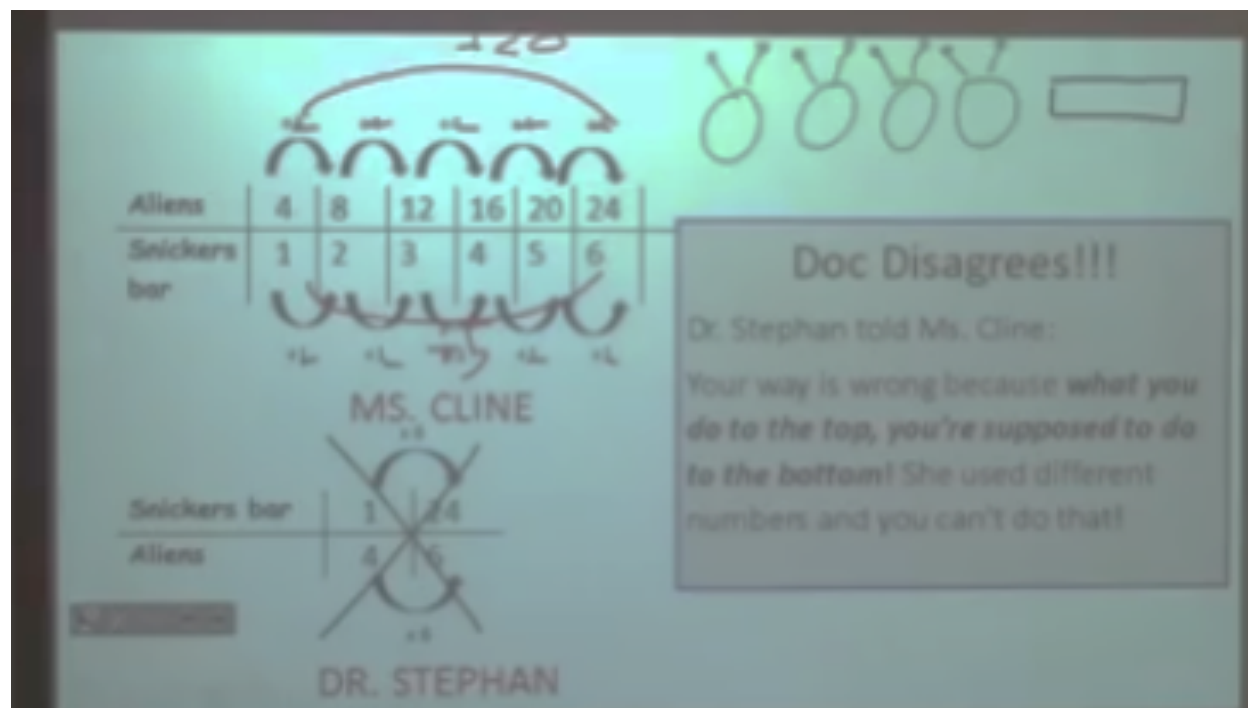


Aliens: establishing meaning for calculations



1. Cline: Natalie had that "Oh!" moment. What do you notice?
2. Natalie: That 4, if you jump, it's like 4×6 . And on the bottom it's 1×6 [Natalie draws arrows and $\times 6$ on bottom and top of ratio table]. Right here is times six and here is times six.
3. Cline: Alright, so let's think about all this math we've just seen. Alright so Thomas saw we started with four. We started with 4 and he said we added 4 and then 4 and then 4 and 4 and 4 to get 24. And he sees that as adding 20. Do you guys see that as adding 20?
4. Students: Yeah.
5. Cline: OK, down here he said we started with 1, we added 1, we added 1, we added 1. And he said we added a total of 5. Do you guys see those 5?
6. Students: Yeah.
7. Cline: But then Natalie comes up here and she's like, "oh, you can take 4×6 to get 24. And you can take 1×6 and get 6.
8. Student: Oh!
9. Student: I see what they're doing.
10. Cline: So, we're getting the same thing right?

11. Students: Yeah [and talk] You're talking about simplicity. You say you've got it. Let's listen to Michael and see if we agree.
12. Michael: Like I see, they just started like sort of lazy. Instead of adding all of those numbers, like add up, like into a shorter, like into a small table instead of making a huge table.
13. Stephan: Did you call me lazy? (smiling)
14. Michael: No.
15. Cline: Yes. [laughter] Sam, were you gonna add to that or were you just stretching?
16. Sam: I was just stretching.
17. Cline: So, I think what I heard you saying is I was doing +4, +4, +4 over and over. And you said Dr. Stephan was lazy, so instead of doing +4 she just multiplied by x6. [Thomas has his hand raised]. I'm gonna let you and then I have a question for y'all.
18. Thomas: There's also 6 numbers [underlines each of the numbers in the top of the ratio table].
19. Cline: Oh.
20. Thomas: If you were to have 5 numbers, you would multiply by 5. Four times five is twenty and one times five is five. One times five is five.
21. Cline: OK. Woooo! So, what in the world does this times 6 represent?
22. Thomas: The $4 + 4 + 4 \dots$
23. Cline: How many +4s do we see? [holds up her hand to show 5].
24. Students: Five!
25. Cline: There's one hidden somewhere, evidently.
26. Student: It's the one upfront.
27. Cline: He says *this* [circles the 4/1 in the ratio table with her finger] is one of our groups of 4 and one of our groups of 1. So, that's our first one, isn't it? And this is our second one [circles the 2/8 with finger], this is our...
28. Student: third, fourth, fifth, sixth.
29. Cline: So, how many groups of 4 aliens and 1 snickers bar are there?
30. Student: Six.
31. Cline: Alright, I'm a picture person and I want to go back and think about yesterday. Here's four aliens and one snickers bar [draws 4 dots and one line]. That's right here, isn't it [points to 4/1 on table]? And according to...ok, so that's this column right here, isn't it? [circles 4/1 with marker]. I simply added another one [draws 4 more dots and 1 line]. That's the second column....here, you be my column circler [hands marker to Ellie]. So, do I have two snickers and 4 or 8 aliens?

32. Students: Yes.

33. Cline: Alright, what if I did it again [draws four more dots and one line while Ellie circles $12/3$ on table]. So, Michael, let's call Dr. Stephan something other than lazy. Let's call her efficient. If she wanted to go from here to here [4 dots 1 line to 12 dots, 3 lines], what would she have multiplied by?

34. Students: 3.

35. Cline: By 3? Because isn't one times 3, these three? Oh, gentlemen, you're missing out. And four times 3 is 12 and that's what we see in our table?



36. Students: Yes.

37. Cline: Would I keep doing that? Alright, here's our, what number group is this [draws four more dots and one more line]?

38. Students: Four. [Ellie circles fourth column]

39. Cline: What's this one [draws another set of four dots and 1 line]?

40. Students: Five [Ellie circles fifth column].

41. Cline: [draws one more set] Six. [Ellie circles the sixth column]. If I wanted you to count these dots, would you guys go 1, 2, 3, 4, 5, 6, 7, 8?

42. Students: No!

43. Cline: No. What would you probably do?

44. Students: [gesturing] count the rows and the dots.



45. Cline: So, how many's in the first row?

46. Students: 4.

47. Cline: How many rows?

48. Students: 6.

49. Cline: What's 6×4 ?

50. Students: 24.

51. Cline: Who else did 6×4 ?

52. Student: Natalie.

53. Cline: Yeah! And who else really did 6×4 ?

54. Students: Dr. Stephan

55. Cline: OK.