Reasoning With Fractions: Representing and Explaining the How and Why




April 4, 2019

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Mathematics Support Teachers

Elementary Mathematics

Howard County Public School System

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HCPSS Strategic Call to Action

Learning and Leading with Equity

The Fierce Urgency of Now

Vision

Every student and staff member embraces diversity and possesses the skills, knowledge and confidence to positively influence the larger community.

Mission

HCPSS ensures academic success and social-emotional well-being for each student in an inclusive and nurturing environment that closes opportunity gaps.

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Four Overarching Commitments

VALUE

Every HCPSS stakeholder feels happy and rewarded in their roles and takes pride in cultivating the learning community.

ACHIEVE

An individualized focus supports every person in reaching milestones for success.

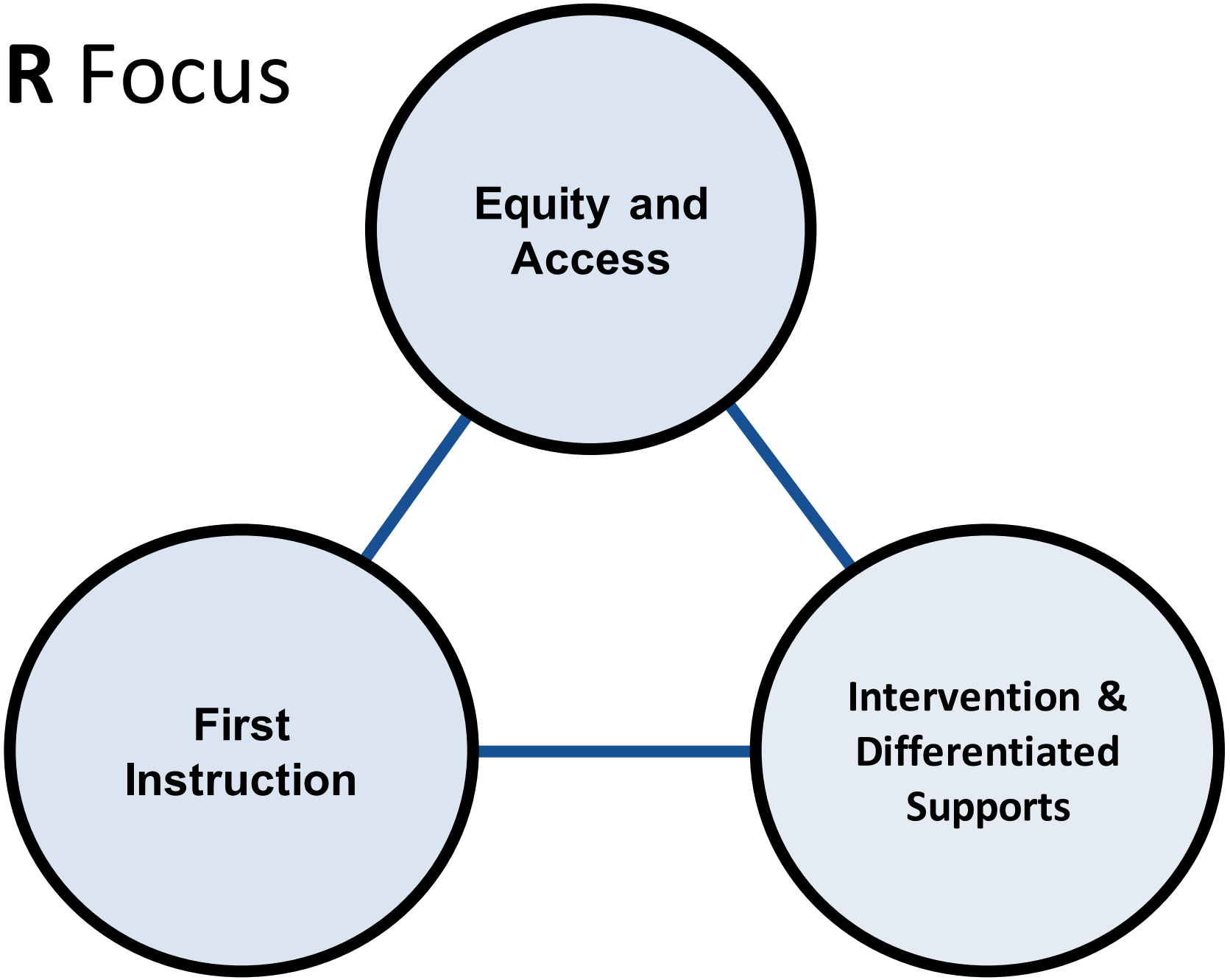
CONNECT

Students and staff thrive in a safe, nurturing and inclusive culture that embraces diversity.

EMPOWER

Schools, families and the community are mutually invested in student achievement and well-being.

OUR Focus



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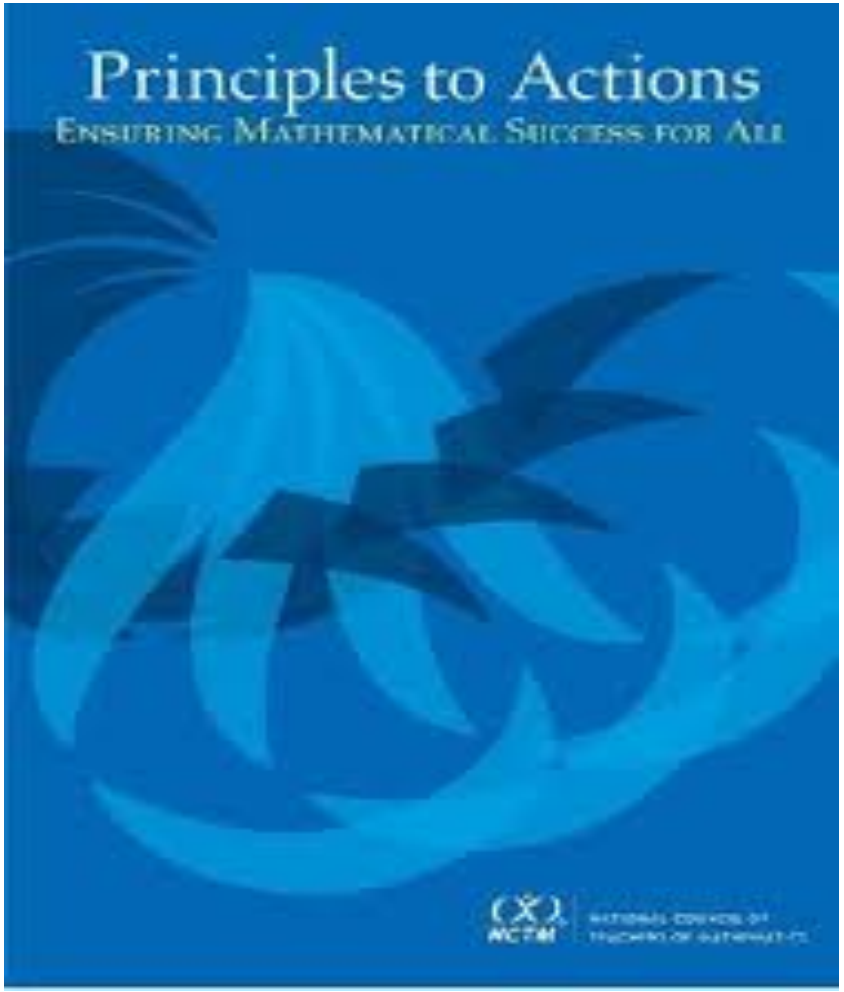
OUR Work is Research-informed



NCTM (2014)

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Standards for Mathematical Practice

1. Make sense of **problems** and persevere in solving them.
2. **Reason** abstractly and quantitatively.
3. Construct viable **arguments** and critique the reasoning of others.
4. **Model** with mathematics.
5. Use appropriate **tools** strategically.
6. Attend to **precision**.
7. Look for and make use of **structure**.
8. Look for and express **regularity** in repeated reasoning.

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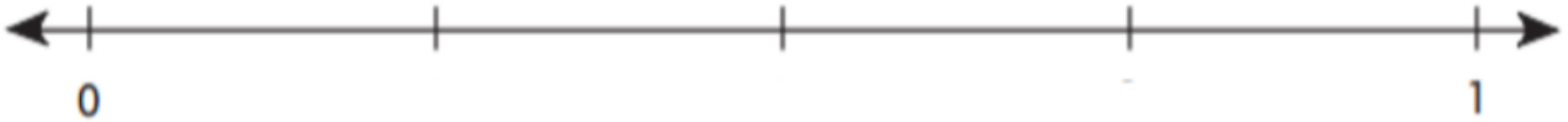
Session Outcomes

Participants will be able to:

- Represent fractional concepts on a number line.
- Represent fractional concepts with Cuisenaire rods.
- Explain different fractional concepts based on a visual models.
- Reflect on current instructional practices and how we can improve them.

Place the Fraction $\frac{3}{4}$ on the Number Line



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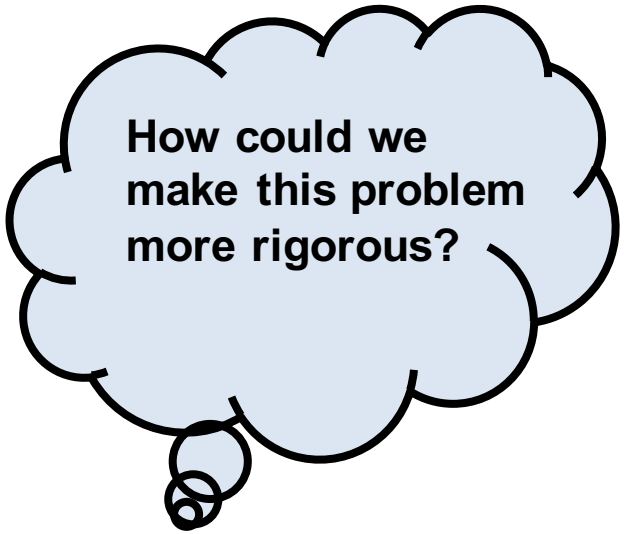
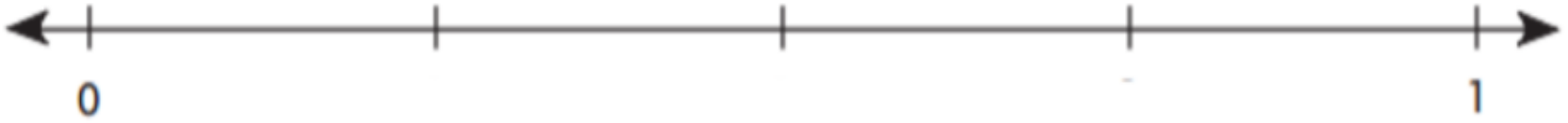
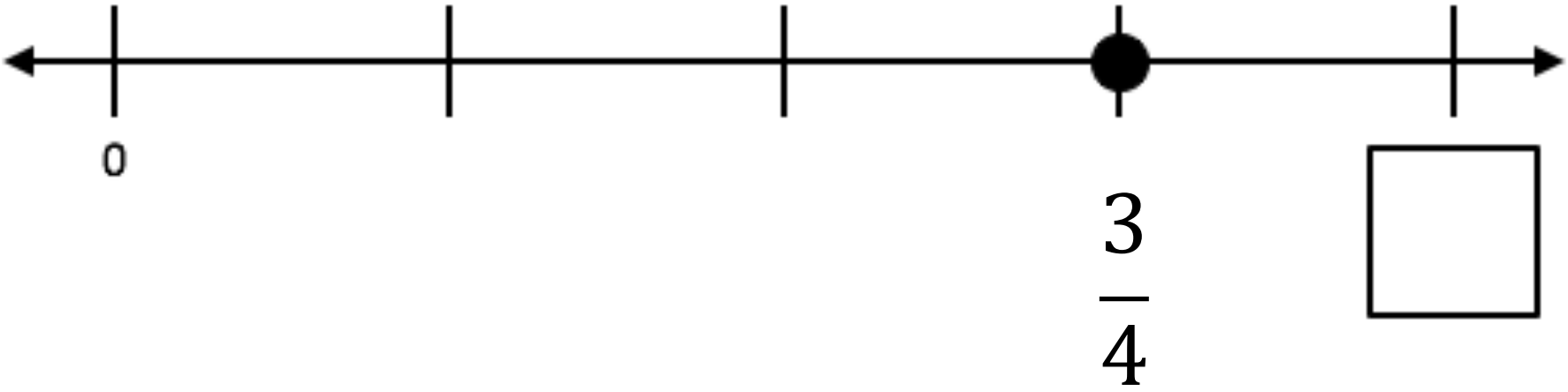




What is the Endpoint ?



