

Constructing a Linear Function from a Graph

You know how to construct a linear function from a table of values, but constructing a linear function from a graph follows almost the exact same process, but you will have to determine the value pairs yourself from looking at the graph. To construct a linear function from a graph, begin by determining the rate of change, or slope, m . Next, determine the initial value, or y -intercept, b . Lastly, write the function in the form y equals mx plus b . Let's see this process play out in an example.

Here we have a graph of a line with a negative slope, but that's all we know about it. In order to determine the equation for this line, we will need to do some calculations. We begin by finding the slope, or rate of change, m . We can do that using the slope formula, m equals y_2 minus y_1 over x_2 minus x_1 . To use this equation, we need to identify two points that lie on this line. We'll pick this point here as our first point. It has coordinates of negative 4, 3. And we'll pick this point here as our second point. It has coordinates of 6, negative 2. Now we substitute and solve. We substitute in the x and y values from our first point as x_1 and y_1 , and then the x and y values from our second point as x_2 and y_2 . If we perform this subtraction in the numerator and denominator, then this simplifies to negative 5 over 10, which is equal to negative one half.

Now that we know the slope, m , we can use this to solve for the initial value, or y -intercept, b . To do this, we will substitute known values into the slope-intercept form equation y equals mx plus b . We know that the slope of this line, m , is negative one half, so we can begin by substituting that value in. Now, we just need to pick a point on the line, and substitute in the x - and y -coordinate for x and y in this equation. Let's use the first point we picked, negative 4, 3. We substitute 3 in for y in the equation, and negative 4 in for x . Now we just need to solve for b . Negative one half times negative 4 is 2. If we subtract 2 from both sides, we get 1 equals b , which is the same as b equals 1. This is our initial value, or y -intercept, for this function.

Now in this case, we could have just looked at the graph, and seen that this line crosses the y -axis at a value of 1, but we will not always be able to see the y -axis with a problem like this, so it's usually best practice to solve for that value, and use the graph to verify our answer when possible. So, after all that, our final linear function that represents this line is y equals negative one half x plus 1.

Let's go over to the whiteboard to work on a few more examples.

This first question asks, "The graph below represents the distance a motorcycle travels as a function of time. Construct the linear function that is shown on the graph." Well, let's begin by calculating the rate of change, or the slope, m . And we can use the slope formula, which is y_2 minus y_1 over x_2 minus x_1 . And in order to use this formula, we have to be able to identify two points that this line passes through. So let's do that. First, I guess we can use this point here, which is 2, 5, and then this point here, which is 4, 8. So our y_2 is going to be 8, minus our y_1 , which is 5, divided by our x_2 which is 4 minus our x_1 , which is 2. 8 minus 5 is 3, and 4 minus 2 is 2. And we can't reduce this fraction anymore, so our rate of change, m , is equal to three halves or 1.5. Now we can use this information to help us find the initial value. And we can do that by substituting into the slope intercept form, which is y equals mx , and we have a value for m , it's 3 over 2. So 3 over 2 x plus b . Now we need to substitute in an x - y pair of values that is on this line into this equation and we can solve for b . So let's use this point here, 4, 8. The y -value is 8, so 8 equals 3 halves times the x -value, which is 4, plus b . So that simplifies to 8 equals, 3 halves times 4 is 6,

plus b . And if we subtract 6 from both sides of this equation then we get 2 equals b , or b equals 2 . Now that we know the slope and the initial value, we have enough information to put together our slope intercept equation. That it's going to be y equals 3 halves x plus our initial value of 2 . Alright, let's look at another one.

This one reads, "Construct the linear function that is represented in the graph." Just like the other one, we're going to start by calculating the rate of change, and for that, we need to identify two points that are on this line. So I see one point right here at negative 2 , 0 , and then one point right here at 2 , negative 5 . So let's write those points down: negative 2 , 0 and 2 , negative 5 . Now we're going to substitute these values into the slope formula, which is m equals y_2 minus y_1 over x_2 minus x_1 . Our y_2 value is negative 5 minus our y_1 value is 0 . Our x_2 value is 2 minus our x_1 value is negative 2 . Negative 5 minus 0 is negative 5 , and 2 minus negative 2 is the same as 2 plus positive 2 , which is 4 . So our rate of change, m , is equal to negative 5 fourths or negative 1.25 . Now we can substitute this rate of change value plus one of these ordered pairs into the slope intercept form to solve for b . So that's y equals negative $1.25x$ plus b . And let's substitute in this first ordered pair, negative 2 , 0 . So the y -value is 0 equals negative 1.25 times the x -value, which is negative 2 , plus B . That becomes 0 equals, negative 1.25 times negative two is 2.5 , plus b . And if we subtract 2.5 from both sides of the equation, then we get negative 2.5 equals b , or b equals negative 2.5 . And looking at our graph, that looks to be correct. So we now have the rate of change, the slope, and the initial value, the y -intercept, which is enough information to construct this equation, and that's going to be y equals negative $1.25x$ minus 2.5 . Alright, let's look at one last example.

This one reads, "The graph below represents the relationship between the average rainfall that occurs in a jungle during the winter over a given number of days. Construct the linear function that is shown on the graph." Well, again, we're going to start by finding our rate of change, m , which is going to be y_2 minus y_1 over x_2 minus x_1 . So in order to apply this formula, we need to identify two points on this graph. So I see one point right here at 5 , 210 , and then another one right here at 10 , 170 . So let's write those ordered pairs down. That was 5 , 210 , and 10 , 170 . Now let's substitute these x - and y -values into the slope formula. Our y_2 value is 170 , minus our y_1 value is 210 , over our x_2 is 10 minus our x_1 value which is 5 . 170 minus 210 is negative 40 , and 10 minus 5 is 5 . Negative 40 divided 5 is negative 8 , so our rate of change, m , is equal to negative 8 . Now we can substitute this into the slope intercept form to help us solve for the initial value, so that's going to be y equals negative $8x$ plus b . And let's pick an ordered pair that we can substitute in for x and y . So let's use 5 , 210 . That's going to be 210 equals negative 8 times 5 plus b . Negative 8 times 5 is negative 40 , and if we add 40 to both sides of the equation, then we get 250 equals b , or b equals 250 . So now we know the rate of change, the slope, and the initial value, the y -intercept, which is enough for us to construct this function. That's going to be y equals negative $8x$ plus 250 .