

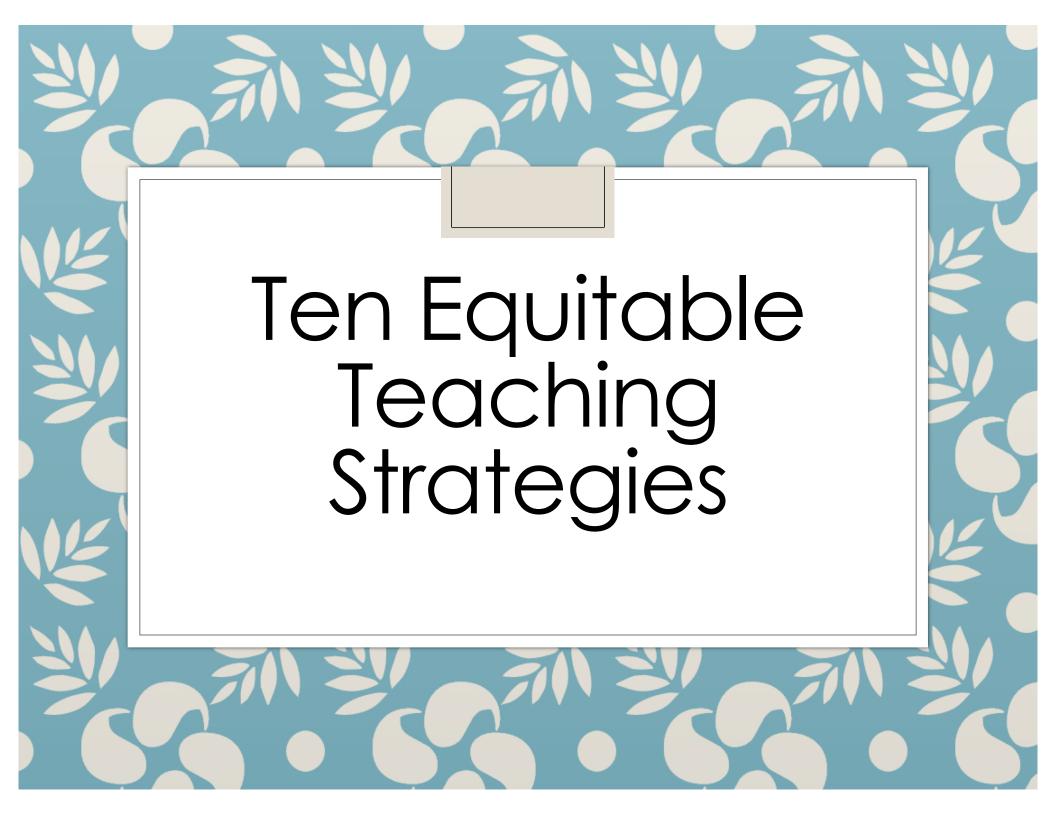
Agenda

 Examine ten equitable teaching practices.

Access and Equity

An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential.

(National Council of Teachers of Mathematics, 2014)



1. Use mathematics autobiographies and other means to get to know your students.

- Prompts or questions may include:
 - What have your math experiences been like so far?
 - My most memorable moment in mathematics is...
 - What do you think mathematics is all about?
 Why is it important to study mathematics?
 - Tell me about your favorite math teacher to date. What kinds of qualities did you like in her/him?

Sixth Grade Student's Responses to Questions about Mathematics.

MS: How would you rate yourself as a mathematics student on a scale of 1 to 10 with 10 being the highest?

Nikki: An 8.

MS: An 8, why would you say an eight?

Nikki: Because I don't make all hundreds and stuff like that on all my papers. But I would give myself either an eight or a nine.

MS: How do you feel about mathematics?

Nikki: It's my favorite subject besides spelling it's the best thing that I can do. Because in spelling and mathematics, I always make A's and stuff like that, so that's why I like mathematics.

MS: Do you know what it is about mathematics that you like?

Nikki: I like to work with numbers and write. I like to write a lot.

(Strutchens, 1993)

Sample 8th Grade Student's Response

Cordell: Math is my favorite subject because it is my easiest subject. Math is interesting and fun because in math you have to think and keep trying until you get it right. I was first drawn to math in the third grade when we started to learn how to multiply. I knew I was good because I learned to multiply earlier than the other kids in my class. I am glad that I was good at math at a young age, because that put me ahead of the other kids in my class. My third-grade teacher divided the class into groups, and I was with the group that got the harder problems. This made me feel like I was smart.

