Simple Harmonic Motion

We've now studied both linear and parabolic motion. Now, we're ready to look at another type of motion, called simple harmonic motion. One of the great things about studying this type of motion is that you can often use your intuition to help determine what is occurring in these situations. So to help build that intuition, let's begin by looking at what exactly simple harmonic motion is.

Simple harmonic motion is the periodic motion that results from a restoring force. Two important terms in that definition are, first, periodic, and that means this motion repeats over and over. And restoring force, that means there's a force that's trying to bring the object back to an original position. The resulting motion of this is an oscillation, that is to say, a back and forth movement.

So some examples of this would be a playground swing, or a wrecking ball, or a hypnotist's watch. These are all examples of pendula. There's also another type of object that engages in simple harmonic motion called a mass spring system. A mass spring system is exactly what it sounds like. It's a mass connected to a spring. And every mass spring system has an equilibrium position. At this position, the spring isn't pulling the mass in the negative direction or pushing it in the positive direction. It's exerting no force whatsoever on the mass.

But if we were to pull that mass in the positive direction, stretching the spring out, at this point, we've displaced the mass distance x. And as a result of this, the spring is pulling the mass in the negative direction with force F, the restorative force. If, by contrast to this, we were to compress the spring, like that, we now have displacement x in the negative direction. But as a result, the spring is trying to push the mass back in the positive direction with restorative force F.

If we were to further compress the spring, like this, we now have both a greater displacement as well as a corresponding greater force. And this is a result of Hooke's Law. Hooke's Law says F equals negative kx. In this equation, F is the restorative force measured in newtons. That's the force trying to bring the mass back to its equilibrium position.

k is the stiffness coefficient of the spring. Springs have different stiffnesses. The springs and the shocks of a car will be extremely stiff, whereas the spring inside of a watch will be not stiff at all. And we measure the stiffness of a spring in newtons per meter. And lastly in this equation, x represents the displacement measured in meters.

It's important to note the minus sign in Hooke's Law. This ensures that the force vector and the displacement vector are always in opposite directions. If the displacement is in the negative direction, the force is in the positive direction, and vice versa.

Now, let's look and see how Hooke's Law applies to our mass-spring system. If we begin by pulling the mass in the positive direction, then the instant we let go of that mass, the displacement is at a maximum in the positive direction. Force is also at a maximum, but because force and displacement are always in opposite directions, it's at a maximum in the negative direction.

Newton's second law tells us that F equals ma. So since force is at a maximum in the negative direction, acceleration is also at a maximum in the negative direction. And the instant we let go of this mass, the

velocity is 0. So even there's a strong force and a strong acceleration and a big displacement, at the instant we let go, the mass isn't moving.

But a moment later, it gets back to its equilibrium position. At this moment, displacement is 0. And from Hooke's Law, we know, if displacement is 0, then force must also be 0. And from Newton's second law, if force is 0, then acceleration is 0.

But at the moment that the mass is passing through its equilibrium position, velocity is at a maximum. In this case, it's moving in the negative direction, and it will continue that motion until the spring is compressed. When the spring is fully compressed, displacement is at a maximum again, but this time in the negative direction.

Force is at a maximum, and it's in the opposite direction as displacement. So the spring is pushing the mass in the positive direction. Acceleration is also at a maximum in the positive direction, and velocity at this exact moment is 0.

Again, the spring returns to its equilibrium position, except this time, while displacement, force, and acceleration are all 0, velocity is at a maximum in the positive direction. And in the absence of friction, this process will continue indefinitely, with the mass oscillating between maxima and minima values.