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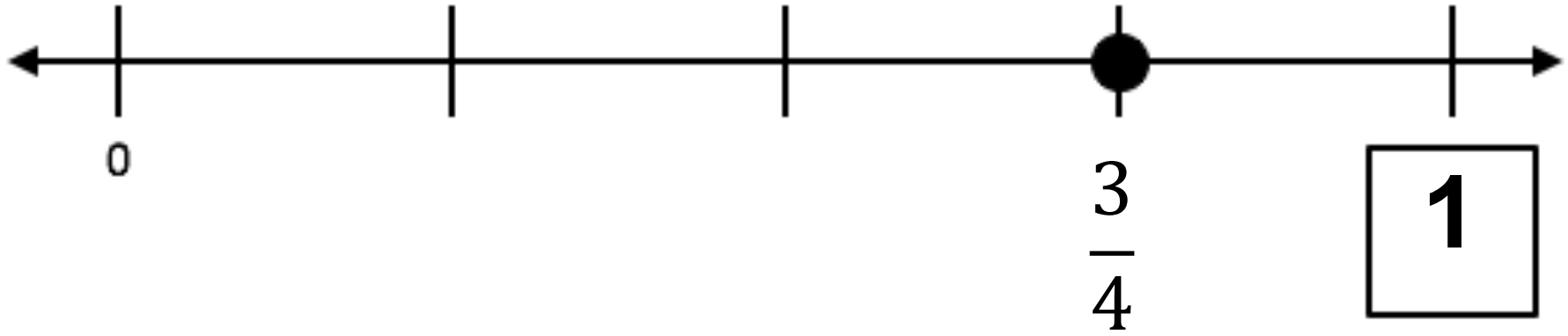

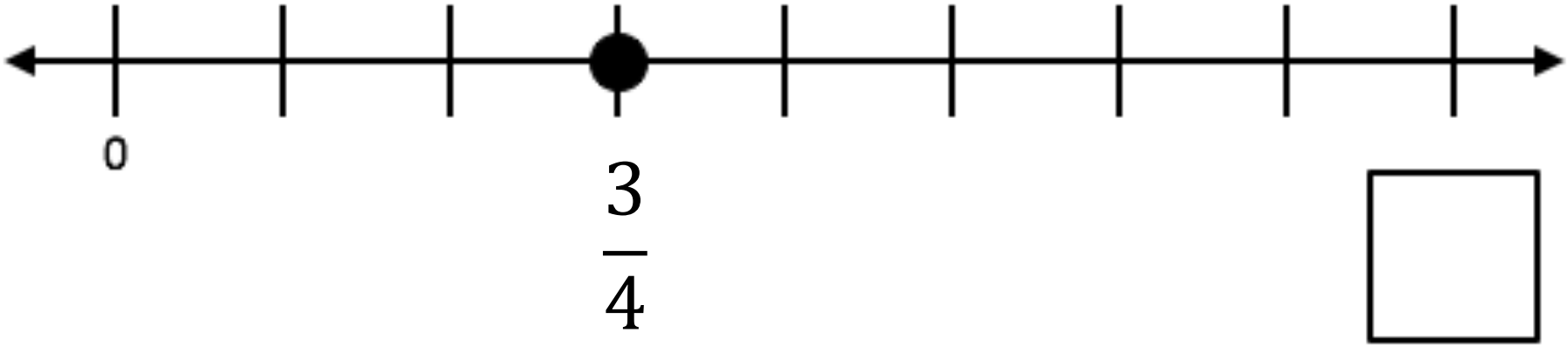

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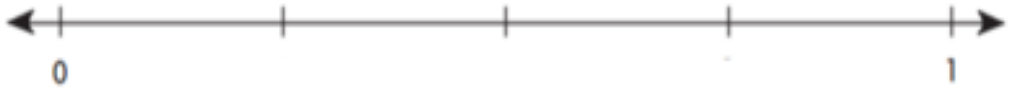
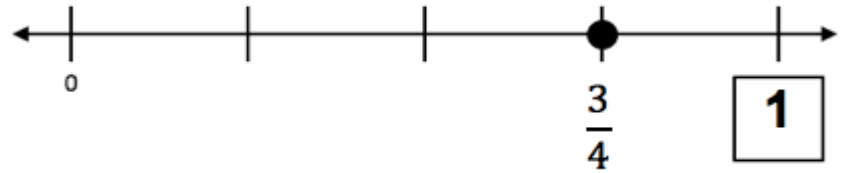
Level of Rigor

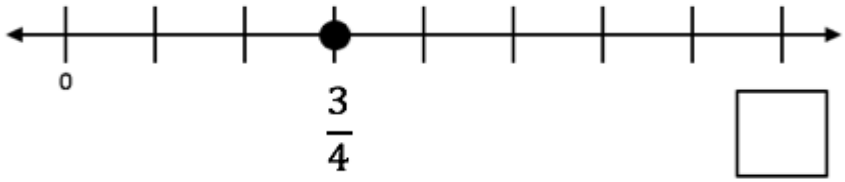
Place the Fraction $\frac{3}{4}$ on the Number Line



Which question is the most rigorous?

What is the Endpoint?





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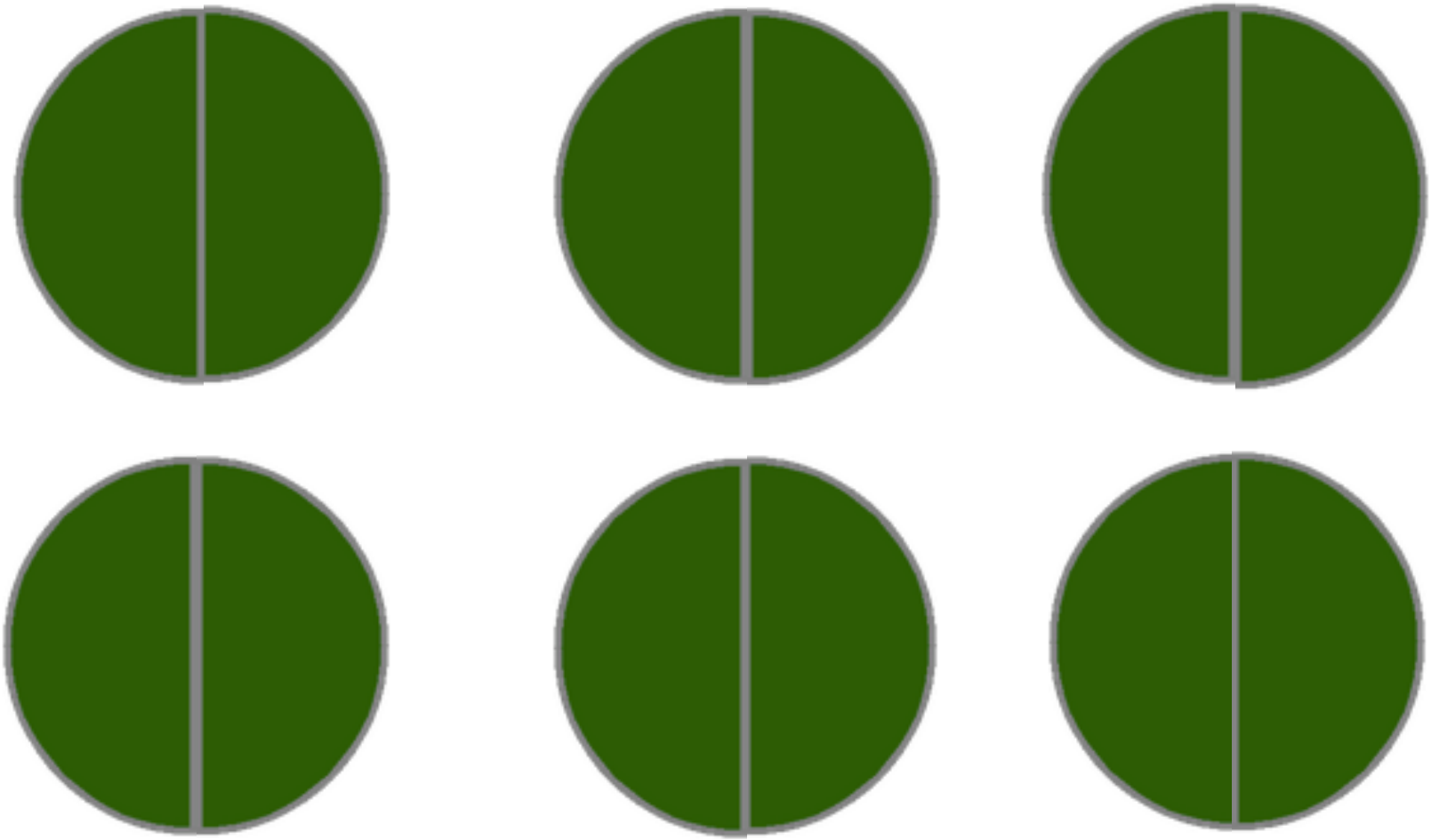
Count Around the Circle



