Finding Interior Angles with Algebraic Expressions

This first question reads, “Solve for x and then determine the angle measurements for angle A, angle B, and angle C.” Well we're going to begin solving for x by applying the triangle sum theorem. So we're going to sum these three algebraic expressions and set it equal to 180. So let's do that. That's going to be x plus 10 plus 120 minus x plus 2x plus 26 equals 180. Now let's combine like terms, starting with our variable terms. X minus x plus 2x is 2x, and 10 plus 120 plus 26 is 156, and that's equal to 180. Then we can subtract 156 from both sides of the equation, and that gives us 2x equals 24. Dividing both sides of the equation by 2 reveals that x equals 12. So that's the first thing we wanted to find, was the value for x, and that's 12. Now let's solve for the angle measures, starting with angle A. That's equal to x plus 10, and x is 12, so that's 12 plus 10. So the measure of angle A is equal to 22 degrees. Now let's do angle B. That's equal to 120 minus x, or 120 minus 12. So the measure of angle B is equal to 108 degrees. Lastly let's do angle C, which is 2x plus 26, or 2 times 12 plus 26. 2 times 12 is 24, and 24 plus 26 is 50, so the measure of angle C is 50 degrees. We can check our work by summing these three values together: 22 plus 108 plus 50. 22 plus 108 is 130, plus 50 is indeed equal to 180, which that checks out. Let's look at another example.

This question reads, “In a right triangle, 2 interior angle measurements are 33 minus x degrees and 5x minus 3 degrees as shown below. Solve for x. What are the interior angle measurements of this triangle?” Well we're going to solve for x again by applying the triangle sum theorem, and that tells us that 33 minus x plus 5x minus 3 plus, and because this is a right triangle we know that this third angle has a measure of 90 degrees, and those sum to 180. Alright let's combine like terms starting with our variable terms. Negative x plus 5x is 4x, and 33 minus 3 plus 90 is 120, and that's equal to 180. We can subtract 120 from both sides of the equation and that gives us 4x equals 60, and then dividing both sides of the equation by 4 reveals that x equals 15. Alright that's the first thing we're trying to find, now let's find the angle measures in this triangle. And we can do that by substituting this value for x back into these algebraic expressions. Let's start with the expression 33 minus x. That becomes 33 minus 15, which is equal to 18, so we’ll write that in our triangle here, 18 degrees. The next one is 5x minus 3, so that’s 5 times 15 minus 3. 5 times 15 is 75, minus 3 is equal to 72. So this angle has a measure of 72 degrees. So the measures of the three angles of this triangle are 18 degrees, 72 degrees, and 90 degrees. Alright let's look at one last example.

This one reads, “Solve for x. What are the interior angle measurements for this triangle?” Now here we only have two algebraic expressions, so it might seem like we can't solve this one, but also notice that

this is an isosceles triangle, and the base angles on an isosceles triangle are going to be congruent, that is they're going to have the same measure, so we can also write over here 2x plus 12, because it's going to be congruent to this angle over here. Now, just like the other ones, we're going to apply the triangle sum theorem, and that's going to be 2x plus 12 plus 28x minus 84 plus 2x plus 12 equals 180. Now let's combine like terms, starting with our variable terms. 2x plus 28x plus 2x is 32x, and 12 minus 84 plus 12 is minus 60, and that's equal to 180. We can add 60 to both sides of the equation, and that gives us 32x equals 240. If we divide both sides of this equation by 32, then we get x equals 7.5. So that's the first thing we were looking for. Now let's find what these angle measures actually are. We'll start by finding the measures of the base angles, which are represented with the expression 2x plus 12. Now that we know the value for x, you can write that as 2 times 7.5 plus 12. 2 times 7.5 is 15 plus 12 is 27, so these have a measure of 27 degrees. Now let's look at this top angle, which is represented with the expression 28x minus 84, so that's 28 times 7.5 minus 84. 28 times 7.5 is 210, and 210 minus 84 is 126, so the angle measures on this triangle are 27 degrees, 27 degrees, and 126 degrees. We can check our work by summing those three values. 27 plus 27 plus 126. 27 plus 27 is 54, and 54 plus 126 is indeed 180, so we did that correctly.