Inverses of functions and relations

Hello, in this video, I'd like to share some information about function inverses. Function inverses essentially allow us to work backwards through a function in order to help solve for unknown variables. Let me explain. A function, is a mathematical device that takes an input value, applies a series of mathematical operations and maps it to a unique output value. For example, a collection of input values… would map to a collection of output values… with this function, $f(x) = \frac{1}{2}x^2 - 3$. We use the function to determine where each input maps to, and you might notice that some outputs were mapped by different inputs, which is okay. But no input maps to more than one input – this is required of all functions. If a mathematical operation maps an input to more than one output, it would be considered a relation, not a function.

The question for now, is how would we work backwards through a function? What if I know a function's output, and I'd like to find the input that mapped to it? Generally, we use the inverse of a function to work backwards. Although there are methods that use graphs to do the same.

Let's start with our function from above and find its inverse. A common method for finding an inverse is to first rewrite our function using "y equals" notation rather than function notation. This makes our work a little bit easier to follow on paper. Then we switch the x and y values… We do this since we are looking for the opposite of the original function. Since x is sometimes considered the opposite of y, this switch allows us to proceed towards the inverse, by solving for the new y value. To solve for y in this case, we must isolate it by first adding three to both sides of the equation… Then multiply by two on both sides of the equation to eliminate the one-half… Technically, this equation is the inverse of the original, although it is written in an implicit form. In other words, neither x nor y are completely isolated. However, if we take the square root of each side to solve for y… you'll notice this is not a true function since you must include both a positive and negative result of the root to satisfy the domain and range of the original equation. When the inverse is finally determined, you should record it using function inverse notation, which includes a negative one written between the f and x. Again, since the remaining operation maps some inputs to more than one output, this inverse is not considered a function. It is a relation.

If we try another, $f(x) = x^3 - 7$, we would rewrite using "y equals" notation… Switch the x and y… Then add seven to both sides of the equation… And finally apply a cube root to both sides of the equation and write using function inverse notation…$\left[f^{-1}(x) = \sqrt[3]{x+7}\right]$ Since a cube root doesn't have the same domain restrictions as a square root, we don't use a positive and negative form of the root.

To check your work, we can verify if two functions are inverses if they compose with one another back to the identity, or x. Let's check with our most recent example. The function f composed with f-inverse would require something cubed minus seven to act as our outer function… and the cube root of the quantity x plus seven will act as the interior function… If we simplify the function… we see a result of x. Now check the composition in the other direction… and it too simplifies to x. It is important to check the composition in both directions to verify the inverses.

To finish the video, I'll provide two sample functions and show how to derive their inverses. I would like you to graph each pair of functions and inverses from this video to see if you can understand how the graph of functions and inverses can be used to check your work. Good luck!