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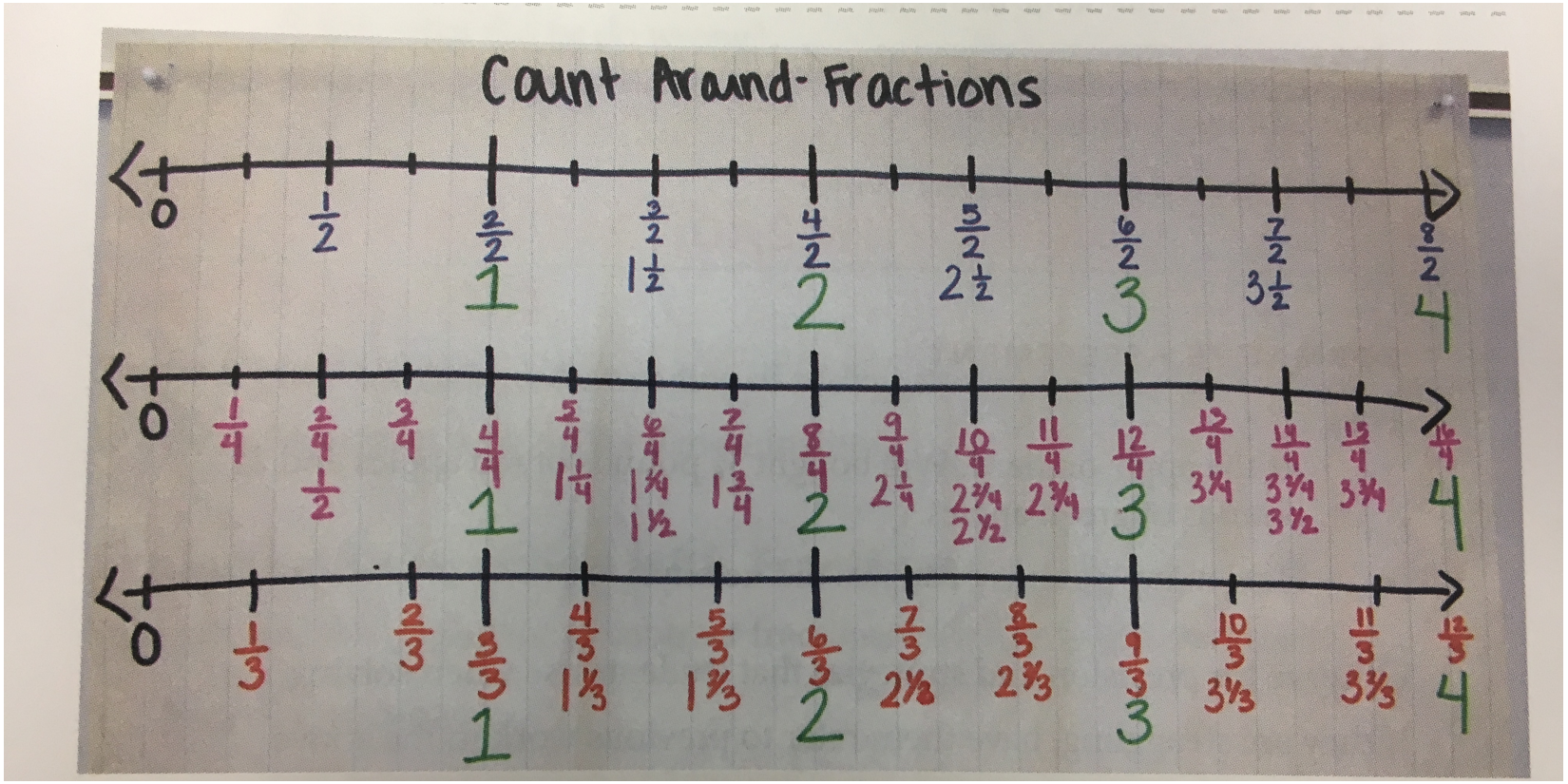

Picture taken from “Math In Practice”

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Fractions…Where to Start?

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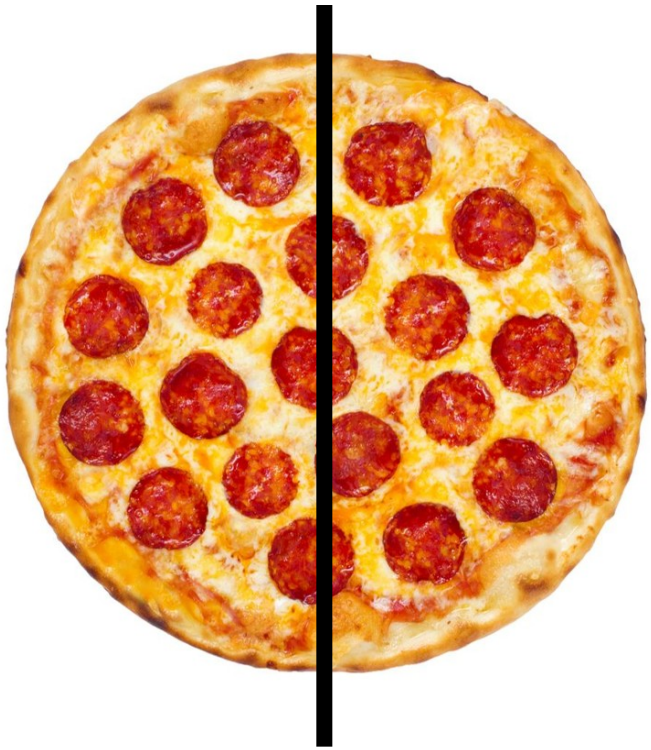
- Understanding that the size of the whole matters.
  Ex: A half is not always equal to a half.



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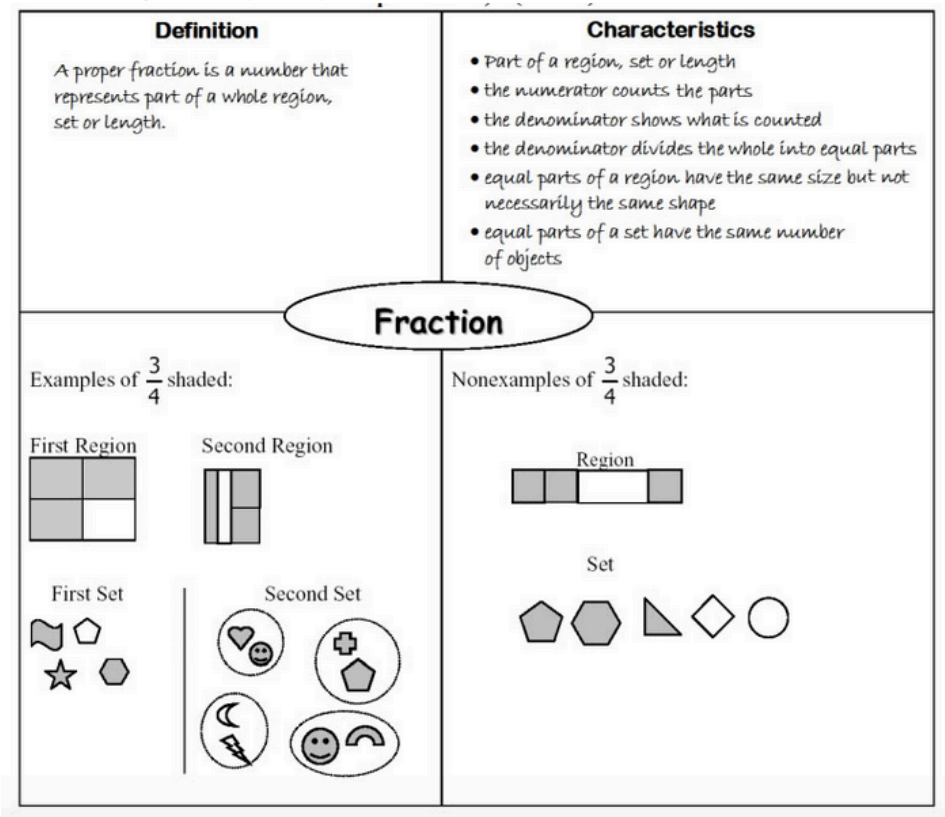
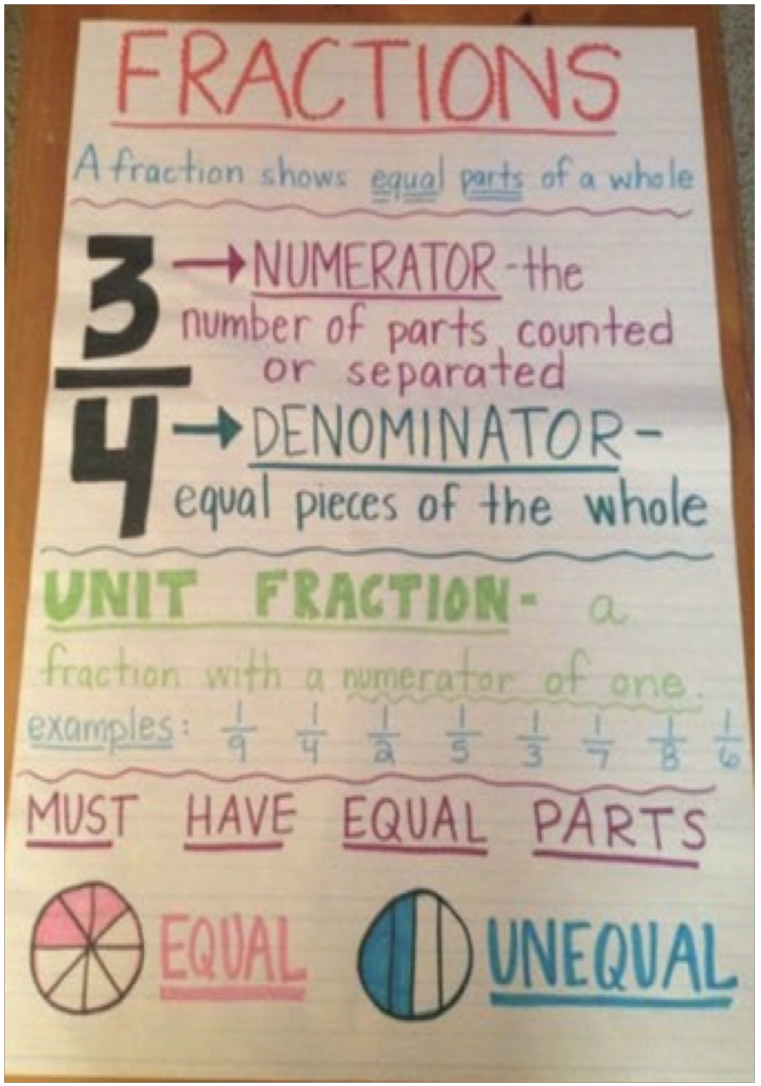
Fractions…Where to Start?

Basic Vocabulary





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Partitioning

- Why don't they just get it?!
- We often times introduce this with folding paper.
  - Fold the paper in half
  - Now fourths
  - Now eighths
- This seems to be easy. When does it get hard?

Partitioning with Odd Numbers

- Instead of folding, what if we partitioned with moveable lines?
- Take your sentence strip and partition it into fourths using the popsicle sticks.
  - What technique did you use?
- Now partition your sentence strip into fifths.
  - How does partitioning these two fractions differ?
- What representation does this lead perfectly to?

Using Multiple Colors

- With your popsicle sticks, break the sentence strip into fourths.
- Using a different color, break the same sentence strip into eighths.
- What did you notice?
- What idea does this lead students to understand?

