Simple Pendula

We've seen how mass spring systems exhibit simple harmonic motion. Now we're going to use these systems to better understand not only the concept of simple harmonic motion, but also other objects that exhibit this behavior. Perhaps the most important of these is the simple pendulum. Let's begin by seeing how a mass spring system behaves similarly to a simple pendulum.

In this diagram, we see that both the mass spring system and the pendulum both have an equilibrium position. That is a position in which there is no restorative force acting on the mass. But with either of these systems, if we were to displace or deflect the mass in one direction, then we do get that restorative force.

So with either of these, the instant we were to let go of the mass in this position, the velocity is going to be 0 because we've just let go of it. The displacement at that exact moment is going to be at a maximum in the positive direction, meaning it will never be more displaced than it is at this moment. And the restorative force, and thus the acceleration vector acting on either of these masses, is also going to be at a maximum, but in the opposite direction as a displacement.

As either of these objects continues their motion, they'll return back to their equilibrium position. At this position, velocity is at a maximum at this point in the negative direction, while displacement, force, and acceleration are all 0. As either of these masses continue their motion, they move in the negative direction. And when they reach their maximum there, velocity will be 0, displacement will be at a maximum in the negative direction, and thus, force and acceleration will be at a maximum in the positive direction. The result of this is that the masses returned back to their equilibrium position, where velocity is once again at a maximum, and displacement, force, and acceleration are 0 again, with this pattern repeating over and over.

So let's talk about how we measure simple harmonic motion. There are three main attributes that we look for. The first is amplitude. Amplitude is the maximum displacement from equilibrium position. This can be measured in meters, like we sometimes do with the mass spring systems, or degrees or radians, like we sometimes do with pendula.

The next thing we measure is frequency. Frequency is the number of oscillations per unit time, usually per second. Sometimes you hear frequency given as cycles per second, oscillations per second, or just hertz. These are all measuring the exact same attribute.

The third and final attribute that we measure with simple harmonic motion is period. Period is the time it takes to complete an entire cycle of motion. Period and frequency are related by the equation period equals 1 over frequency. Or another way of writing this is frequency equals 1 over period.

And this is one way that we can calculate the period of a pendulum is to simply take 1 and divide it by the frequency. But that's only useful if we're given the frequency, which we usually aren't. If we're just given the physical attributes of the pendulum, then we have to use this equation-- period equals 2 pi times the square root of L over g. In this equation, L is the length of the pendulum, and g is the acceleration of gravity. Let's go over to the whiteboard and look at a couple of examples of this.

The first example reads, what is the period of a pendulum that is 1.44 meters long and has a 3.40 kilogram bob at the end. The equation for the period of a pendulum is period T is equal to 2 pi times the square root of length over gravity. We can see by looking at this equation that the period of a pendulum only depends on length, not the mass of the bob.

So let's go ahead and enter the values that we know and solve for the period of this pendulum. Period is going to be equal to 2 pi times the square root of the length of this pendulum, which is 1.44 meters, divided by the acceleration of gravity, which we're going to write as a positive number, 9.81m/s. If we simplify what's inside that radical, we get that the period is equal to 2 pi times the square root of 0.147 seconds squared.

If you enter that square root into your calculator, you'll find that the period is equal to 2 pi times 0.383 seconds. And multiplying that all out gives us that the period for this pendulum is 2.41 seconds. That's how long it's going to take this pendulum to complete an entire cycle.

Let's look at one more example. This example reads, Lawrence is designing a time-keeping device. He wants a pendulum that oscillates once every 1.00 seconds. How long should the pendulum be?

To solve this problem, we're going to use the same equation as before. T equals 2 pi times the square root of length over gravity. Entering in the values that we know, we get 1.00 seconds is equal to 2 pi times the square root of length, which we're solving for, divided by gravity, which is 9.81m/s.

We can divide both sides of this equation by 2 pi. And that gives us 0.159 seconds is equal to the square root of length over 9.81m/s. We can get rid of that square root by squaring both sides of the equation. That gives us 0.0253 seconds squared is equal to length divided by 9.81m/s. If we multiply both sides of this equation by 9.81m/s, we'll find that the length of this pendulum has to be 0.248 meters in order to get a period of 1.00 seconds per oscillation.