamella Flock (Tamke *et al.* [9]) and framework desgined at the University of Kassel (Günther and Proll [5])

3. Digital workflow

This paper explores how free-form structures can be fabricated and assembled following a reciprocal frame pattern by designing algorithmically their segmentation into mass-customized planar members. The digital workflow that implements this study has been generated in Rhinoceros using Grasshopper and the plug-ins Kangaroo, Millipede and HAL along with custom C# components.

3.1. Computational design

The first phase of the digital workflow takes a surface as an input. Given the parametrically-controlled generative curves, the lofted surface is discretized into a mesh, where the user inputs the number of divisions in each direction. The computational definition takes the edges of each face and rotates them an angle defined by the user to get a reciprocal frame following the underlying surface (Piker [9]). The eccentricity between each pair of elements is also defined in this step and dynamic relaxation is run. The centerlines are translated into solids as thin rectangular extrusions. The default orientation of the panels follows the original layout of each bar on the mesh after rotating each element, which is normal to the mesh at a point in the middle of the line. A script on alignment is run so that the panels are oriented normal to the mesh (Figure 7).

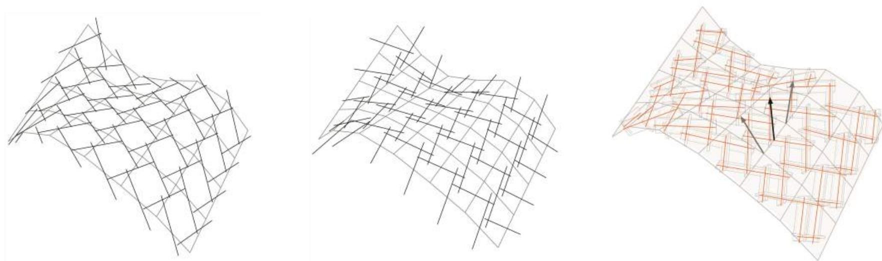

Figure 7: RF options by adjusting the angle (left and center) and panel orientation according to normal vectors (right)

3.2. Assembly

The complexity of fabricating elements of a reciprocal frame structure which feature different length and notching qualities can be simplified by tracking each member and understanding their relationship with every other element with which they intersect. In this workflow, mesh topology data has been organized and scripts allow for inspection of the larger aggregations (green) and the smaller ones (orange) within a RF assembly (Figure 8).

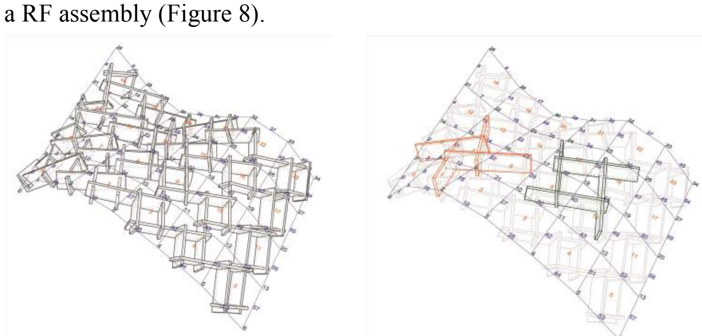

Figure 8: RF topology data

An algorithm written in C# has been defined to tag the members and facilitate assembly. For each unit forming a circular arrangement, the script shows the elements that can be assembled unchanged and highlights the last member that must be adapted to avoid collisions. It is understood that the assembly of the last member occurs by rotating it. By running an interference check, overlapping and conflicting areas are highlighted. The digital workflow allows for finding the angle at which the rotated member would collide with the others and uses it to make an angled notch in the member subject of study (Figure 9).

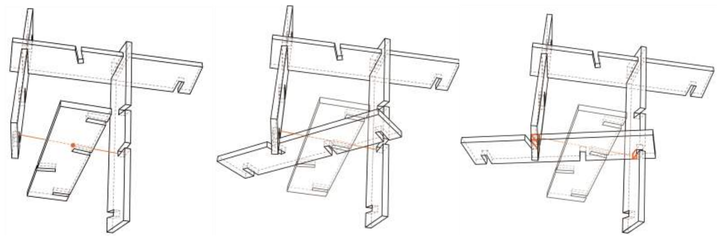

Figure 9: Rotation of member to check collisions

3.3. Structural Optimization

Having organized all the data into manageable items, each member can now be optimized to meet structural requirements. Properties of the section of the members, reactions, forces and moments, member stresses and deformations and overall deflection can be calculated. The RF member model for structural studies, using finite-element analysis, consists of as many segments as there are connections occurring along its length. The eccentricity between bars needs to be modeled as well to ensure connectivity and transmission of loads. This has been done supposing the eccentricity lines are steel bars. The section gets reduced too (in orange) to run calculations on the safe side (Figure 10).