Partitioning on a Number line

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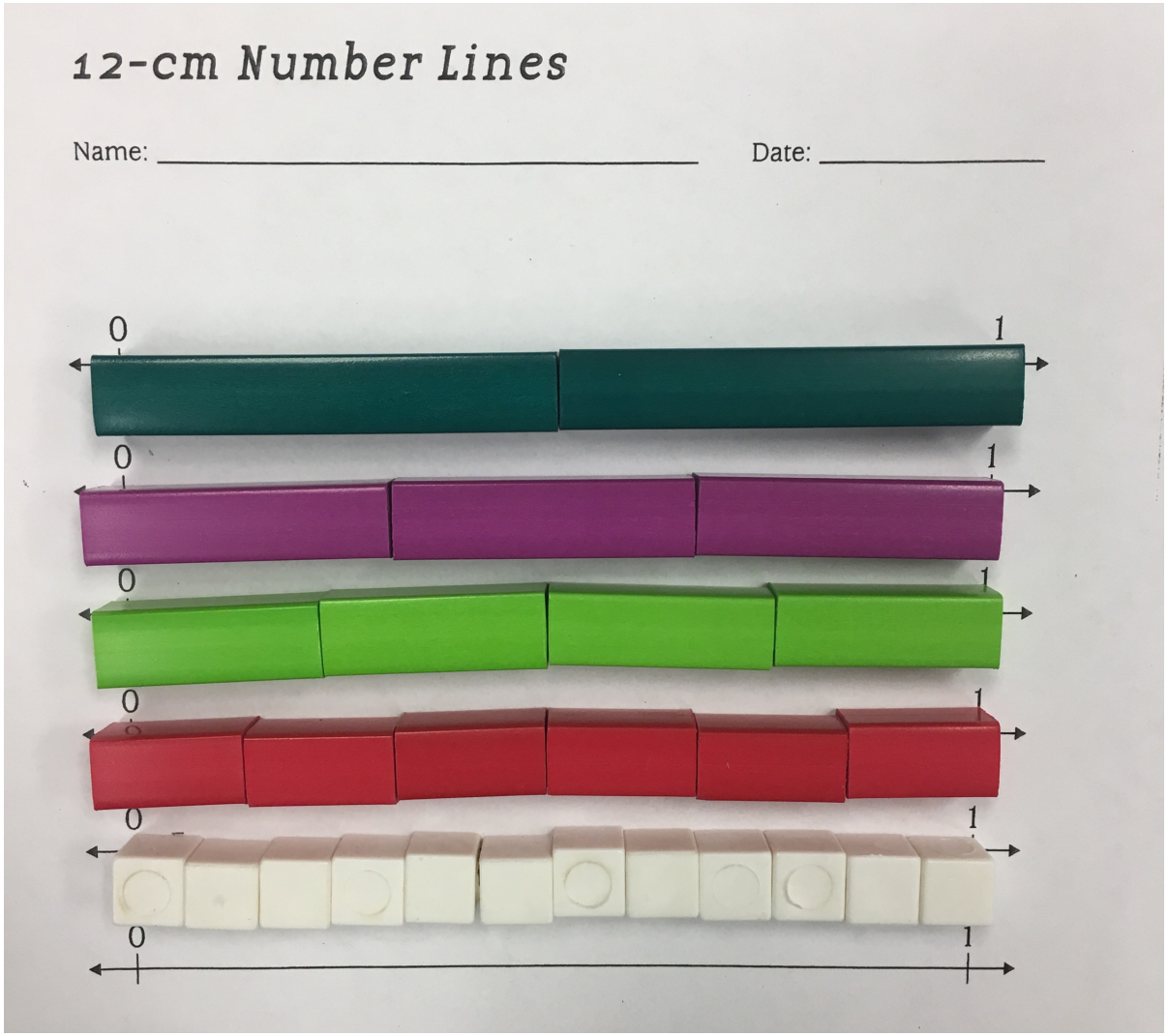
How can we use the number line to show equivalence?

Fraction Equivalence on a Number Line



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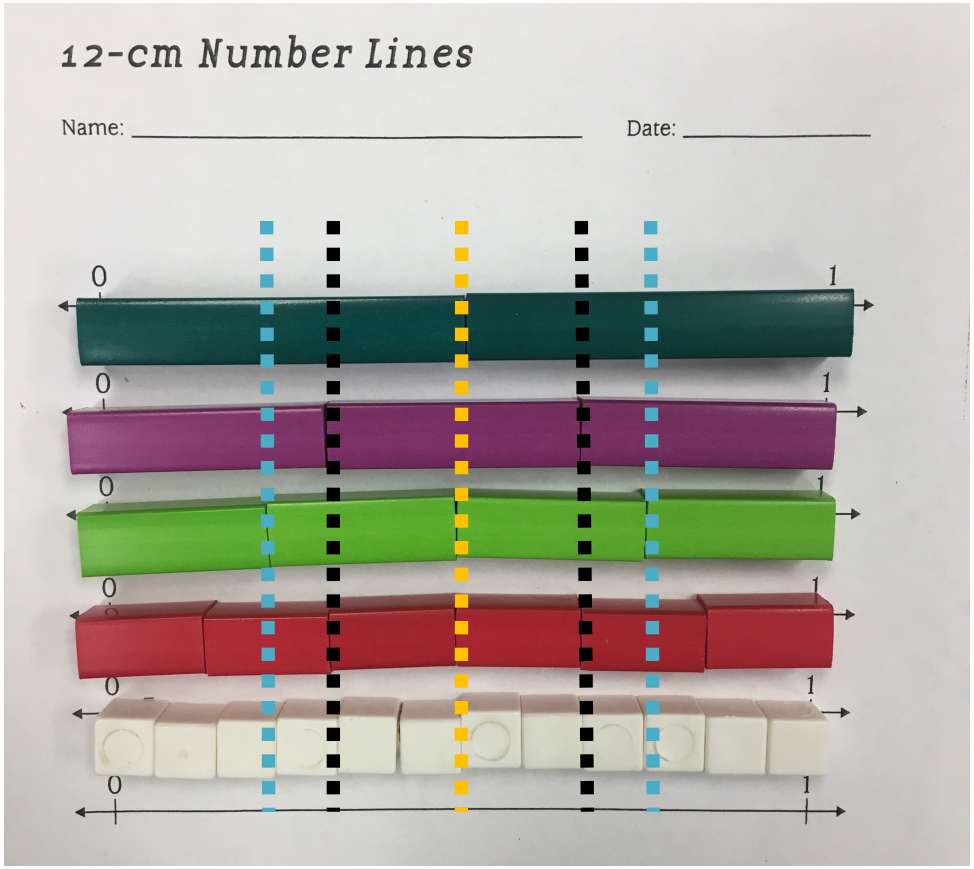
Fraction Equivalence on a Number Line



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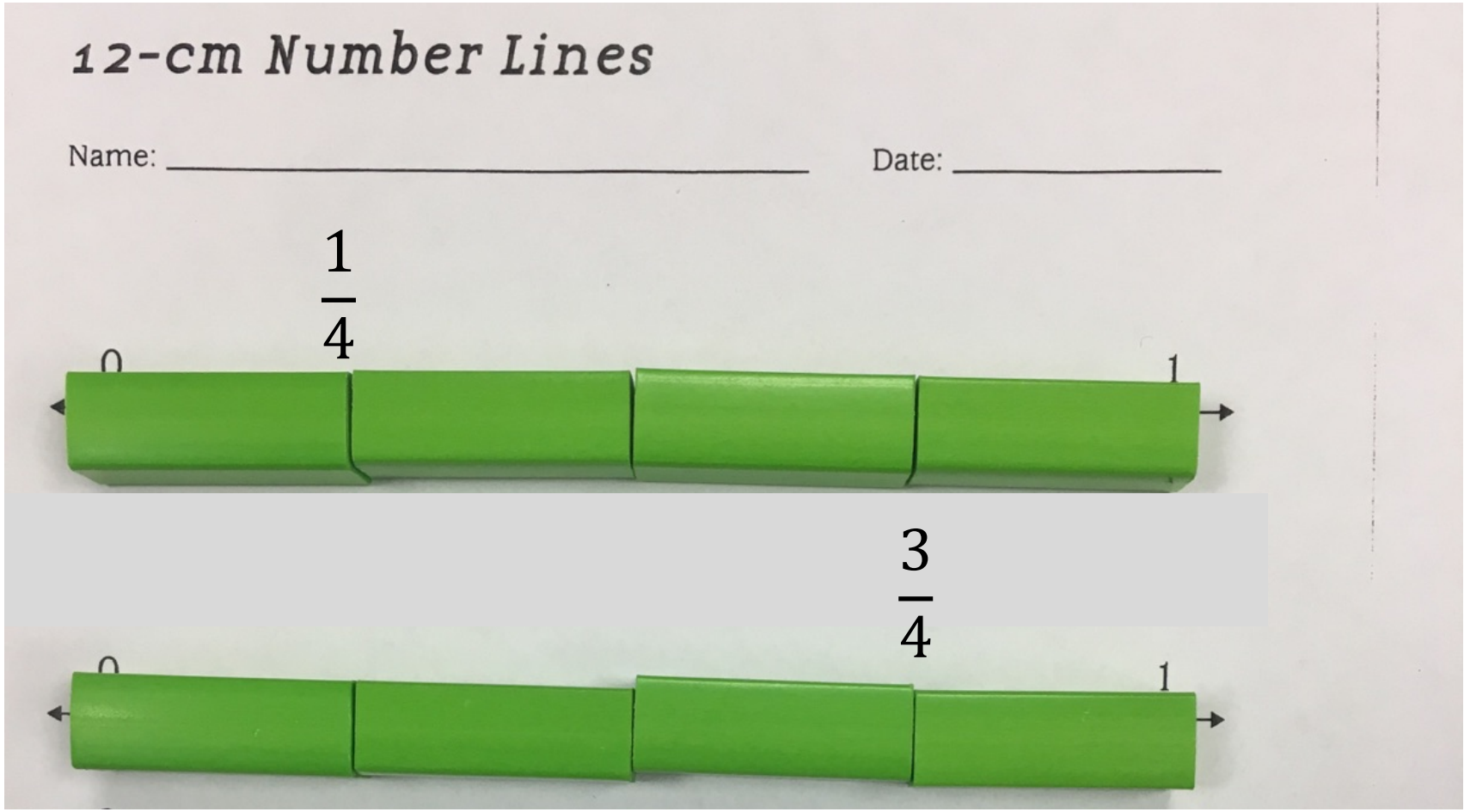
Comparing Fractions

- What are some strategies for comparing fractions?
  - Denominators the same, compare the numerators.
  - Numerators the same, compare the denominators.
  - Use benchmark of ½.
- How can we model these on the number line and use Cuisenaire rods to show understanding?

