Solving Inequalities with Variables on Both Sides

Solving inequalities with variables on both sides is very similar to solving equations with variables on both sides, but instead of applying the properties of equality, we apply the properties of inequality, and we can check our work by checking the shading when we graph the inequality. Let's quick look at what the steps are to solve an inequality like this.

First, we use the distributive property, if needed. Then we collect like terms on each side of the inequality, if needed. Next, we use the properties of inequality to collect all the variable terms on one side of the inequality, and all the constant terms on the other side. We then use inverse operations and the properties of inequality to finish solving. We finish by graphing our answer on a number line and checking the shading. Let's look at how this works in an example.

The question asks us to solve 1 plus 2 times x plus 5 is greater than 6 plus 3x and graph the solution on a number line. To begin, we will distribute this two over the two terms inside the parentheses. Doing that gives us the inequality 1 plus 2x plus 10 is greater than 6 plus 3x. We want to combine like terms, but our only like terms are the 1 and 10 on the left hand side. Combining those gives us the inequality 2x plus 11 is greater than 6 plus 3x.

Now we want to move all variable terms to one side, and all constant terms to the other. So let's move our variables to the right hand side by subtracting 2x from both sides, an application of the subtraction property of inequality. Doing that gives us the inequality 11 is greater than 6 plus x. Now we want to move all the constant terms to the left-hand side. We accomplish that by subtracting 6 from both sides, again applying the subtraction property of inequality. That gives us the inequality 5 is greater than x, which can also be written as x is less than 5.

This is our solution, so now let's graph it on a number line. We begin by plotting the value we found, which is 5. Since this is a strict inequality, we are going to plot an open circle there, which means that 5 itself is not a solution to this inequality. Then to shade this, since the variable is on the left side of the inequality, we'll shade in the direction that the inequality points, so this side of our open circle.

We can check that our shading is correct by picking a shaded point, and substituting it into our original inequality. An easy point for us to pick is 0. If we substitute 0 into our original inequality, we get this, which simplifies to 11 is greater than 6. This is a true statement, so we have shaded the correct side on the number line.

Now let's head over to the whiteboard to work on a couple more examples.

This first question reads, "Solve negative 8x minus the quantity x plus 1 is greater than or equal to negative 7x plus 3, and graph the solution on a number line." So to begin, let's distribute this minus sign across both of the terms inside these parentheses. Doing that gives us negative 8x minus x minus 1 is greater than or equal to negative 7x plus 3. The only like terms we have on the same side of the inequality are these two terms here, so combining those gives us negative 9x minus 1 is greater than or equal to negative 7x plus 3. Let's move all of our variable terms to the right-hand side of the inequality by adding 9x to both sides. Doing that gives us negative 1 is greater than or equal to 2x plus 3. We begin isolating x by subtracting 3 from both sides of the inequality, giving us negative 4 is greater than or equal to 2x. Then we can get x by itself by dividing both sides of the inequality by 2 and that gives us negative

2 is greater than or equal to x. So there's our solution there. Now let's plot this solution. We begin by plotting the point negative 2, and because this is a non-strict inequality, we're going to use a closed circle at that point indicating that negative 2 is part of our solution set. Now we need to determine which side of this value we shade. And because x, our variable is on the right-hand side of this inequality, we're going to shade in the direction opposite to where the inequality points. The inequality points to the right, but we're going to shade to the left like this, all values less than negative 2.

We can check our work by substituting in one of these shaded values in for x in our original inequality, so let's use negative 4. Substituting negative 4 for x into this inequality gives us the expression negative 8 times negative 4 minus negative 4 plus 1 is greater than or equal to negative 7 times negative 4 plus 3. Alright, let's do some simplification here. Negative 8 times negative 4 is 32 minus negative 4 plus 1 is negative 3, that's greater than or equal to negative 7 times negative 4 is 28, plus 3. And this subtracting a negative becomes a positive, so this becomes 32 plus 3 is greater than or equal to 28 plus 3. 32 plus 3 is 35 is greater than or equal to 28 plus 3 is 31. 35 is greater than or equal to 31, that's a true statement, so we shaded the correct side of this value. Alright, let's look at another example.

This one reads, “Solve negative 8x is greater than negative 12 times the quantity one half x plus 2 plus 8 and graph the solution on a number line.” So again, let's begin by distributing this coefficient negative 12 across both of these terms inside the parentheses. Doing that gives us negative 8x is greater than negative 12 times one half x is negative 6x and negative 12 times 2 is minus 24 plus 8. Alright, let's combine like terms. The only like terms we have are the 2 constant terms on the right-hand side of the inequality. So that gives us negative 8x is greater than negative 6x and negative 24 plus 8 is negative 16. Now let's move all the variable terms to the left-hand side of the inequality. We do that by adding 6x to both sides. Negative 8x plus 6x is negative 2x. That's greater than negative 6x minus 16 plus 6x is minus 16. Now we can finally isolate x by dividing both sides of the inequality by negative 2. Negative 2x divided by negative 2 is x. Because we're dividing by a negative number, we have to flip the direction of our inequality, so now it's less than. And negative 16 divided by negative 2 is 8, so x is less than 8. That is our solution to this inequality. Now let's shade this on the number line. We start by going to the value we solved for, and we're going to use an open circle this time, because 8 is not included in this strict inequality. So because the variable is on the left-hand side of this inequality, we're going to shade the direction the inequality points, and it points left, so we're into shade all points left of 8 on the number line like that. Now we can check our work by substituting a shaded value into our original inequality and verifying that it produces a true statement. So let's choose zero. Substituting zero in, we get negative 8 times 0 is greater than negative 12 times one half times 0 plus 2, all that plus 8. Alright, let's simplify this down. Negative 8 times 0 is just zero. That's greater than negative 12 times one half times 0 is 0 plus 2 is 2 plus 8. That is zero is greater than negative 12 times 2 is negative 24 plus 8, and that gives us zero is greater than negative 16, which is a true statement. So we shaded the correct side of 8.