(Aguirre, Mayfield-Ingram, & Martin, 2013, p. 15)

Students' Mathematics Identities

Are how students see themselves and how they are seen by others, including teachers, parents, and peers, as doers of mathematics.

(Aguirre, Mayfield-Ingram, & Martin, 2013)

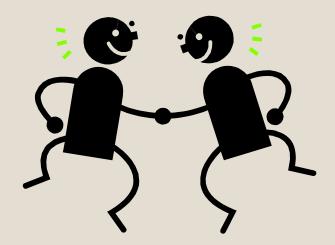
Mathematics Identity

- Mathematics identity includes:
 - Beliefs about one's self as a mathematics learner;
 - One's perceptions of how others perceive him or her as a mathematics learner,
 - Beliefs about the nature of mathematics,
 - Engagement in mathematics, and
 - Perception of self as a potential participant in mathematics.

(Solomon, 2009)

2. Use multiple entry level tasks.

Hoop Greeting



A group of 10 kids got together at the playground to play basketball. Before the game, every kid shook hands with each of the other kids exactly once. How many handshakes took place?

Tasks

- Read the problem.
- Discuss the problem with your table mates.
- Solve the problem.
- Be prepare to discuss your solution.





- In 2014, she became the first woman in 78 years to be awarded the Fields Medal, the most prestigious honor in mathematics.
- She said, "my older brother was the person who got me interested in science in general. He used to tell me what he learned in school. My first memory of mathematics is probably the time that he told me about the problem of adding numbers from 1 to 100. I think he had read in a popular science journal <a href="https://www.how.gov.numbers.number

https://www.theguardian.com/science/2014/aug/13/interview-maryam-mirzakhani-fields-medal-winner-mathematician

Maryam Mirzakhani



- What advice would you give those who would like to know more about mathematics – what it is, what its role in society has been, and so on?
 - This is a difficult question. I don't think that everyone should become a mathematician, but I do believe that many students don't give mathematics a real chance. I did poorly in math for a couple of years in middle school; I was just not interested in thinking about it. I can see that without being excited mathematics can look pointless and cold. The beauty of mathematics only shows itself to more patient followers.

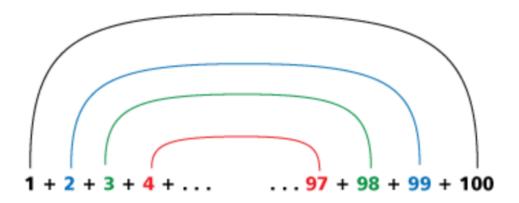
https://www.theguardian.com/science/2014/aug/13/interview-maryam-mirzakhani-fields-medal-winner-mathematician

The Story of Gauss

By Jane M. Wilburne, Posted October 10, 2014 -



I love the story of Carl Friedrich Gauss—who, as an elementary student in the late 1700s, amazed his teacher with how quickly he found the sum of the integers from 1 to 100 to be 5,050. Gauss recognized he had fifty pairs of numbers when he added the first and last number in the series, the second and second-last number in the series, and so on. For example: (1 + 100), (2 + 99), (3 + 98), . . . , and each pair has a sum of 101.



50 pairs \times 101 (the sum of each pair) = 5,050.

Multiple Entry Level Tasks

Allow for multiple entry levels where everyone can make a start and where students may approach the problem in multiple ways.

(Van de Walle, Bay Williams, Lovin, & Karp, 2013)

Types of Multiple Entry Level Tasks

- Open-Ended Tasks have many avenues of access and allow students to respond in a variety of ways. They include tasks that require students to explain answers, solve nonroutine problems, make conjectures, justify their answers, and make predictions.
- Open-Middle Tasks require one correct answer; however, students may provide different paths to the answer.

(Bush & Greer, 1999; Stenmark, 1991)

Worthwhile Tasks

"Regardless of the context, worthwhile tasks should be intriguing, with a level of challenge that invites speculation and hard work. Such tasks often can be approached in more than one way, such as using an arithmetic counting approach, drawing a geometric diagram and enumerating possibilities, or using algebraic equations, which makes the tasks accessible to students with varied prior knowledge and experience."

(NCTM, 2000, p. 19).