Compare $\frac{1}{4}$ and $\frac{3}{4}$

Denominators are the same, compare the numerators



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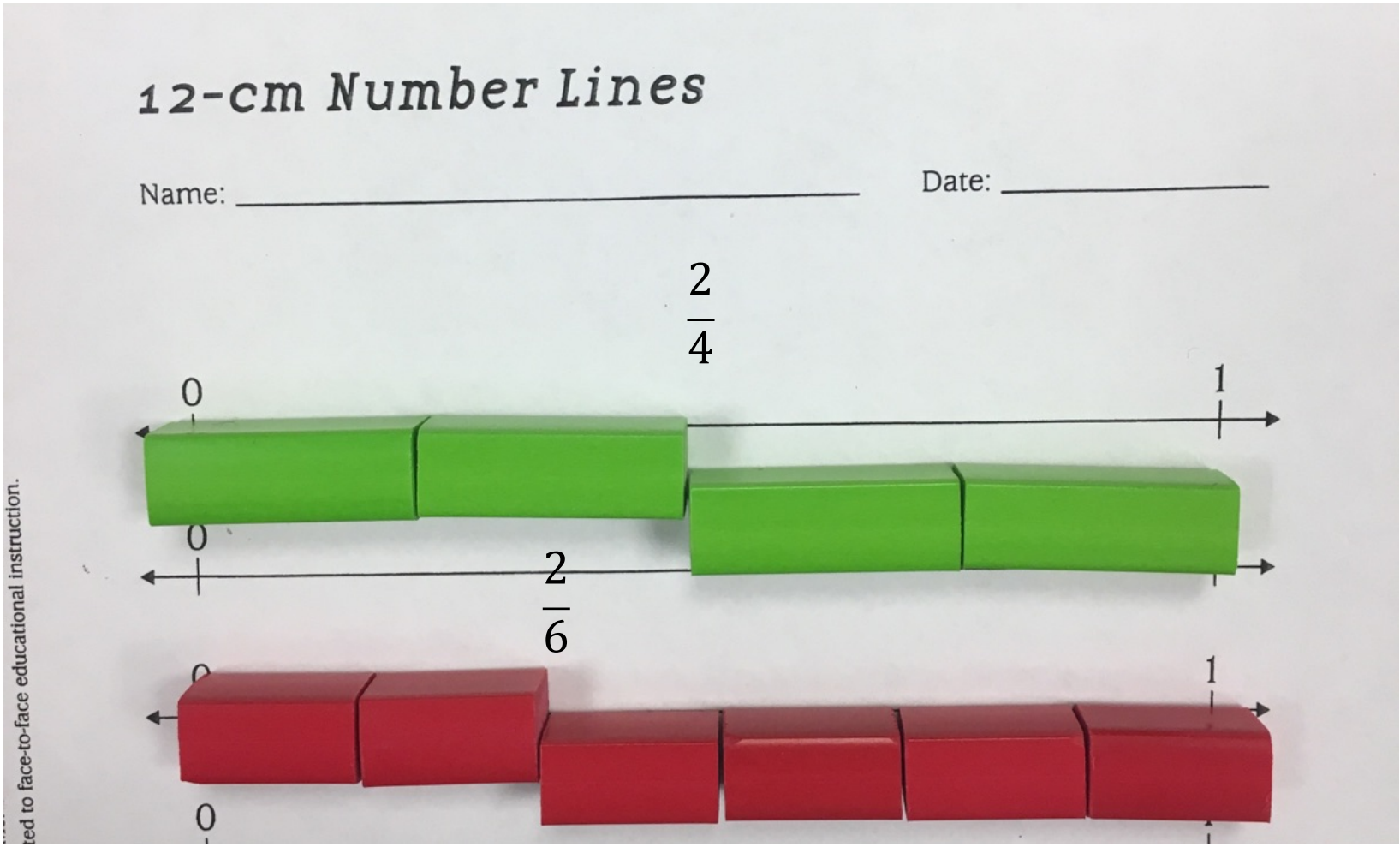
Compare $\frac{2}{4}$ and $\frac{2}{6}$

Numerators are the same, compare the denominators.



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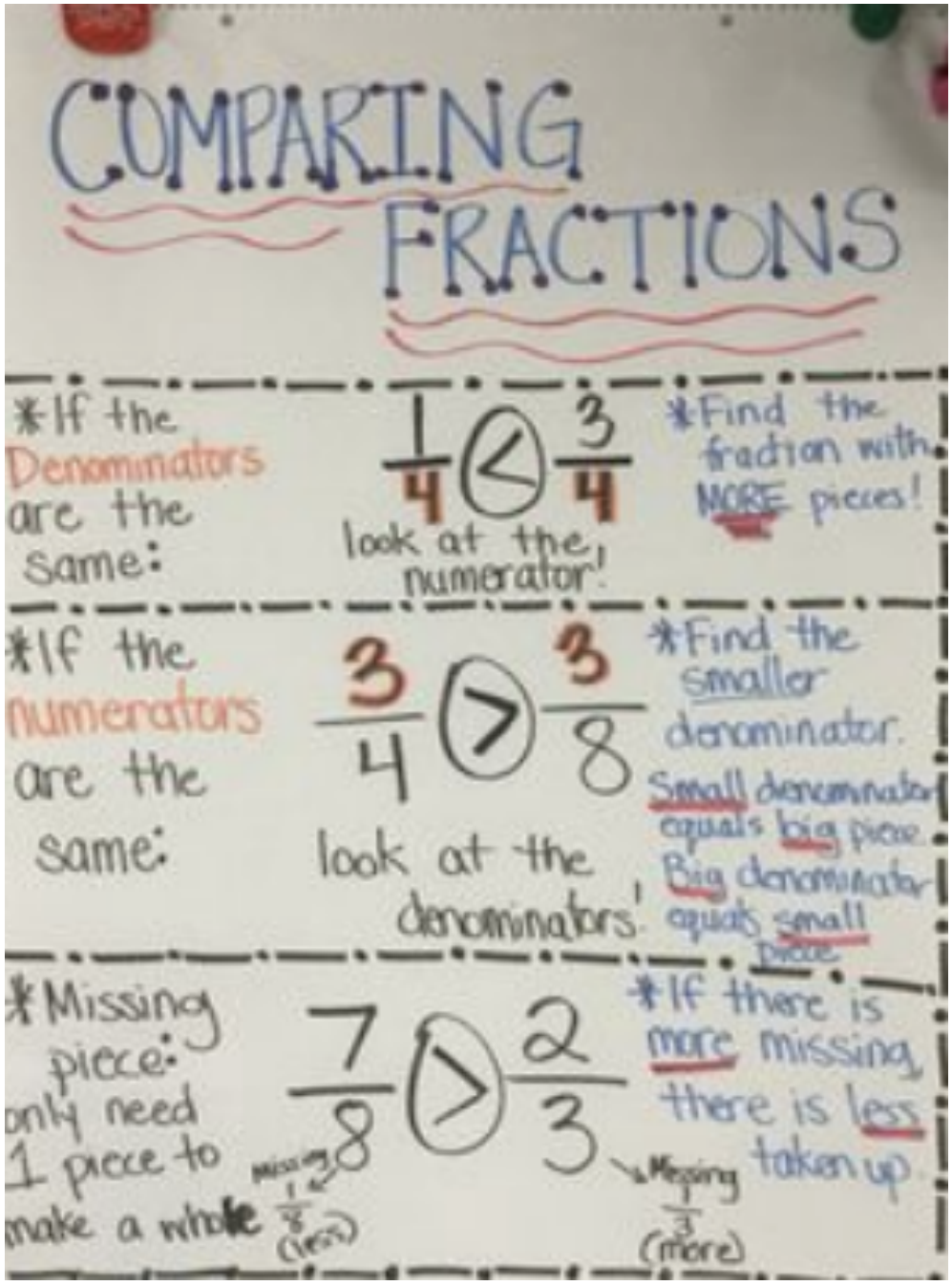
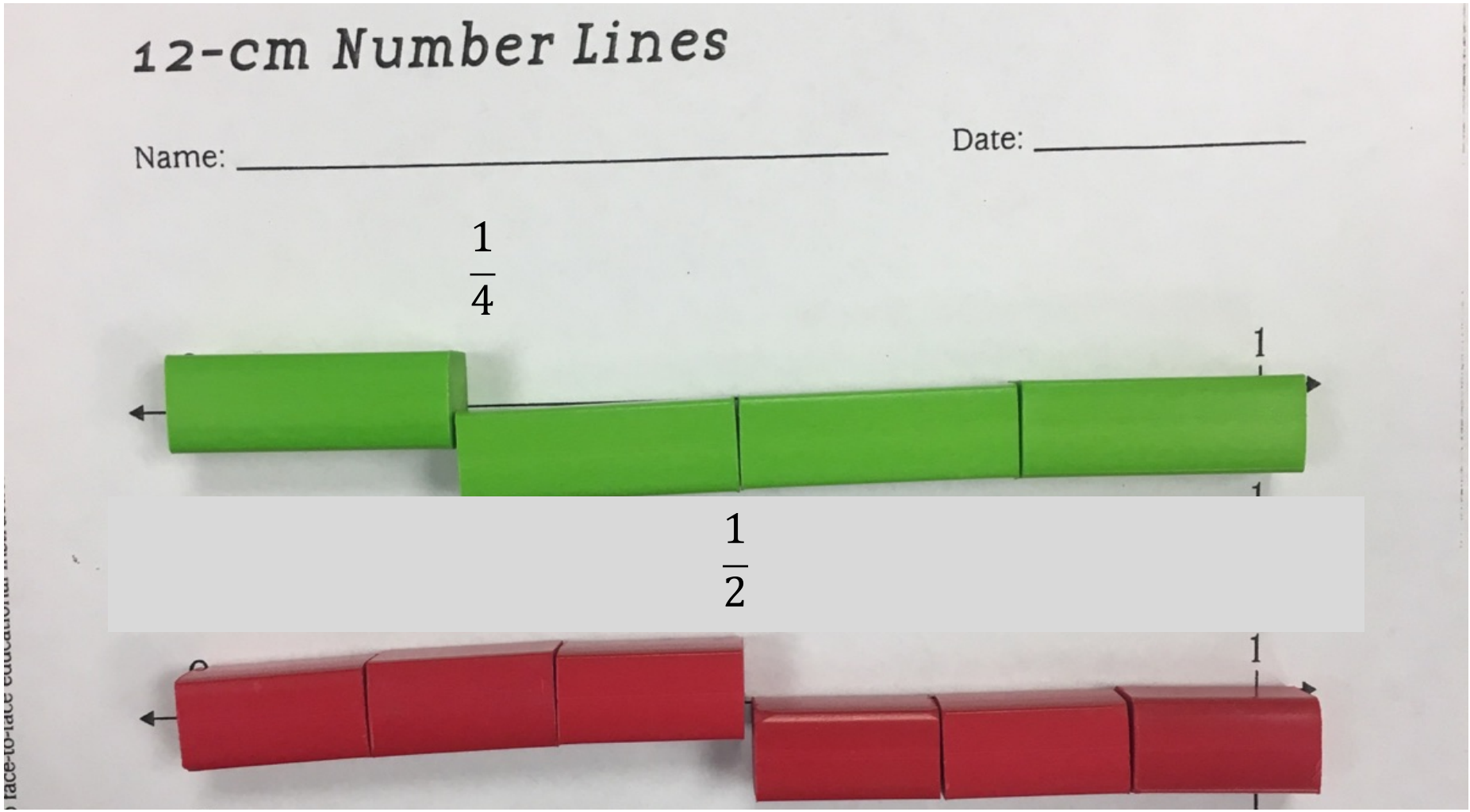


Compare $\frac{1}{4}$ and $\frac{1}{2}$

Look for the benchmark of ½.