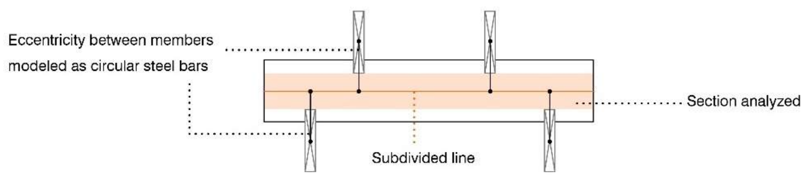

Figure 10: Structural model of planar reciprocal frame member

For example, using a free-form shape with six supports – two pins and four rollers - the Grasshopper plug-in Millipede allows to retrieve the results for each element and map them to visualize bending moments and deflection (Figure 11). The numeric results describing internal forces, stress, and displacements generated by the loadings will define the final section to be used. The advantage of using a digital parametric workflow is the possibility of going back in the process to review and update the design with the most appropriate solution. The previous assembly studies can be run again over the new sections and the geometric adaptions will be automatically updated.

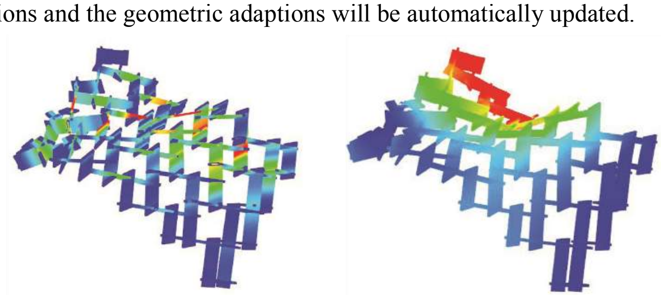

Figure 11: Bending moment and deflection visualization

3.4. Mass customization

Parametric design capabilities are implemented in this workflow by establishing 'parent-child' relationships. The model is hierarchically structured so that the 'parent' model is composed of rectangles representing the members and geometric features and other components are added later as 'children', which inherit dimensions and geometric relationships from the parent. If the parent is modified - the depth or length values are increased or decreased - the children follow these changes. For example, the rectangular shape can be replaced by an oval-like one, where the radius becomes a parameter (Figure 12).

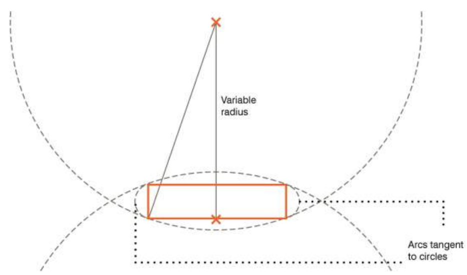

Figure 12: Oval shape from rectangle

Given a free-form surface and the corresponding members making a reciprocal frame, the curvature can be calculated by projecting the lines into the surface and getting the value for the center point. Making the radius of each adaptive component that of the curvature and considering the convexity or concavity of the curve, each member is different from one another (Figure 13)

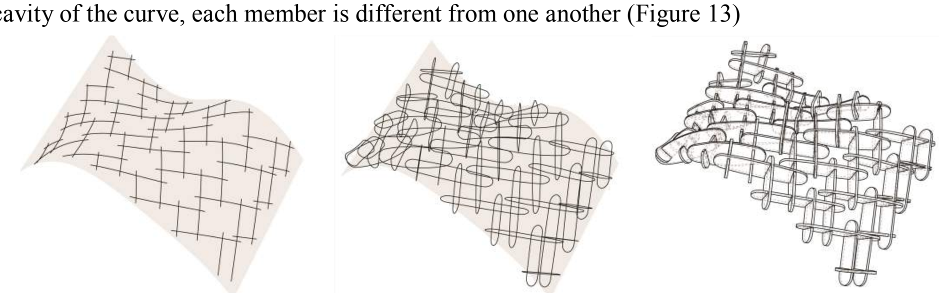

Figure 13. RF projection of lines on surface, adaptive components as curves and as solids

In the next phase in the digital workflow the structural elements get arranged for fabrication and they are tagged with their identification number and those of the pieces that intersect with them (Figure 14 and Figure 15).