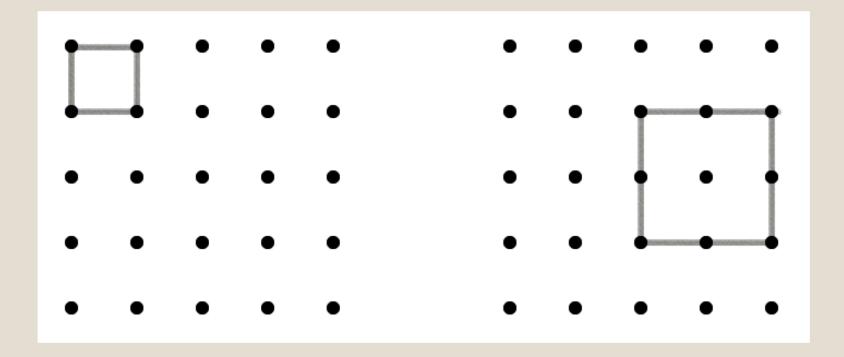
3. Implement NCTM's eight mathematics teaching practices.

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

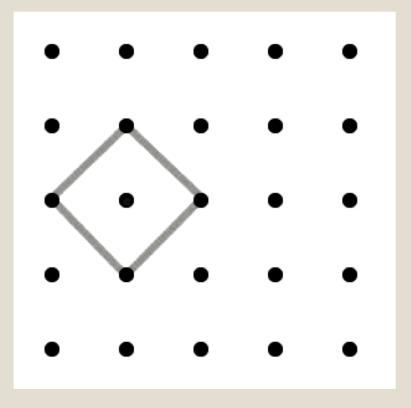
(National Council of Teachers of Mathematics, 2014, p.10)