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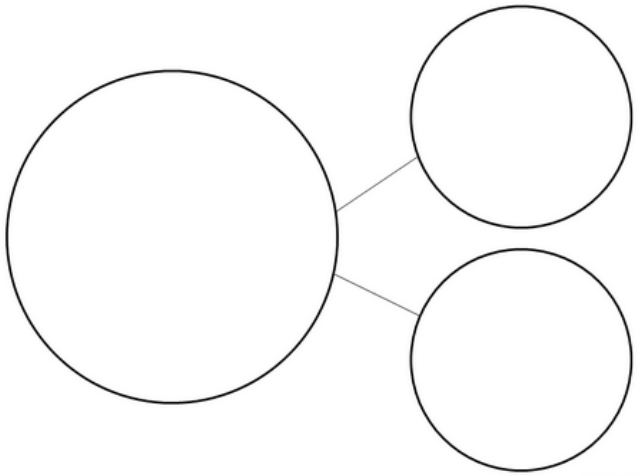
Decomposing

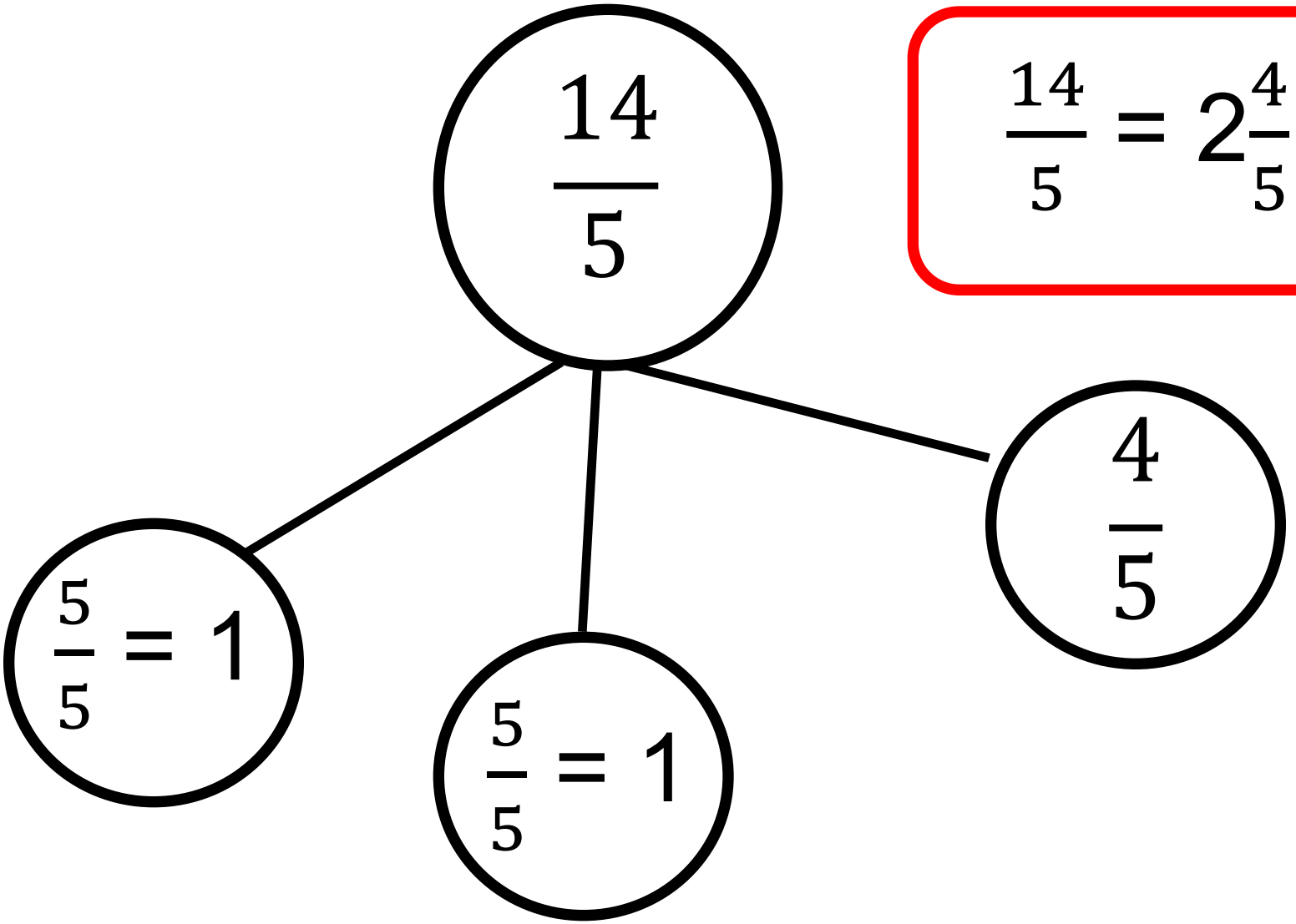
- Number Bonds:
  - If the orange rod is your whole, what color is fifths?
  - How could we break apart our whole into different fifths?
  - How does this representation prove we are correct?



Decomposing

- Complete the Whole
  - If the brown is my whole, what color is half?
  - How could I complete the whole?
  - Is this the only way?
- How do the rods prove we are correct?
- How could this transfer over to the number line?

Decomposing Fractions



$$\frac{14}{5} = 2\frac{4}{5}$$

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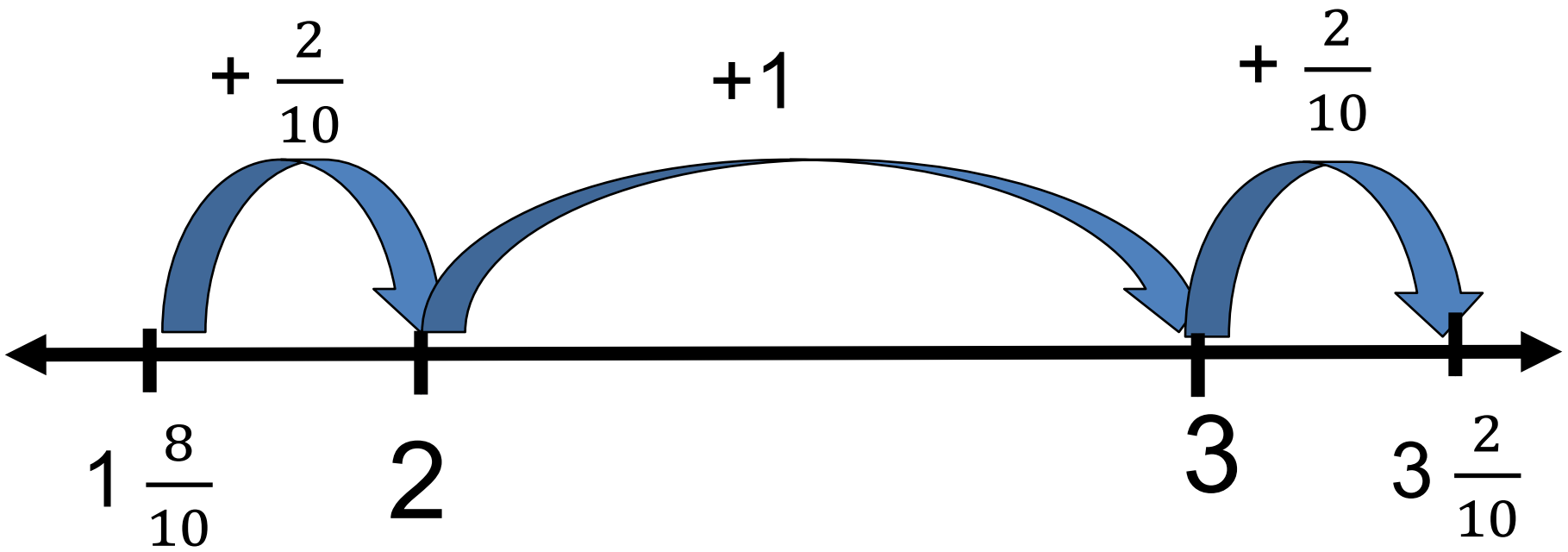
Adding and Subtracting Fractions

Students can use their prior experience of adding whole numbers and apply it to fractions.

Adding Up on an Open Number Line

$$3\frac{2}{10} - 1\frac{8}{10} = ?$$



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Constant Difference

$$3\frac{2}{10} - 1\frac{8}{10} = ?$$

We can add the same amount
to the subtrahend and minuend
and the difference will remain the same..

Constant Difference

$$3\frac{2}{10} - 1\frac{8}{10} = ?$$

What could we add to each fraction to make it simpler to solve?

$$\left(3\frac{2}{10} + \frac{2}{10}\right) - \left(1\frac{8}{10} + \frac{2}{10}\right) = ?$$

$$3\frac{4}{10} - 2 = 1\frac{4}{10}$$

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Same or Different?

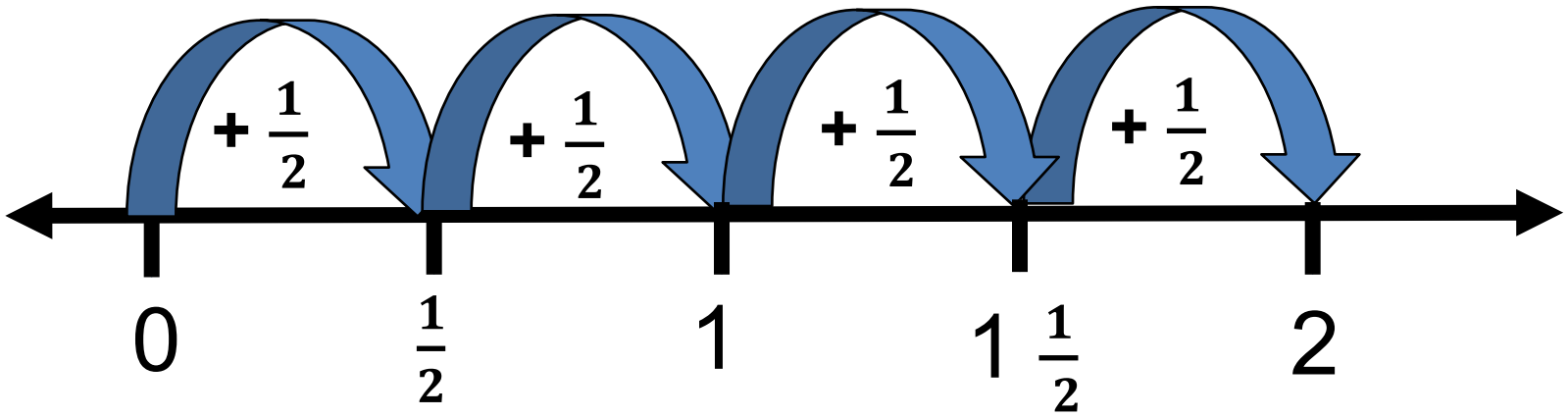
$4 \times \frac{1}{2}$

$\frac{1}{2} \times 4$

Multiplying Fractions

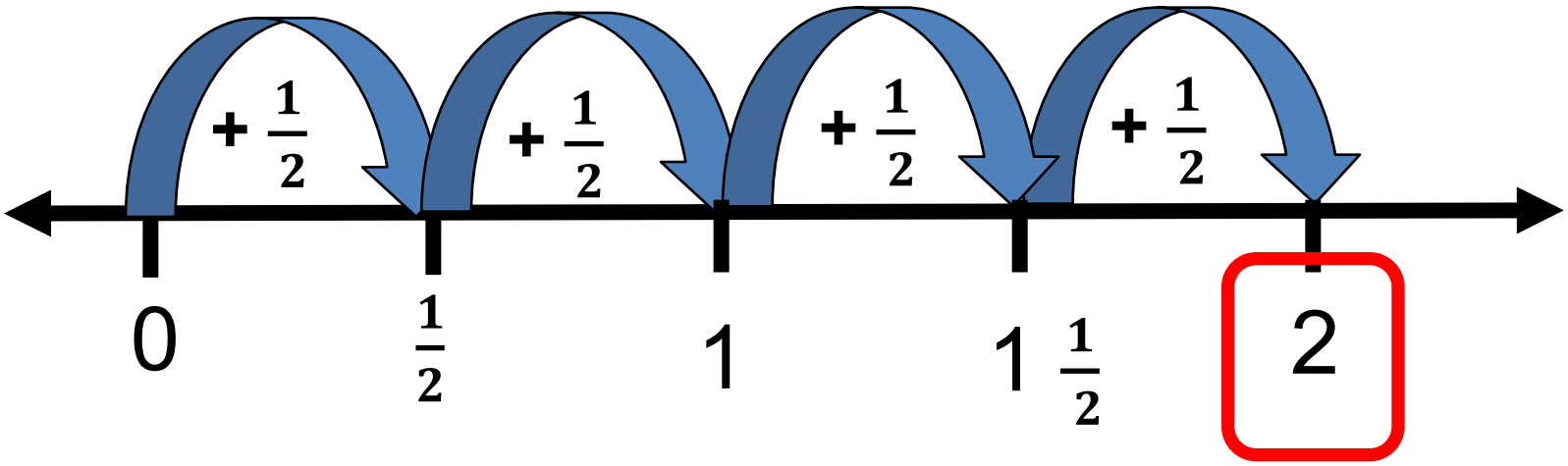
- Students need to use multiple representations and understand that while the product remains the same, the representations look very different.

