

Leveraging students strengths with mathematical modeling in cultural/community contexts grades 3-5

Mary Q. Foote
Queens College, CUNY
Erin E. Turner
University of Arizona

Julia M. Aguirre
University of Washington, Tacoma
Amy Roth McDuffie
Washington State University

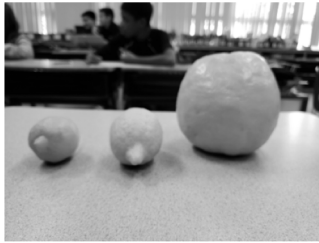


National Council of Teachers of Mathematics
Annual Meeting
San Diego - April 6, 2019

EHR: #1561274

Overview of Session

- Overview of Project
- Introduction to Math Modeling Cycle
- Modeling Routines and Tasks
 - Mathematizing the world-routines
 - Community-based Modeling Task
- Lesson Planning Tools
- Whole Group Discussion

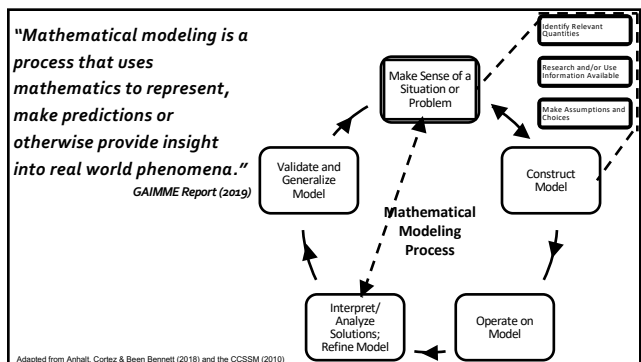


What do you notice?
What do you wonder?

What questions do you have
that you could answer using
mathematics?

"Mathematical modeling is a process that uses mathematics to represent, make predictions or otherwise provide insight into real world phenomena."

GAIMME Report (2019)



Adapted from Anhalt, Cortez & Ben Bennett (2018) and the CCSSM (2010)

Principles for our Work

- Mathematical modeling *is accessible* to children in upper elementary grades and students from a diverse range of mathematical and cultural backgrounds.

(Carlson et al., 2018; English et al., 2005; English 2014; Plumb et al., 2017; Suh et al., 2017)

- Contexts matter for supporting student learning of mathematical modeling

(Anhalt, 2014; Greer, 1997; Verschaffel, De Corte & Borghart, 1997)

- Culturally responsive, community-based approaches to teaching mathematics have added benefits, particularly for students from underrepresented groups in STEM fields

(Aguirre & Zavala, 2013; Civil, 2007; Gay, 2001; Ladson-Billings, 2009; Lipka et al., 2005; Turner et al., 2008)

Project Sites

- Pacific Northwest, Southwest
- Grades 3-5 Teachers and Math Coaches (15 per site per year, for 2 years)
- Mix of novice and veteran teachers
- Teachers attend professional development (summer and academic year) in school-based teams
- Schools serve multi-racial, multi-lingual, multicultural communities, and working class communities



Mathematizing the World Routines

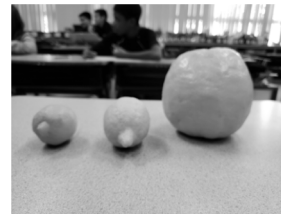


Entry Point to Mathematical Modeling for Elementary Students

- Mathematizing the World - Open Ended
- Pose an image, video, table or graph, a story, or object
 - Ask:
 - What do you notice? What do you wonder?
 - What questions do you have?
 - What do you need to answer those questions?

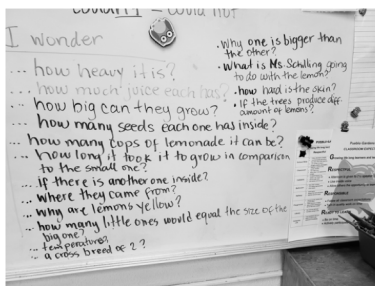
Entry Points to Mathematical Modeling for Elementary Students

- Mathematizing the World – Specific Question
 - Pose a specific, mathematical question related to image, photo or object
 - Question can come from students, or be posed by the teacher
- Discussion focuses on what one would do to answer the question:
 - What do we know? What do we need to know?
 - What can we do to figure this out?
 - What assumptions will we have to make?