Looking for Squares



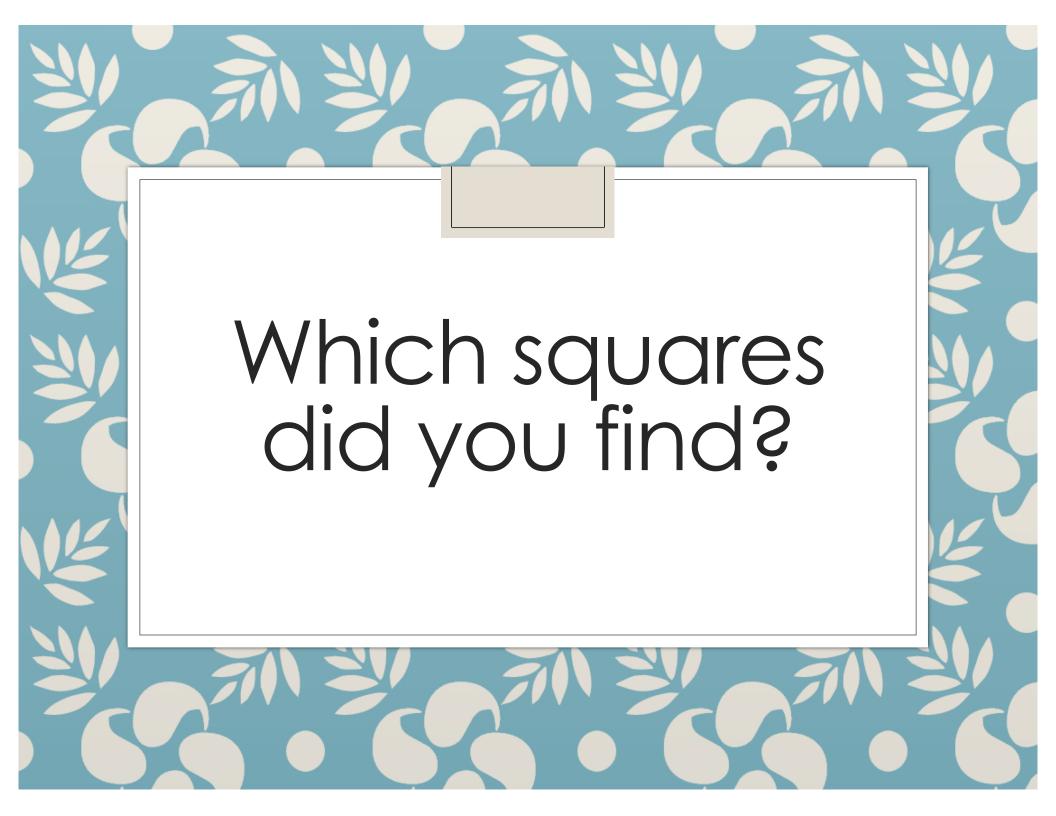
Tilted Square



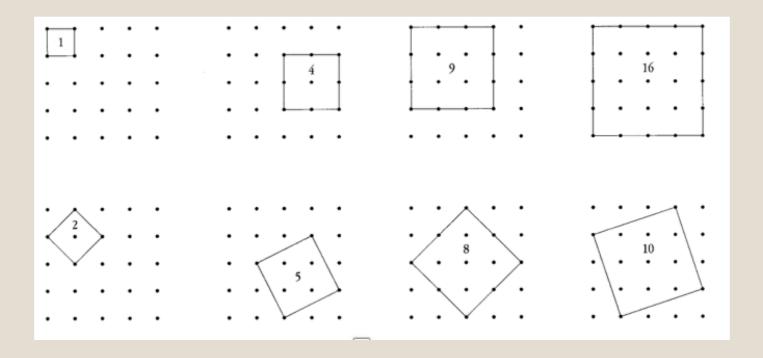
Task

- On the 5-dot-by-5-dot grids handout, draw squares of various sizes by connecting dots. Try to draw squares with as many different areas as possible. Label each square with its area.
- Be prepared to tell how you found each square and determined its area.

Connected Mathematics



Solutions



Video Excerpt

Connected Mathematics Project: Looking for Squares

Teacher: Lisa Brown

Grade: 8

School: Kealing Middle School, TX

The goal of this lesson is for students to develop an early understanding of the concept of square root as the length of a side of a square. They begin by finding the area of a square and looking at the dimensions. Students work in pairs to find as many squares as possible that will fit in a 5x5 grid. Finding areas for squares that are not vertical to the grid is perplexing to some students.

Video Discussion Questions

- Look for the teacher's use of the eight mathematics teaching practices (NCTM, 2014).
- Equity lens questions:
 - Who has access to the learning that is occurring?
 - Are all students able to participate in the learning process?
 - Who has access to the resources that support learning?

(Rousseau-Anderson, 2007)

Looking for Squares Video

Video Discussion Questions

- Look for the teacher's use of the eight mathematics teaching practices (NCTM, 2014).
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 - Who has access to the learning that is occurring?
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(Rousseau-Anderson, 2007)

4. Use formative assessment.

Formative Assessment is students and teachers using evidence of learning to adapt teaching and learning to meet immediate learning needs minute-to-minute and day-by-day.

(Thompson & Wiliam, 2008)

Formative Assessment Strategies

- Clarifying and sharing learning intentions and criteria for success.
- Engineering effective classroom discussions, questions, and learning tasks.
- Providing feedback that moves learners forward.
- Activating students as the owners of their own learning.
- Activating students as instructional resources for one another.

(Leahy et al., 2005)

The Hexagon Task



Trains 1, 2, 3, and 4 are the first 4 trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added. For the hexagon pattern:

- 1. Compute the perimeter for the first 4 trains.
- 2. Determine the perimeter for the tenth train without constructing it.
- 3. Write a description that could be used to compute the perimeter of any train in the pattern. Explain how you know it will always work. (Use the edge length of any pattern block or the length of a side of a hexagon as your unit of measure.)



Possible Solutions

- \circ Tops and bottoms plus ends: 2n + 2n + 2 or 2(2n) + 2.
- \circ Tops and bottoms of each plus ends: 4n + 2, or (2 + 2)n + 2
- \circ Insides and Outsides: 5 + 4(n-2) + 5, or 4(n-2) + 5 + 5, or 4(n-2) + 10.
- \circ Total minus shared sides: 6n 2(n 1)

The Hexagon Task Video

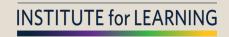
School: Austin Independent School District

Teacher: Ms. Patricia Rossman

Class: 6th grade

Many of the students in the classroom have recently arrived to the United States. The students are in a dual language program.

This task is an introduction to a unit titled Number Sense, Patterns, and Algebraic Thinking. It is the first time students have been asked to engage in a task that requires them to analyze and look for patterns in a visual diagram.



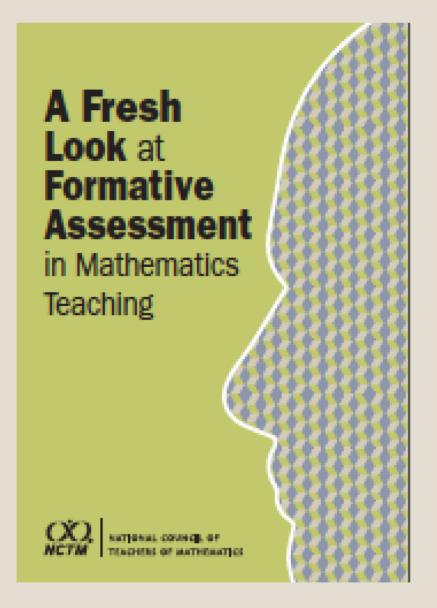


As you watch the video look for the teacher's use of the formative assessment strategies:

- Clarifying and sharing learning intentions and criteria for success.
- Engineering effective classroom discussions, questions, and learning tasks.
- Providing feedback that moves learners forward.
- Activating students as the owners of their own learning.
- Activating students as instructional resources for one another.