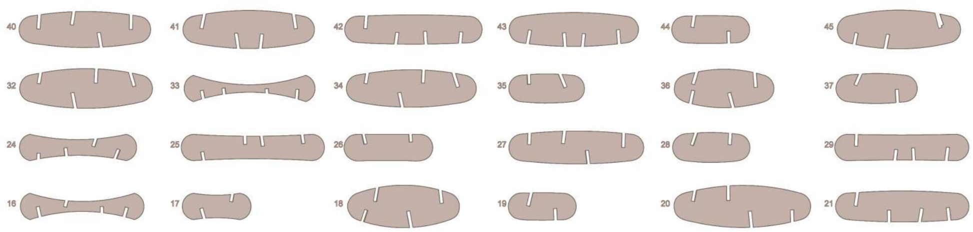

Figure 14. Mass customization of RF members

However, not every piece created is eligible for fabrication and some changes must be made. The digital workflow incorporates a script to measure the total depth of the member, the depth of the element at the center point and the length of the lines where the members intersect. A minimum value can be set and a coefficient is applied if the depth results are below it. This will ensure that the elements are strong enough to support the loading conditions (Figure 15).

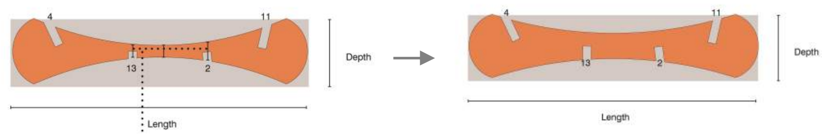

Figure 15. Minimum depth check and fix on a concave member

The overall structural analysis can be run once again using these new shapes. Additionally, each member can be now analyzed in relation to the other elements that are intersecting with it. Here, structural optimization explorations are based on multiple load cases that consider each member to be a finely-divided mesh. Millipede returns the stiffness factor, the optimization-calculated factor that determines the relative strength of each face of the mesh within the piece. This is visualized as a system with less material where the areas in grey could be carved out. The results are illustrated below for one of the pieces in the system (Figure 16, Figure 17 and Figure 18).

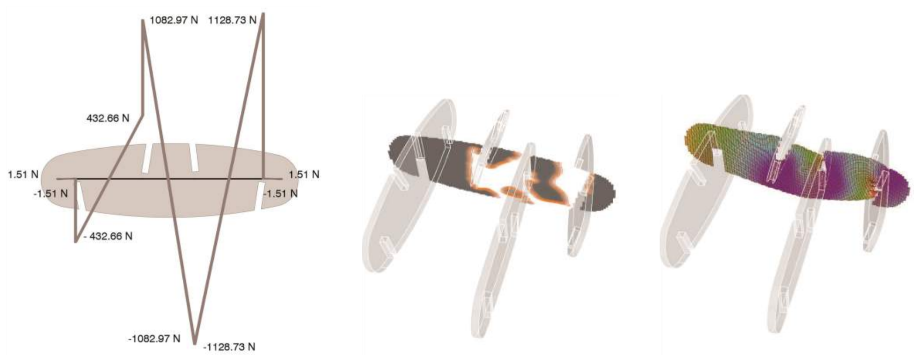

Figure 16. Tension forces extracted from the structural analysis, and stiffness and deflection visualizations

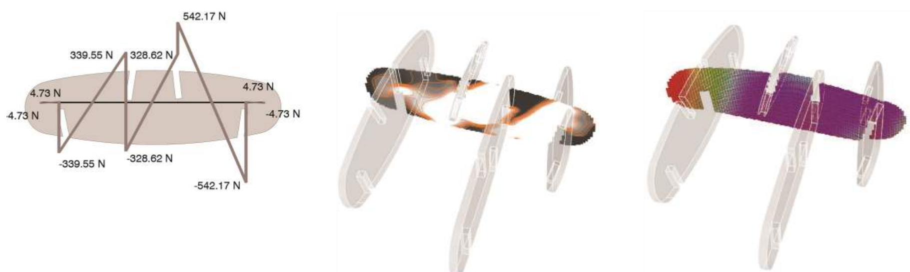

Figure 17. Torsion forces extracted from the structural analysis, and stiffness and deflection visualizations

Figure 18. Axial forces extracted from the structural analysis, and stiffness and deflection visualizations

4. Built prototypes

Informed by the structural analysis performed, the overall RF system is translated into a set of instructions appropriate for manufacturing the members with a 6-axis robot, which deals with the three-dimensional nature of the notches. In order to serve as case study for this research, proof-of-concept free-form frameworks have been constructed consisting of a limited number of elements.

