Using Classroom-Based Scenarios to Build Mathematical Knowledge for Middle and Secondary Teaching

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The Prompts

From *Facilitator's Guidebook for Use of Mathematics Situations in Professional Learning*, NCSM & IAP, 2018, pp. 9, 31, 59, 115, 137.

Situation 1. Division Involving Zero

On the first day of class, pre-service middle school teachers were asked to evaluate $\frac{2}{0}$, $\frac{0}{0}$, and $\frac{0}{2}$ and to explain their answers. There was some disagreement among their answers for $\frac{0}{0}$ (potentially 0, 1, undefined, and impossible) and quite a bit of disagreement among their explanations:

- Because any number over 0 is undefined;
- Because you cannot divide by 0;
- Because 0 cannot be in the denominator;
- Because 0 divided by anything is 0; and
- Because a number divided by itself is 1.

Situation 2. The Product of Two Negative Numbers

A question commonly asked by students in middle school and secondary mathematics classes is "Why is it that when you multiply two negative numbers together, you get a positive number?"

Situation 21. Graphing Quadratic Functions

When preparing a lesson on graphing quadratic functions, a student teacher found that the textbook for the class claimed that $x = \frac{-b}{2a}$ was the equation for the line of symmetry of a parabola $y = ax^2 + bx + c$. The student teacher wondered how this equation was derived.

Situation 35. Calculation of Sine of 32 Degrees

After completing a discussion on special right triangles (30°–60°–90° and 45°–45°–90°), the teacher showed students how to calculate the sine of various angles using a calculator.

A student then asked, "How could I calculate the sin(32°) if I do not have a calculator?"

Prompts continue on the next page.

Situation 38. Mean and Median

The following task was given to students at the end of the year in an AP Statistics class.

Consider the box plots and five-number summaries[1] for two distributions, each of which is comprised of a finite number of data values (see Figure 44.1 and Figure 44.2). Which of the distributions (Data Set 1 or Data Set 2) has the greater mean?

One student's approach to this problem was to construct what he thought were probability distributions for each data set and to compare the corresponding expected values to determine which data set had the greatest mean. The student formed four intervals using the five-number summaries and calculated the midpoint of each interval (i.e., he defined the intervals as the four quarters of the distributions, with each quarter containing 25% of the values for the distribution). Using the midpoint of each interval as the X-value of that interval, he then calculated the weighted mean for each probability distribution (see Figure 44.3). After completing his calculations, the student responded that the second data set had the larger mean.

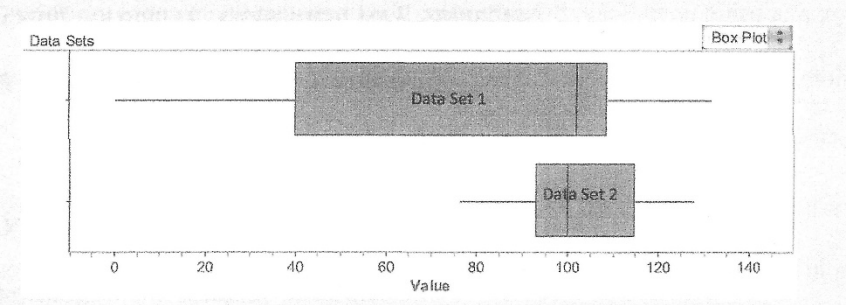

Figure 4.1. Box plots for Data Set 1 and Data Set 2.

Data Sets

	Group	
	one	two
Value	0	76
	40	93
	102	100
	109	115
	132	128

S1 = min ( )
S2 = Q1 ( )
S3 = median ( )
S4 = Q3 ( )
S5 = max ( )

Figure 44.2 Five-number summaries for Data Set 1 and Data Set 2

Data Set 1:

E(X) = 79.25

	0 – 40	40 – 102	102 – 109	109 – 132
X	20	71	105.5	120.5
P(X)	0.25	0.25	0.25	0.25

Data Set 2:

E(X) = 102.75

	76 – 93	93 – 100	100 – 115	115 – 128
X	84.5	96.5	107.5	121.5
P(X)	0.25	0.25	0.25	0.25

Figure 44.3. The student's calculations for the two data sets.

[1] Box plots are sometimes referred to as boxplots or box-and-whisker-plots. The box plot is a visual display of the five statistics values that comprise the five-number summary.

Framework for Mathematical Understanding for Secondary Teaching (MUST)

1. **Mathematical Proficiency**
   - Conceptual understanding
   - Procedural fluency
   - Strategic competence
   - Adaptive reasoning
   - Productive disposition
   - Historical and cultural knowledge

2. **Mathematical Activity**
   - Mathematical noticing
     - Structure of mathematical systems
     - Symbolic form
     - Form of an argument
     - Connect within and outside of mathematics
   - Mathematical reasoning
     - Justifying/proving
     - Reasoning when conjecturing and generalizing
     - Constraining and extending
   - Mathematical creating
     - Representing
     - Defining
     - Modifying/transforming/manipulating
   - Integrating strands of Mathematical Activity

3. **Mathematical Context of Teaching**
   - Probe mathematical ideas
   - Access and understand the mathematical thinking of learners
   - Know and use the curriculum
   - Assess the mathematical knowledge of learners
   - Reflect on the mathematics of practice

Framework for Mathematical Understanding for Secondary Teaching, from Heid, M. K., & Wilson, P. S., with Blume, G. W. (Eds.) (2015). *Mathematical understanding for secondary teaching: A framework and classroom-based situations.* Charlotte, NC: Information Age Publishing, p. 14.

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