Verifying Trigonometric Identities

Hello! In this video we will practice verifying various Trigonometric Identities. To verify an identity means to prove that a statement about equivalent expressions is true. Since it is common to use identities to simplify more complicated expressions, it is important to show that the substitutions made during the process are indeed permissible. In a sense, you have already been verifying identities in this course. Each time you simplify a Trigonometric expression, you are essentially verifying an identity.

While there is no one specific technique that must be used to verify identities, mathematicians tend to follow these guidelines. First, we usually pick one side of the identity equation and try to transform it into the other side. It is usually recommended to start with the more complicated side, and try to show how it simplifies to the other side. Secondly, we will use algebra, and other verified identities, such as the Fundamental Identities, to rewrite the complicated side. Common algebra techniques include distribution, factoring, determining common denominators, et cetera. Thirdly, and not necessarily in this order, it is often helpful to rewrite trig functions in terms of the Sine and Cosine functions. In a way, this provides common ground for the transformation of expressions.

It is also important to note that not all algebraic processes are permissible. For example, squaring both sides of an equation is not recommended when verifying identities since the process is not wholly reversible. Certain values must be excluded from the domain in order to reverse squares and square roots.

Let's start with this first example. We would like to verify that the Cosine of an angle measure times the quantity of Secant of the angle measure minus Cosine of the angle measure is the same as Sine-squared of the angle measure. Perhaps we see the expression on the left side within a larger equation, and would find it helpful to rewrite as the expression on the right. In order to do so, we must verify this equation. I would recommend starting on the left, since it is more complicated. I'll rewrite Secant in terms of Cosine <break>. Then I'll distribute Cosine into the group <break>. In doing so, we have a fraction that reduces to one minus Cosine-squared. Technically, we're finished. Since one of our Fundamental Identities stated that Sine-squared plus Cosine-squared equals one, it is fair to say that one minus Cosine-squared is equal to Sine-squared. This identity has been verified.

In this next example, I would again recommend starting with the left side of the equation to see if we can make it match the right side. If we square the binomial by FOILing, we would see the following <break>. Here again, one of our Fundamental Pythagorean Identities states that Sine-squared plus Cosine-squared equals one <break>. Very quickly we see the expression on the left transform into the same as the expression on the right. And this identity has been verified.

We can try a different method for the final example. Sometimes, we do transform both expressions in the hopes of making them match along the way. If we change all expressions to their equivalent form in terms of Sine and Cosine using Fundamental identities we would see the following <break>. The numerator now simplifies to one, and when divided by the fraction of the denominator, we could multiply by the reciprocal <break>. Since, one times Cosine over Sine is the same as Cosine over Sine, we have verified the original identity.

It is important to practice simplifying trigonometric expressions in order to successfully verify identities. I would recommend using the Fundamental identities to rewrite existing Trig expressions in a variety of ways. Perhaps you'll find patterns and establish techniques that will help you in the future. Good luck!