4.1. Rectangular boards

The first prototype has been built using the parent geometries – the rectangular boards - generated in the digital workflow. It has been fabricated with a robot cutting across structural panels with a depth dimension of 22 cm (Figure 19).

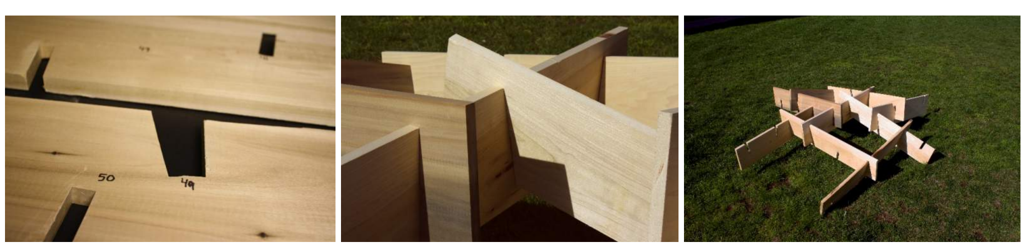

Figure 19. Framework built with rectangular boards

4.2. Oval-like boards

Each member in this framework represents the topology of the original surface at that location by using the value of the curvature as its generative radius. To fabricate these adaptive components, the robot has milled perpendicular to the main surface of the board. A detail was introduced in the notch to ensure that the parts meet at a level seat (Figure 20)

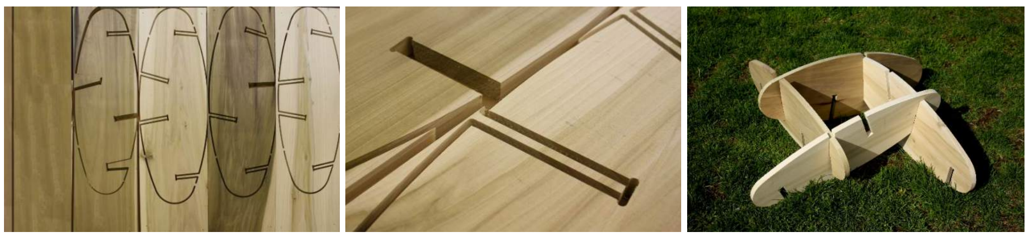

Figure 20. Framework built with oval-like boards

4.3. Mass-customized boards

A study of all the load cases' results obtained in the Mass Customization section leads the way to design strategies for material removal. Where material is not needed, the structure can be made lighter. Two prototypes have been built by interpreting the orange curves reading the stiffness values. The first implements a fairly literal approach by removing the lower part of the member, while leaving the ends untouched to keep the overall shape of the part. For the second one, an algorithm was written to allocate circular holes of different diameters depending on the stiffness values (

Figure 21).

Figure 21. All load cases stiffness visualization and mass-customized boards

5. Conclusions and discussion

Simple to prefabricate in the past as a repetitive system, this highly-engineered version of a reciprocal frame structure enhances wood craftsmanship by delegating the tedious task of differentiated detailing of hundreds of pieces to a robot. Mass-customized parametric wood construction is possible thanks to a flexible digital workflow that takes into consideration structural requirements, fabrication constraints and assembly methods. If a large planar RF structure were to be designed (Figure 22), the information embedded in this workflow would provide accuracy and short throughput time to get it built, emphasizing the importance of having a highly-defined model for digital fabrication.

Figure 22: View of a reciprocal frame pavilion built with planar members

5.1. Limitations

The main limitation of this workflow is that it is computationally expensive. The more subdivided the meshed version of the input surface, the longer the time the algorithm takes to compute the solution. Additionally, the current state of this digital workflow can only ensure fine results using a quad mesh. Small variations of the algorithm would have to be performed to make it work for triangular and hexagonal meshes.

5.2. Further studies

The current tool is not able to deal with timber-specific material parameters such as the fiber direction or the humidity content. These features could be explored in future developments.

In terms of using adaptive components on planar RF structures, other geometries could be explored along with defining rules and settings to determine the range of feasible parametric design solutions.

Regarding adapted pieces for assembly, they should be analyzed in detail, since their modification to meet assembly requirements might impact the structural behavior of the adjacent pieces negatively (Figure 23). The analysis of these members might return high values for the torsion forces.

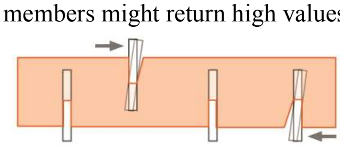

Figure 23. Structural implications of adapted members

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