Mass Spring Systems

Just as we can calculate the period of a pendulum based on its physical characteristics, we can also calculate the period of motion for a mass-spring system based on similar characteristics. Let's look at how we accomplish that. Recall that in simple harmonic motion, we're primarily interested in measuring three values.

In the case of a mass-spring system, amplitude is the measure of the maximum displacement. Period is the measure of how long it takes for the mass to complete an entire oscillation, for example, going from maximum positive displacement to negative and back to positive. That is a complete oscillation or cycle. And frequency is the measure of how many of those cycles occur per second.

The relationship between period and frequency holds for all periodic motion. But if we want to calculate the period for a given mass spring system, then we will again have to use a different equation. The equation for the period of a mass-spring system is T equals 2 pi times the square root of m over k.

While this equation looks similar to the pendulum equation, the difference lies inside the square root. In this equation, the period of motion depends on both the mass of the object as well as the stiffness of the spring. Let's go over to the whiteboard and look at a couple examples of this in order to better understand it.

The first example reads, Clara is building a mass spring system. She only has access to one spring, and it has a stiffness coefficient of 65.0 newtons per meter. If Clara wants the system to have a period of motion of 1.45 seconds, how large a mass should she attach to the spring? We're going to use the period equation for a mass-spring system to solve this. And that equation is period T is equal to 2 pi times the square root of the mass divided by the stiffness of the spring.

So let's plug in the values we know and solve. The period of motion we want to be 1.45 seconds. That's equal to 2 pi times the square root of mass M, which we're solving for divided, by the stiffness of the spring, which is 65.0 newtons per meter. Dividing both sides of this equation by 2 pi gives us 0.231 seconds is equal to the square root of mass M divided by 65.0 newtons per meter.

To get rid of this radical sign, we can square both sides of the equation like that. And that gives us 0.0533 seconds squared is equal to mass M divided by 65.0 newtons per meter. If we multiply both sides of this equation by 65.0 newtons per meter, we'll find that in order to get that period of oscillation, we'll need a mass of 3.46 kilograms attached to that spring. Let's look at one more problem.

This problem reads, what is the frequency of a mass-spring system consisting of a spring with a stiffness of 12 newtons per meter and a 1.05 kilogram mass? Now, this problem is asking for frequency, but we're going to begin by using that same equation that we used last time that T, the period, is equal to 2 pi times the square root of mass divided by stiffness. So let's use this equation to solve for period.

Period is equal to 2 pi times the square root of mass, which is 1.05 kilograms, divided by stiffness, which is 12 newtons per meter. If we simplify what's inside that radical, we get that the period is equal to 2 pi times the square root of 0.0875 seconds squared. If you enter that into a calculator, you'll find that the period of motion for this mass spring system is 1.86 seconds.

But the question didn't ask for period. The question asked for frequency. The relationship between period and frequency is that frequency is equal to 1 over period. So in this situation, frequency is going to be equal to 1 divided by 1.86 seconds. And that gives us that the frequency of motion for this mass-spring system is 0.538 cycles per second, which we just call hertz.