(Leahy et al., 2005)

Hexagon Video



Strutchens, M.E. & Silver, E.
 A. (April, 2018). Formative
 Assessment and Equitable
 Mathematics Classrooms:
 Probing the Intersection
 (Chapter)

Interconnectedness
Between Instructional
Frameworks, Tools, &
Approaches and
Formative
Assessment

Classroom Discussions & Discourse Tools

Cognitively Guided Instruction

Culturally Responsive Pedagogy

Mathematical Tasks Framework

Response to Intervention

(Silver & Mills, 2018)

5. Foster relational equity among students.



(Boaler & Staples, 2008; Boaler 2008)

Complex Instruction (CI)

- In CI, teachers use cooperative group work to teach at a high academic level in diverse classrooms.
- They assign open-ended, interdependent group tasks and organize the classroom to maximize student interaction.
- In their small groups, students serve as academic and linguistic resources for one another.
- When implementing CI, teachers pay particular attention to unequal participation of students and employ strategies to address such status problems.

(Cohen, Lotan, Scarloss, & Arellano, 1999; Boaler & Staples, 2008; Boaler 2008, Horn, 2013)

Student's Voice

"I think it helps, because it helps with learning to get out of your comfort zone, cause whenever you learn, you're not always going to learn the exact way, so to be able to learn different types of ways, if someone interprets something the way they do, and then you look at it and you're like: "oh look at this", and you see it their ways, you never know later on when you might have to change your interpretation or something. So it allows you to come out of like your comfort zone. "
(Ayana, Y4)

(Boaler, 2011)

6. Provide students with the opportunities to explore the purposes of mathematics

- 1. Expand Professional Opportunity
- 2. Understand and Critique the World
- 3. Experience Wonder, Joy, and Beauty

(NCTM, 2018)

Career	Mathematics Required	Mathematics Used	
High School Mathematics Teacher	Calculus I and II, Linear Algebra, Calculus of Several Variables, Ordinary Differential Equations, Theory of Analysis, Abstract Algebra, Statistics, Recommended: Graph Theory, Combinatorics, Number Theory	al sample problems in class, create assignments for students, and write and grade tests.	
Market Research Analyst	College Algebra, Trigonometry, Geometry, Calculus, Mathematical Economics, Statistics	Market research analysts use math every day as they perform the following tasks: analyze statistical data on past sales to predict future sales. gather data on competitors and analyze prices, sales, and methods of marketing and distribution, devise methods and procedures for collecting data, evaluate product and consumer data, and make recommendations to the firm's management or client so that decisions can be made on the promotion, distribution, design, and pricing of goods and services.	
Political Scientist	College Algebra, Geometry, Trigonometry, Calculus I and II, Statistics	Political scientists use math and statistics to predict the behavior of a group of people. They must keep track of the social, political, and monetary implications of a community's opinions and actions. Political scientists study the population using many different applications of math, including computer science, database management, statistics, and economics.	

6a. Expand Professional Opportunity

Resources: http://weusemath.org

Career	Mathematics Required	Mathematics Used
Epidemiologists	College Algebra, Trigonometry, Calculus I and II, Applied Data Analysis, Survey and Research Methods, Mathematical Statistics, Biostatistics	Epidemiologists use mathematical models in order to track the progress of most infectious diseases. They may also discover the likely outcome of an epidemic or to help manage them by vaccination. Some specific areas that epidemiologists may track are as follows: transmission, spread and control of infection; persistence of pathogens within hosts; immunoepidemiology; virulence; strain structure and interactions; evolution and spread of resistance. One specific type of mathematical model used for many infectious diseases, such as measles, mumps, and rubella, is the SIR model. This model consists of three variables: S (for susceptible), I (for infectious) and R (for recovered).
Nurses	Statistics, small number of institutions require college algebra alone or in addition to statistics	Nurses administer medications and each dosage must be customized to the patient. Math formulas are used to determine how much to administer by IV drip, injection, or other methods. Nurses use math to make sure the medication amount is appropriate and that patients do not receive too little or too much.

Resources: http://weusemath.org and http://work.chron.com/rns-use-math-jobs-15760.html

6b. Understand and Critique the World

Two Americans were detained by a Border Patrol agent after he heard them speaking Spanish



Sitting While Black? Crowds protest arrest of 2 Black men sitting in Starbucks

Racial Profiling Headlines



Definition

Driving While Black" is word play on the name of a real U.S. crime, driving while intoxicated. The phrase implies that a motorist may be pulled over by a police officer simply because he or she is black, and then questioned, searched, and/or charged with a trivial offense. This concept stems from a long history of racism in the United States, United Kingdom, and other countries. The term refers to racial profiling, which is said to be used by police and other law enforcement officials

(http://en.wikipedia.org/wiki/Driving_While_Black).