$4 \times \frac{1}{2}$ means 4 groups of $\frac{1}{2}$



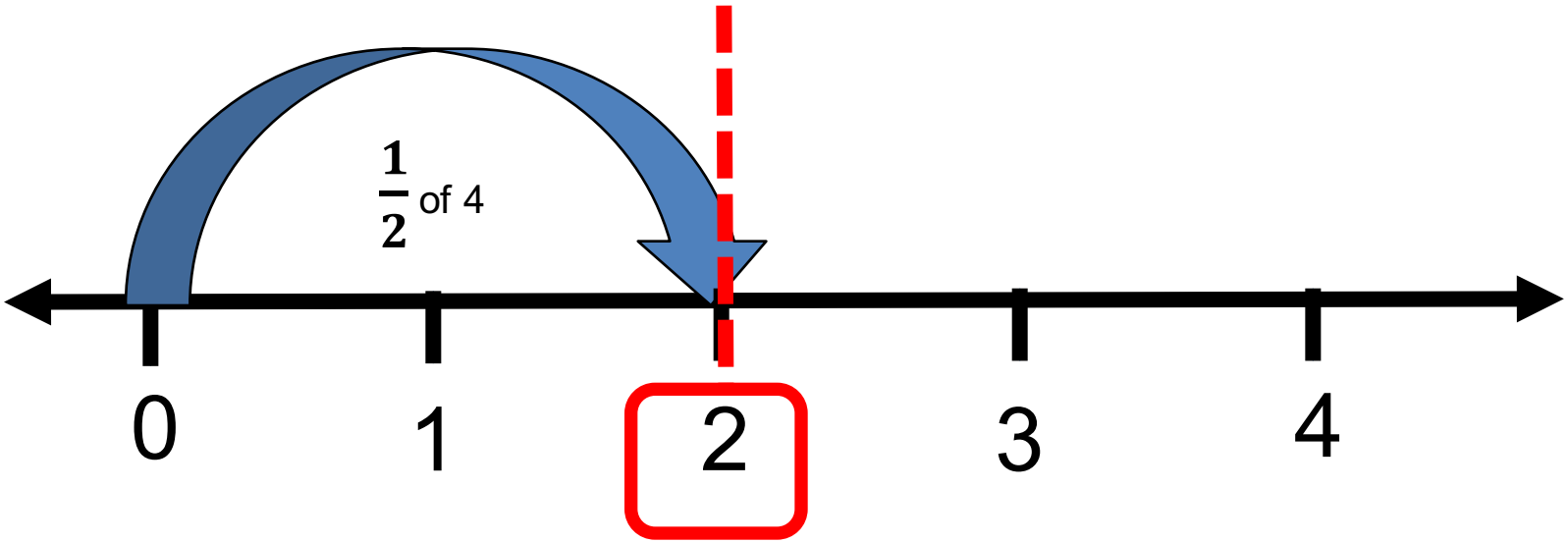
Multiplying Fractions

4 days after work, Jessica ran $\frac{1}{2}$ mile. What was the total amount of miles she ran?



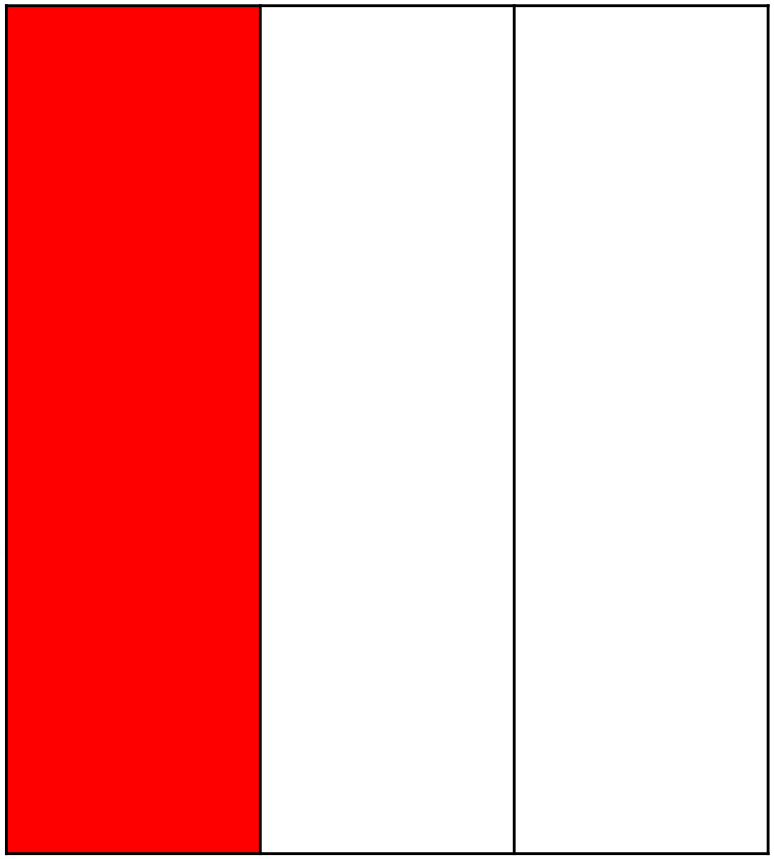
Multiplying Fractions

Jessica ran in a 4 mile race. She stopped exactly halfway through the race to get some water. How many miles had she run before she stopped?



Multiplying Fractions

$$\frac{1}{6} \times \frac{1}{3}$$



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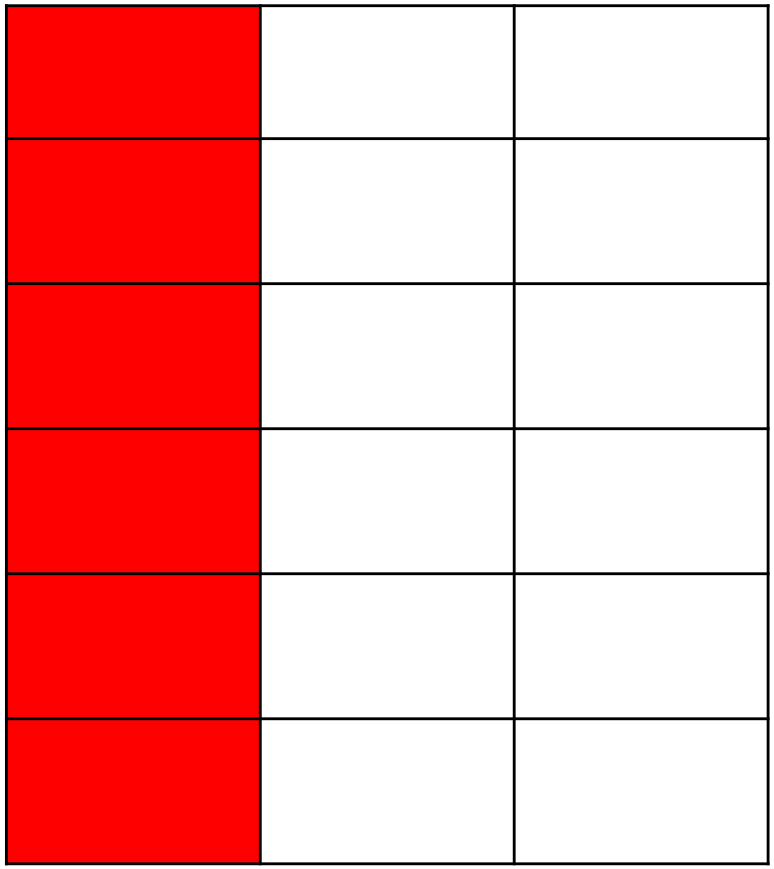

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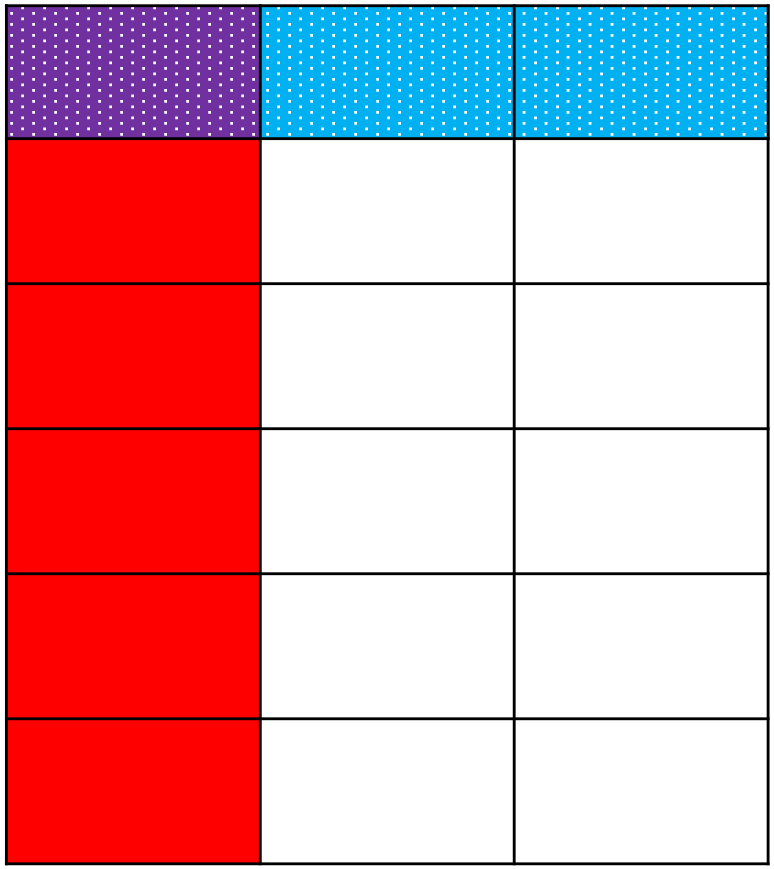

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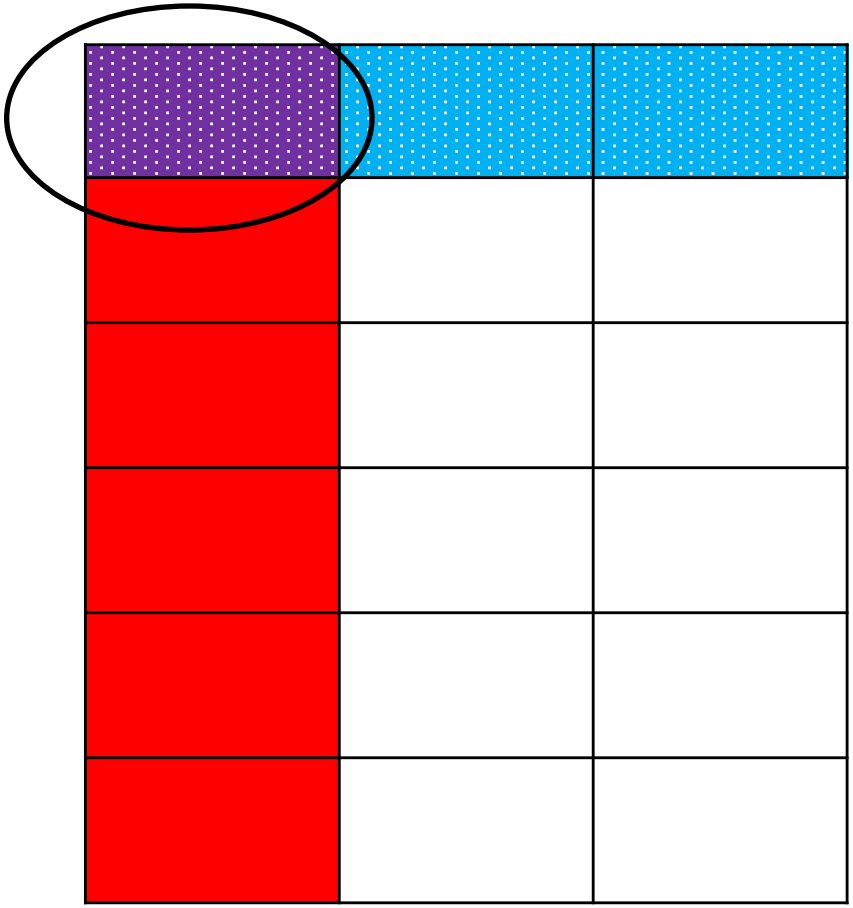

Dividing with Fractions

How can we show $9 \div \frac{1}{3}$ on a number line?

