What Must Be True?

Building Early Understanding of Mathematical Truth in Middle School

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What is Mathematical Truth?

For mathematicians:
Truth is about the meaning of the words and the properties of the objects and operations you are given.
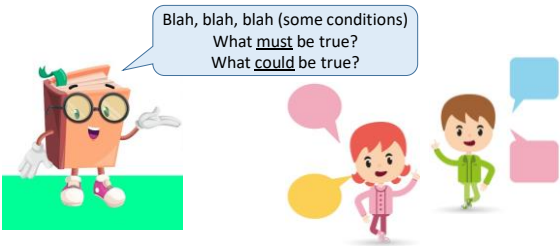
For students:
Truth is about what the teacher told them, what an older student told them, what the textbook says, what they learned last year…

Idea: Mathematical Conversation

Give students some mathematical conditions.

Ask them, "What MUST be true?"

Discuss until consensus is reached.

A "What Must Be True?" Discussion
Listening, Pondering, Talking

> Blah, blah, blah (some conditions)
> What must be true?
> What could be true?

A "What Must Be True?" Discussion
Recording, Reciting

	What Must Be True?	D = disagree A = agree Recorder: Rick
9/15	Blah, blah, blah (some conditions)	
	What must be true? What could be true?	
	Student Statements	

A "What Must Be True?" Discussion
Protocols

- Oral statements must be clearly formulated.
- Recorder writes statements *exactly* as stated.
- Try for consensus on Agree, Disagree, or Not Sure

Example 1: Integers

I have two mystery numbers. Their sum is 0.

What <u>must</u> be true?
What <u>could</u> be true?

	What Must Be True?	D = disagree A = agree Recorder: Rick
Sept 15	I have two mystery numbers. Their sum is 0.	
	What <u>must</u> be true? What <u>could</u> be true?	
	Student Statements	
	Jack: Um… one number must be negative D	
	Jonas: The answer is any pair of numbers that is equal distance from zero D	
	Melanie: Zero bisects the amount between 2 numbers A	
	Caleb: Besides 0 + 0, one of the numbers must be negative A	

Example 2: Measures of Center

I have a set of 10 data values. Their mean is 6.5.

What <u>must</u> be true?
What <u>could</u> be true?

	What Must Be True?	D = disagree A = agree Recorder: Ava
Mar 18	I have a set of 10 data values. Their mean is 6.5.	
	What <u>must</u> be true? What <u>could</u> be true?	
	Student Statements	
	The 6.5 has to be half of two numbers D	
	The total of all the values added up must be 65 A	
	The values must be rational D	
	A number could be negative or positive A	

	What Must Be True?	D = disagree A = agree Recorder: Ava
Mar 18	I have a set of 10 data values. Their mean is 6.5.	
	What <u>must</u> be true? What <u>could</u> be true?	
	Student Statements (continued)	
	All the numbers must not all be negative A	
	At least one number must be positive A	
	All of the number must have the same sign D	
	There has to be at least one number that is higher than or equal to 6.5 A	

Example 3: Rational Numbers

I have two mystery numbers. They are whole numbers and their product is a power of 10.

What <u>must</u> be true?
What <u>could</u> be true?

		D = disagree A = agree
	What Must Be True?	**Recorder: Bryce**
Mar 30	I have two mystery numbers. They are whole numbers and their product is a power of 10.	
	What must be true? What could be true?	
	Student Statements	
	The product must end in 0 A	
	Both of the numbers must be divisible by ten D	
	One of the numbers must be a factor or product of 10 D	
	The two numbers could be even A	

		D = disagree A = agree
	What Must Be True?	**Recorder: Ella**
Apr 1	I have two mystery numbers. They are whole numbers and their product is a power of 10. (Day 2)	
	What must be true? What could be true?	
	Student Statements	
	The product must start with a 1 and end in a 0 A	
	The first number must be 1 and the rest zero A	
	The prime factors of the ~~product~~ factors must be 2 and 5 A	
	There must be an infinite number of answers A	

Biggest Wins

1. Connection to the Mathematical Practices
- MP2: Reason abstractly and quantitatively
- MP3: Construct viable arguments and critique the reasoning of others
- MP6: Attend to precision
- MP7: Make use of structure

2. Students of many different levels can participate productively.

3. This is VERY EASY to prep!

Designing These for Your Classes

- Start with reasoning about numbers and operations.
- Turn an algebraic equation into a condition on one or two mystery numbers.
- Take a big concept and think about smaller pieces of that, and turn those smaller pieces into WMBT discussions
- Consider making pairs or sets of WMBT discussions – mathematical truths that illuminate one another when juxtaposed
- Consider making strings of WMBT discussions – think about all the linked mathematical truths that form a proof

Thank You!

Questions?

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