6b. Understand and Critique the World

"Driving while Black or Brown," encouraged students to use statistics related to racial profiling to determine whether injustices had occurred and then suggest what steps should be taken next.

Information related to the Task

This is a sample of Illinois data based on police reports from 1987-1997. In an area of about 1,000,000 motorists, approximately 28,000 were Latinx. Over a certain period of time, state police made 14,750 discretionary traffic stops (e.g. if a driver changes lanes without signaling, or drives 1-5 mph over the speed limit, police may stop the driver but do not have to). Of these stops 3,100 were of Latino/a drivers.

(Gutstein, 2006)

Questions Related to the Driving While Black or Brown Situation

- What percentage of the motorists discussed in the data were Latinx?
- 2. What percentage of the discretionary traffic stops made by police were Latinx drivers?
- 3. How could you simulate the number of possible stops that might occur based on the population by drawing cubes from a bag? What portion and color would represent the rest of the population and what portion and color would represent Latinx drivers?
- 4. Next students conduct the experiments to see what happens.
 - Students draw cubes from the bag with replacement each time to see what happens.
 - They will also tally what happens each time.
- 5. How many Latinx drivers were picked out of 100 picks, and what percentage is that?
- 6. Do your results from the simulation experiment support the claim of racial profiling? Why or why not?

(Gutstein, 2006)

Social Justice Lessons

- Goals for students:
 - Sociopolitical consciousness
 - An awareness of the social, political, economic, historical, and cultural contexts of their lives, society, and world.
 - A sense of social agency
 - A view of themselves as people able to effect change in the world
 - Positive cultural and social identities
 - Youth who are strongly rooted in their home languages and cultures, and have the confidence and capacities to stand up for that which they believe

(Gutstein, 2006)

6c. Experience Wonder, Joy and Beauty

Great ideas of mathematics are as beautiful as great works of art, and just as in the study of art, students can learn to see mathematics as expressions of beauty fashioned by drama and struggle.

(NCTM, 2018)

Success Stories

Former Penn State Nittany Lion and retired Baltimore Raven, John Urschel earned bachelor's and master's degrees in mathematics (with a 4.0 GPA) in four years, has been published in an international journal of space dynamics, and conducted research on the circular planar threebody problem. During his off season, he worked on a Ph.D. at the Massachusetts Institute of Technology. He is listed as a graduate student for Spectral Graph Theory, Numerical Linear Algebra, and Machine Learning. He is now working on his Ph D fulltime

http://espn.go.com/blog/baltimoreravens/post/_/id/25428/what-offseasonravens-john-urschel-is-starting-his-phd-at-

mit

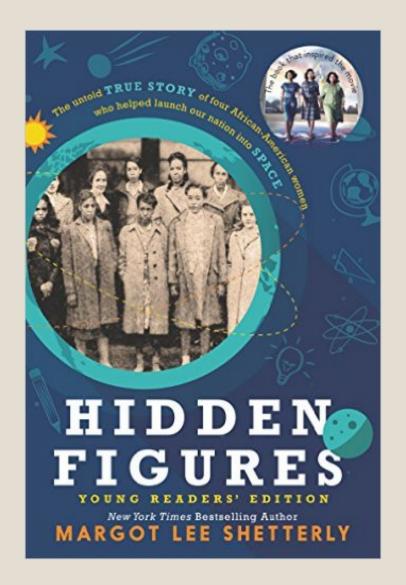
"There is real elegance in math at a higher level, and I just love that it allows me to problem solve and to just work through things to really produce some beautiful things."