It all started when Ms. A brought into the classroom two lemons and showed to the students – first a regular sized lemon that she picked from a tree in the garden, and then a giant lemon called 'ponderosa lemon' which is about the size of a grapefruit

Students' Wonders

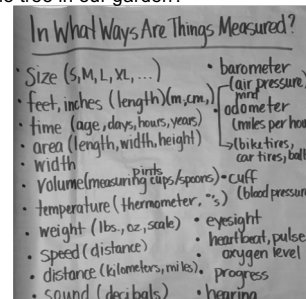


Students shared their notices and wonders based on the two objects in front of them.

"How many small ones make a big one?"

This led to a math modeling task.....

How much bigger is the ponderosa lemon compared to the lemon from the tree in our garden?

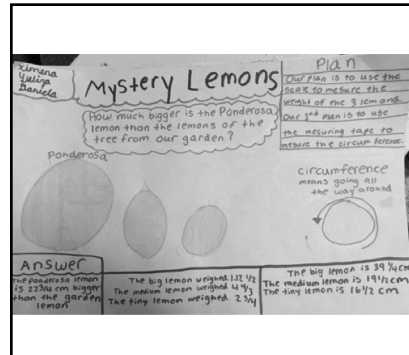


Students generated various ideas about how different things can be measured.

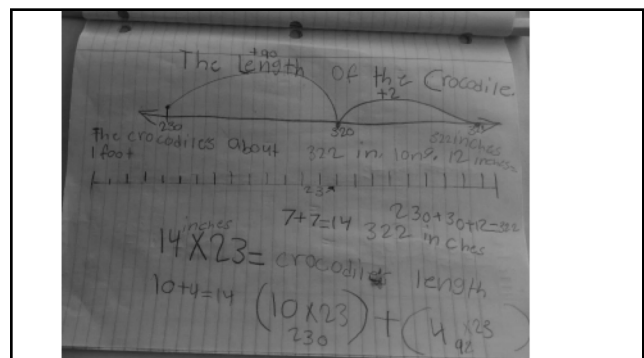
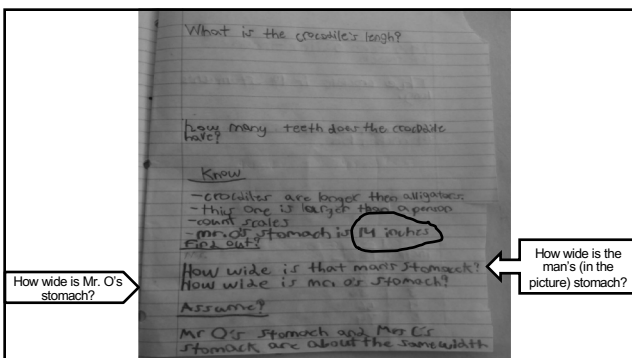
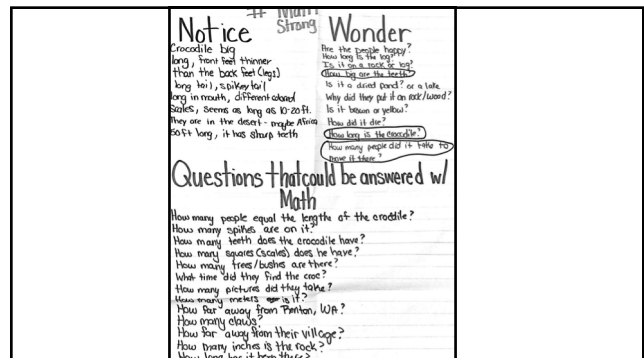
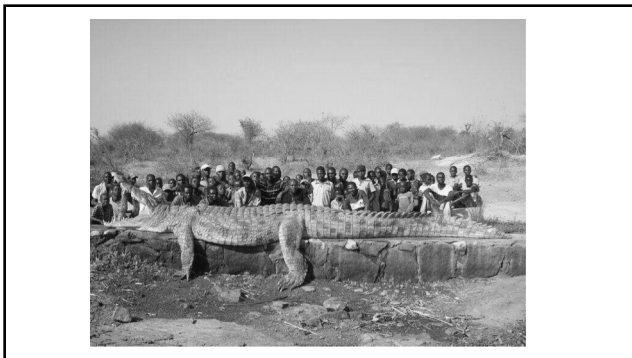


Students came up with different plans to answer the question.