Think: How many one-thirds are in 9?

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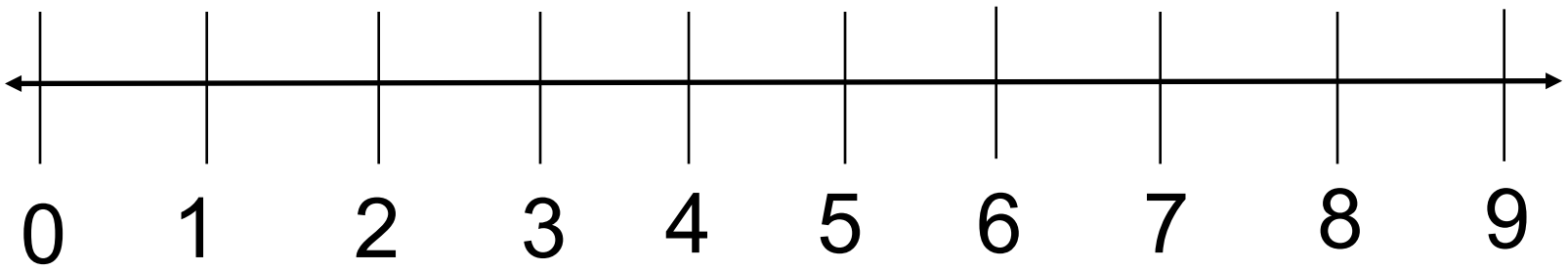

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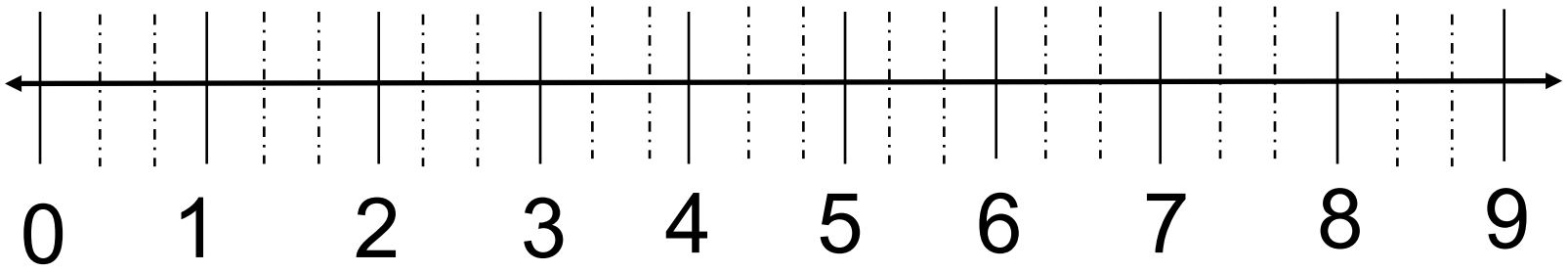

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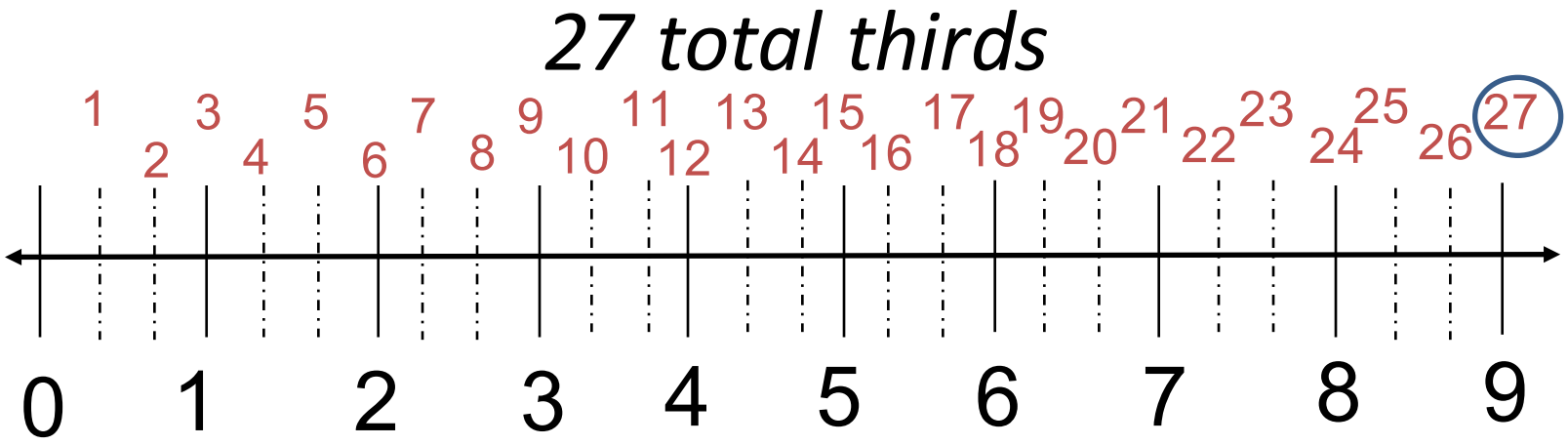

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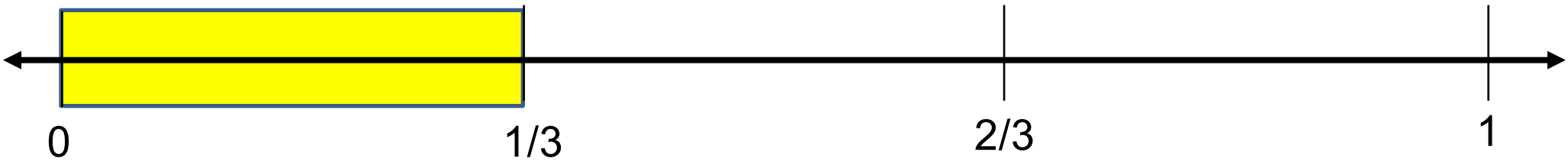

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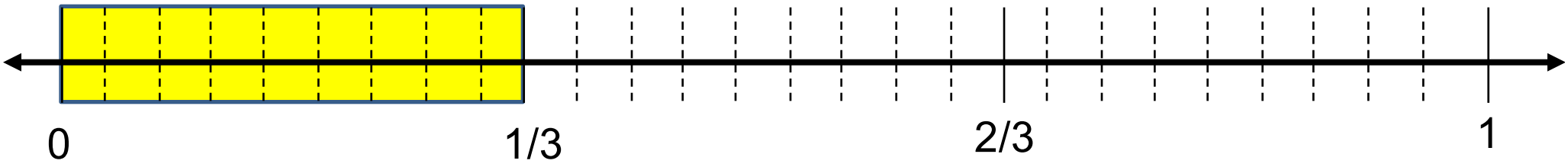

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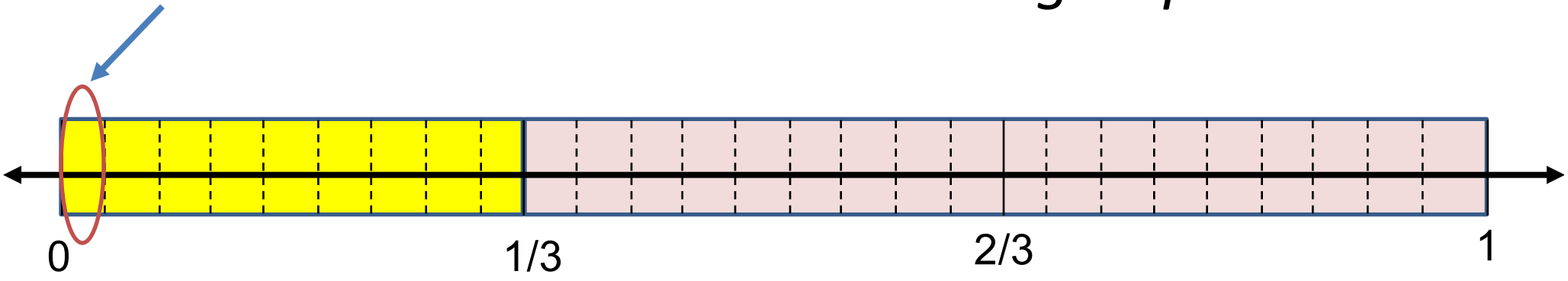
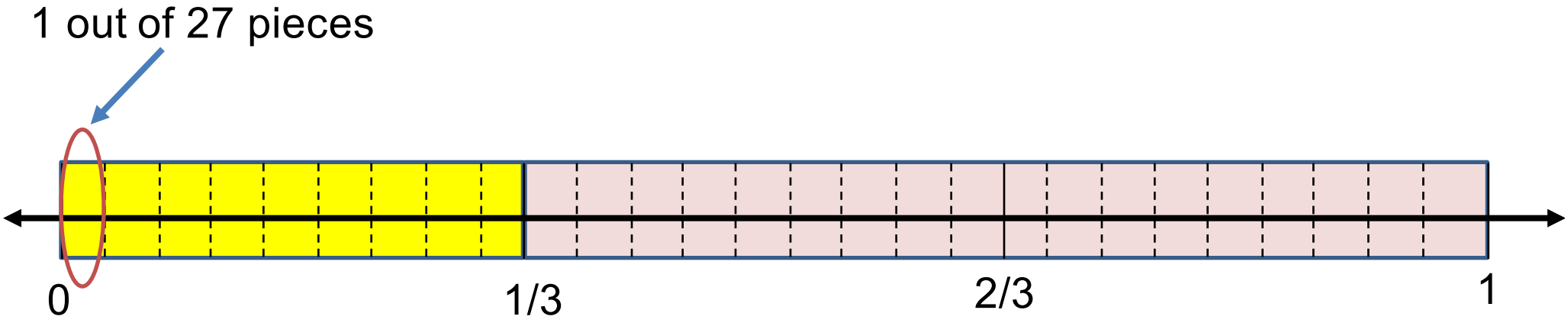


Dividing with Fractions

How can we show $\frac{1}{3} \div 9$ on a number line?

$$\frac{1}{3} \div 9 = \frac{1}{27}$$



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Closure

How does the use of the practices discussed today make learning fractional concepts equitable for all students?

Increase student engagement.

Boost student achievement.

Empower educators.

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Questions?

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Need Additional Info or Resources?

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