Experience Wonder, Joy, and Beauty

$^{\circ}$	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19
20	21	22	23	24
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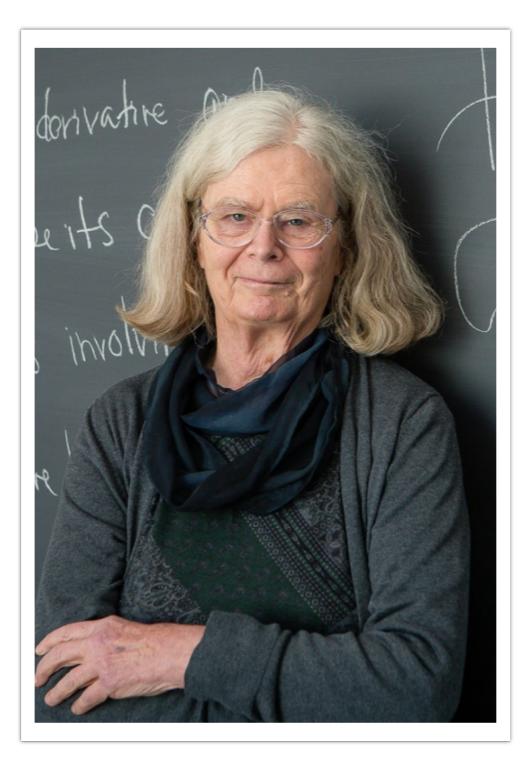


- Starting in World War II and moving through to the Cold War, the Civil Rights Movement and the Space Race, Hidden Figures follows the interwoven accounts of Dorothy Vaughan, Mary Jackson, Katherine Johnson and Christine Darden, four African American women who participated in some of NASA's greatest successes.
- It chronicles their careers over nearly three decades they faced challenges, forged alliances and used their intellect to change their own lives, and their country's future.



NASA names Facility after 'Hidden Figures' Mathematician **Katherine Johnson**. NASA renamed a West Virginia facility in her honor, 57 years after her calculations helped the first American orbit the Farth, Johnson. a West Virginia native, was hired by the National Advisory Committee for Aeronautics (which later became NASA) in 1953. She was part of a team of African-American female mathematicians who manually performed complex calculations for the space agency at the Langley Research Center — all while being largely <u>segregated</u> from their white colleagues. Johnson went on to do trajectory analysis for America's first human spaceflight in May 1961. She played a key role in crunching numbers for the orbital mission of John Glenn in 1962.

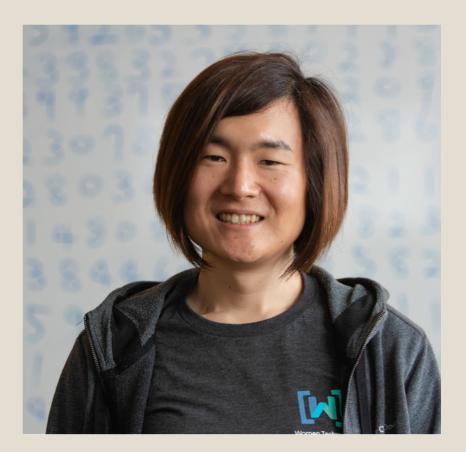
- Recently, I attended a lecture entitled, "Why does Ramanujan, 'The Man Who knew Infinity,' Matter?" by Ken Ono of Emory University.
- Ono (2018) shared that Ramanujan's work was very formulaic and not proof oriented which was not acceptable in higher mathematics.
- He shared how Ramanujan, a selftaught mathematician, created formulas that served as the foundation for many contemporary high technological applications of mathematics.
- The Man Who Knew Infinity has also been made into a movie based on My Search for Ramanujan: How I Learned to Count by Ken Ono and Amir Aczel (2016).



Karen Uhlenbeck

 Is first woman to win the Abel Prize for Mathematics. Dr. Uhlenbeck published many of her major papers in her late 30s and received a MacArthur Fellowship in 1983. Uhlenbeck helped pioneer geometric analysis, developing techniques now commonly used by many mathematicians.

https://www.nytimes.com/2019/03/19/s cience/karen-uhlenbeck-abel-prize.html



Emma Haruka Iwao, a Google employee from Japan has set a new world record for the number of digits of pi calculated. She calculated pi to 31,415,926,535,897 digits, smashing the previous record of 22,459,157,718,361 digits set back in 2016.

https://www.theverge.com/2019/3/14/18265358/pi-calculation-record-31-trillion-google

7. Ask students to develop problems related to their lived experiences.

Students may compare convenience store prices to those in a major grocery store and discuss which store has the better deals and why (Tate, 1995).





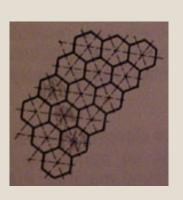
8. Engage students in Ethnomathematics

Ethnomathematics is the mathematics which is practiced among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on.

Intricate Braiding and Weaves in African American Hair

- After observing African American hair stylist for hours, Gloria
 Gilmer began to see patterns and other kinds of mathematics.
- Wrote a paper that showed how the hair-braiding enterprise can contribute to mathematics education, and, conversely, what can mathematics education can contribute to the hairbraiding enterprise?"
- Through a discussion of "tresselations," box braids and triangles, she showed how the activity could, generate many mathematics applications.