They measured and compared different physical attributes of the two lemons, such as the **height**, **weight**, **circumference**, and **volume**, which one group planned to measure by squeezing the juice out of each lemon!



- This group compared the weights and the circumferences of the two lemons
- They calculated the difference in the circumference of the two lemons



Reflection Questions:

- What are the benefits of using this routine with students?
- What are images in your community you could use?
- What do you do in your classroom that draws on real world contexts that might be interesting to the students?

Modeling Task Abuelo's Birthday

Abuelo's Birthday

Discuss in Small Groups

- In what situations have you been involved in sharing costs?
Think about situations with family, friends, siblings, others?
- How do you decide how much each person pays?
- What factors impact your decisions?



Abuelo's 70th Birthday

It is Sr. Aguirre's 70th Birthday.
Four of his grandchildren want to buy him a gift.
They found a Photo Printer on sale for \$119.99.
They want to buy him the printer so he can print photos of family members.

- Alex, a 9th grader, earns between \$15 and \$20 each week from babysitting jobs.
- Sam, a 6th grader, earns \$10 each week taking care of his neighbor's pets.
- Elena, a 4th grader, earns \$5 each week doing jobs for an aunt.
- Jaden, a 1st grader, has no weekly job but has saved \$8 in her piggy bank.

One of the grandchildren says that they should split the cost of the printer among them and each pay the same amount.
Another grandchild says that it is not fair and they should each pay different amounts.

What do you think? What is *fair* in this situation?



TEAM TASK:

Help the children make a plan to share costs in a fair way to buy the gift.

Use mathematics to justify your conclusions.

Your plan should work in other situations where family members want to share costs fairly.

Mathematical Modeling

- What math concepts and practices were needed for this task?
- What funds of knowledge did you bring to this task?

M2C3 Sample student work from modeling tasks

Making Sense of the Task: Group Brainstorm

What we know: <ul style="list-style-type: none"> Printer \$120 Alex earns \$15 a week Sam earns \$10 a week Elena earns \$5 a week Jaden has \$8 Each have \$30 at end of week \$30 per person if they pay the same 	What we need to know: <p>How much will each person pay?</p>
What we need to assume: <ul style="list-style-type: none"> They will not all pay the same 	What has been assumed for us: <p>Time doesn't matter</p> <p>Alex, Sam, and Elena do not have savings</p>

They will pay 120 Dollars They will work for four weeks till they get 120 Dollars. It is fair because they are all paying for the present.

Jaden will only have to pay 3 dollars because she does not have a job like the others.

Alex will pay 60\$.
Sam will pay 40\$.
Elena will pay 17\$.
Jaden will pay 3\$.

60 + 40 = 100
100 + 20 = 120

15 + 15 + 15 + 15 = 60
10 + 10 + 10 + 10 = 40
5 + 5 + 5 + 5 = 20
8 - 5 = 3

Each sibling contributes 4 weeks of earnings, so siblings' contributions are proportional to their weekly earnings.

The youngest sibling contributes \$3 because she does not have a job.

Abuelo's Birthday 1-14-17 • Alex, Sam, Elena, Jaden

Assume

- That each of them can pay for part of the printer.
- Abuelo's birthday is in 3 weeks.
- That the older kids already had saved some money.

Math thinking

90 + 30 = 120 Dollars

Alex will pay \$45 for the printer.
Sam will pay \$30 for the printer.
Elena will pay \$15 for the printer.
Jaden will pay \$8 for the printer.

Some of the older kids may be willing to pay some of the tax already saved.

Math solution

45 + 30 + 15 + 8 = 98
120 - 98 = 22
22 / 2 = 11
11 + 8 = 19
19 + 3 = 22

Each sibling contributes 3 weeks of earnings.

This is not enough money, so the older siblings each contribute more from their savings, with the siblings who earn more contributing more.

The youngest sibling contributes all her savings.

Abuelo's Birthday Present Plan

Assumptions

- They might get it at a store.
- Jaden is going to school.
- Sam has all saved up.
- Elena has all saved up.
- Weeks until Abuelo's birthday.

Mathematical Thinking

30 + 30 + 10 = 70
120 - 70 = 50
50 + 20 = 70
70 + 50 = 120

Each sibling contributes 2 weeks of earnings, plus the same amount from their savings.

The youngest sibling contributes half of her savings (\$4 for tax).

We think it is fair because each person is spending 2 weeks of their allowance and Jaden is spending \$4 which is half of her allowance.

Mathematical Thinking that they had before. How you can use it in a different situation is in real life. Like if it's somebody's birthday and you want to get the same gift as the other person, they can use the same strategy.

Math thinking

4 Weeks is all about 4 weeks
Alex - 2 weeks - 40
Sam - 4 weeks - 40
Elena - 8 weeks - 40
Little kid - 0 weeks - \$4 tax

Math solution

40 x 3 = 120.00
120.00 - 119.99 = 1.01
120 / 3 = 40

Older siblings each contribute the same total amount (\$40), but they each need to save their earnings for different amounts of time (2 weeks, 4 weeks, 8 weeks).

But then students added the weeks (2+4+8=14), not considering that money is earned concurrently.

The youngest sibling contributes \$4 for tax.

Each kid except the first grader gets 40 dollars to help pay for the printer.

Abuelos' Birthday Examination of Student Work

What aspects of modeling were evident in the students' work?

Mathematical Modeling Process

```

graph TD
    A[Identify Relevant Quantities] --> B[Make Sense of a Situation or Problem]
    B --> C[Construct Model]
    C --> D[Operate on Model]
    D --> E[Interpret/Analyze Solutions; Refine Model]
    E --> F[Validate and Generalize Model]
    F --> B
    B -.-> G[Research and/or Use Information Available]
    G -.-> B
    B -.-> H[Make Assumptions and Choices]
    H -.-> B
  
```

Adapted from Anghel, Cortez & Beem Bennett (2018) and the CGSSM (2019)

M2C3

Task Design Tools & Resources

<https://sites.google.com/qc.cuny.edu/m2c3/>

Types of Mathematical Modeling Tasks

DESCRIPTIVE MODELING	PREDICTIVE MODELING
Students are provided with information about a particular scenario, and use math modeling to describe possible outcomes. Possible outcomes depend on assumptions and/or constraints.	Students use math modeling to analyze relationships or trends in a dataset (e.g., rates of increase or decrease over time) to predict additional values or outcomes.
Contexts or Questions: <ul style="list-style-type: none"> How many school buses are needed? How long can this snack last? How much can we earn by selling ____? How much water can we save? How many ____ do we need for ____? 	Contexts or Questions: <ul style="list-style-type: none"> Predict future number of attendees Predict future prices or sales Predict future number Predict future success of athletes Predict future yield (crops, gardens)
Descriptive Modeling with CLAIM probe: Students are provided with a claim about expected outcomes and asked to evaluate whether and under what conditions the claim could be true.	Predictive Modeling with CLAIM probe: Students are provided with a claim about trends, patterns, or future values, and asked to evaluate whether and under what conditions the claim could be true.
OPTIMIZING MODELING	RATING & RANKING MODELING
Students use math modeling to find the "best" option or plan to achieve a specific goal. What is "best" depends on the goal (e.g., cheapest, fastest, cheapest, fastest, longest, smallest).	Students use math modeling to rate and rank different options based on criteria and data. Students decide how to weight criteria and use their ranking to make a selection or selection.
Contexts or Questions: <ul style="list-style-type: none"> The "best" route through a theme park The "best" arrangement for a garden The "best" way to share items The "best" price for a items item The "best" way to package an item 	Contexts or Questions: <ul style="list-style-type: none"> Select players for a team Select a field trip or vacation spot Select a fundraising option Select a carnival game Select a phone or internet plan
Optimizing Model with CLAIM probe: Students are presented with a claim about the "best" option, and asked to evaluate whether the proposed option is the "best" given the goal.	Rating & Ranking Model with CLAIM probe: Students are presented with a claim about the top ranked option, and asked to evaluate whether the ranking criteria are reasonable.
IN ALL MATH MODELING TASKS	
Students generate a plan, communicate, and justify a math model. All plans should: <ul style="list-style-type: none"> Show how the plan/communication works in the specific scenario Describe assumptions, and how those assumptions impact plan or conclusion. Use numbers, words, equations and/or diagrams to explain and justify conclusions. Describe how one would use the plan in other similar situations 	