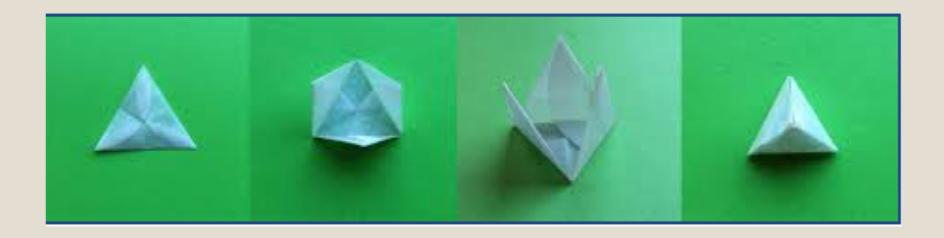
Weiger, P. R. (2000). Re-Calculating math instruction. <u>Black Issues in Higher Education, 17</u>(13), 58 – 63.







Tetrahedron, as explored through Japanese Origami



(Brandt & Chernoff, 2014)

Other Examples of Ethnomathematics

- the examination of ratios, patterns and symmetry in Japanese origami;
- logic of kin relations (e.g., Warlpiri in Australia);
- chance and strategy games and puzzles from various Native American tribes;
- symmetric strip decorations found in Incan and Maori cultures;
- symmetry and concepts of impermanence in the mandalas of the East;
- measuring and ratios in traditional quilting patterns (Presmeg, 1998);
- counting and understanding of time-keeping in the pagan Misseri Calendar, which was created by Icelanders who were greatly influenced by their environment (Bjamadottir, 2010);
- fractals in African design (Eglash, 2007);
- shapes and design in graffiti from hip-hop culture (Eglash, 2012);
- and, the khipus or quipus, which are an ancient Incan system of mathematics and accounting that was based on an elaborate system of tying knots in colored cords of cotton or camelid fibers (Urton, 2012).

(Brandt & Chernoff, 2014)

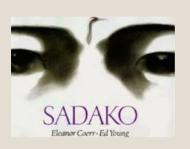
9. Engage families in doing mathematics together.

Multicultural Literature as a Context for Mathematical Problem Solving: Children and Parents Learning Together

- Helps parents understand innovations in mathematics education and how they can reinforce what their children are learning in school at home.
- Teachers facilitate parents and their children reading literature together and solving related mathematical problems based on prepared modules.
- Parents and teachers must work together to ensure that children are developing their full mathematics potential.

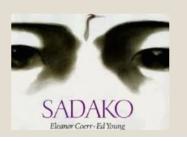
(Strutchens & Perkins)

Sadako and the Thousand Paper Cranes by Eleanor Coerr



Sample Problem

Sadako became sick in February and died in October. By the time of her death, Sadako, her family, and friends had folded 644 cranes. What was the average amount of cranes folded per day from the time Sadako became ill until her death?



Response:

Since I know that there are 8 months from February to October and each month has either 30 or 31 days except for February which has 28 days, I just estimated 30 days for each month and came up with 240 days. Then I thought if she made 1 bird a day in 240 days that would be 240 birds. Then if she made 2 birds a day in 240 days that would be 480 birds which is not enough. 3 birds a day in 240 days would be 720 birds. 720 birds are more than what she made so my scientific answer would be two-ish.

10. Show students that you care.

Amari Mitchell

16 years old, Hoover High School junior. **Black Students Talk about the Achievement Gap in Alabama Schools (Dunigan, 2017)**

Continuum of Caring

- Caring could be used to protect students' emotional and psychological well being.
 - Teachers seek to avoid all risk of adding further to their student's trauma.

Or

- Caring could be used to motivate proactive interventions.
 - Teachers push students to increase their knowledge in order for students to have a variety of options.

(Secada, 2003)

Ten Equitable Pedagogical Strategies

- Use mathematics autobiographies and other means to get to know your students.
- 2. Use multiple entry level tasks.
- 3. Implement NCTM's eight mathematics teaching practices.
- Use formative assessment.
- 5. Foster relational equity among students.
- Allow students to explore the purposes of mathematics.
- 7. Ask students to develop problems related to their lived experiences.
- 8. Engage Students in Ethnomathematics
- 9. Engage families in doing mathematics together.
- 10. Show the students that you care.

Conclusions!

- Use a variety of equitable teaching approaches to help student see the beauty and joy of mathematics.
- Work with your colleagues across the grades to make it happen systemically!

Which of the 10 equitable teaching strategies will you implement in the next week?

Resources