DESCRIPTIVE MODELING
Students are provided with information about a particular scenario, and use math modeling to describe possible outcomes. Possible outcomes depend on assumptions and/or constraints.

Contexts or Questions:

- How many school buses are needed?
- How long can this snack last?
- How much can we earn by selling ____?
- How much water can we save?
- How many ____ do we need for ____?

Descriptive Modeling with CLAIM probe: Students are provided with a claim about expected outcomes and asked to evaluate whether and under what conditions the claim could be true.

Claim probe used frequently in the project

This tool was adapted from/informed by a tool created in the Immersion Project

Traditional Word Problems v. Mathematical Modeling Tasks

Traditional Word Problems	Modeling Problems
Tend to be closed problems	Tend to be open-ended problems
Context is often contrived	Real-life context less contrived
Solutions have defined algorithms	No particular algorithm is defined
All parameters given	Some parameters given, some are assumed
Structure is given	Less structure is given
Assumptions are usually not necessary to solve the problem	Assumptions are part of the process for solving the problem
Usually there is an exact (single) solution	Approximate (and multiple) solutions are acceptable

(Anhalt, 9/2014). Scaffolding in mathematical modeling for ELLs. In *The Common Core State Standards in Mathematics for English Language Learners: Grades K-8*, edited by Mathy, Cook, and Ego-Turner, pp. 111-126. Alexandria, VA: Teachers of English to Speakers of Other Languages.


How do you think these types of modeling tasks might benefit students?

Teacher Reported Benefits of MM

- High Engagement**
 - Especially for students who normally do not participate in math book problems
- Built math stamina and perseverance**
- State testing benefits**
 - Tasks were similar to performance based items
 - Removed students' fear from engaging in performance based items

What we've noticed in other situations that our kids don't have a lot of background knowledge about, they don't have a wide range of background knowledge that connects to a lot of the math that we've seen in other curriculums. So it felt really impactful. It was just an eye-opening to see that they really do have a lot of experience and background knowledge, but we just need to create tasks that elicit that. And then their engagement was way higher and they were very knowledgeable. But maybe not about going skiing on a ski slope, or something. They haven't done that before, but they've played soccer and they have a grandmother, an abuela, whose birthday they celebrated... You just have to, I guess, pay attention to what kids know and what their experiences are.

End of year interview 4th grade teacher



M2C3 Acknowledgements

With Sincere Gratitude

We express our sincere gratitude to our teacher-colleagues and the students in their classrooms.
Without their participation and contributions, our work would not be possible.

Project Members


Erin Turner*, Cynthia Oropesa Anhalt, Marta Civil, Mei-Kuang Chen, Nicole Kersting, Amy Been Bennett, Monica Granillo, Nishaan Ponnuru, University of Arizona
Julia Aguirre*, Erin Caril, Elzena McVicar, University of Washington, Tacoma
Amy Roth McDuffie*, Candace Chappelle, Ryan Seidel, Washington State University
Mary Q. Foote*, Beverly Smith, City University of New York
 *Project PI's

Former Project Members

Christine Leibbrand, Mahtab Nazemi

We thank the National Science Foundation for its support of the project entitled *Mathematical Modeling With Cultural and Community Contexts (M2C3)*, NSF EHR Award #1561274 and the collaboration and support of our team.

Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of NSF.



EHR: #1561274

Project Website (under construction):

<https://sites.google.com/qc.cuny.edu/m2c3/home>