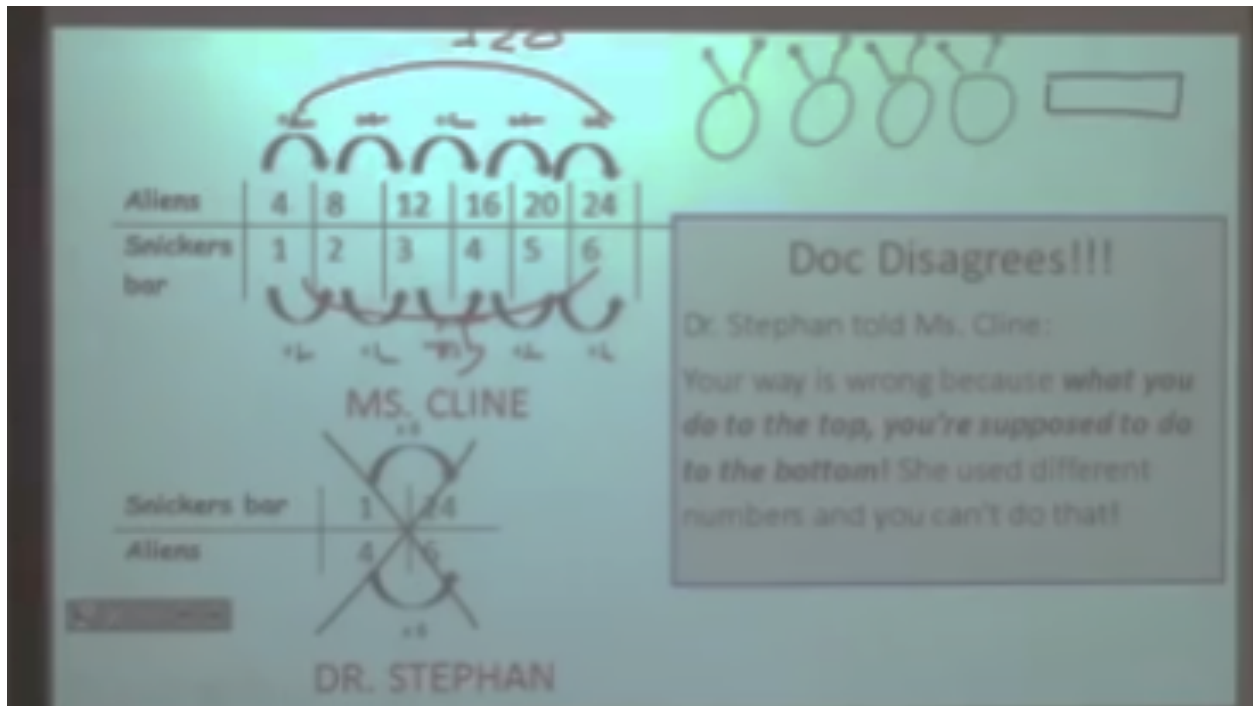Aliens: establishing meaning for calculations

Cline: Natalie had that "Oh!" moment. What do you notice?

Natalie: That 4, if you jump, it's like 4 x 6. And on the bottom it's 1 x 6 [Natalie draws arrows and x 6 on bottom and top of ratio table]. Right here is times six and here is times six.

Cline: Alright, so let's think about all this math we've just seen. Alright so Thomas saw we started with four. We started with 4 and he said we added 4 and then 4 and then 4 and 4 and 4 to get 24. And he sees that as adding 20. Do you guys see that as adding 20?

Students: Yeah.

Cline: OK, down here he said we started with 1, we added 1, we added 1, we added 1. And he said we added a total of 5. Do you guys see those 5?

Students: Yeah.

Cline: But then Natalie comes up here and she's like, "oh, you can take 4 x 6 to get 24. And you can take 1 x 6 and get 6.

Student: Oh!

Student: I see what they're doing.

Cline: So, we're getting the same thing right?

Students:	Yeah [and talk] You're talking about simplicity. You say you've got it. Let's listen to Michael and see if we agree.
Michael:	Like I see, they just started like sort of lazy. Instead of adding all of those numbers, like add up, like into a shorter, like into a small table instead of making a huge table.
Stephan:	Did you call me lazy? (smiling)
Michael:	No.
Cline:	Yes. [laughter] Sam, were you gonna add to that or were you just stretching?
Sam:	I was just stretching.
Cline:	So, I think what I heard you saying is I was doing +4, +4, +4 over and over. And you said Dr. Stephan was lazy, so instead of doing +4 she just multiplied by x6. [Thomas has his hand raised]. I'm gonna let you and then I have a question for y'all.
Thomas:	There's also 6 numbers [underlines each of the numbers in the top of the ratio table].
Cline:	Oh.
Thomas:	If you were to have 5 numbers, you would multiply by 5. Four times five is twenty and one times five is five. One times five is five.
Cline:	OK. Woooo! So, what in the world does this times 6 represent?
Thomas:	The 4 + 4 + 4…
Cline:	How many +4s do we see? [holds up her hand to show 5].
Students:	Five!
Cline:	There's one hidden somewhere, evidently.
Student:	It's the one upfront.
Cline:	He says *this* [circles the 4/1 in the ratio table with her finger] is one of our groups of 4 and one of our groups of 1. So, that's our first one, isn't it? And this is our second one [circles the 2/8 with finger], this is our…
Student:	third, fourth, fifth, sixth.
Cline:	So, how many groups of 4 aliens and 1 snickers bar are there?
Student:	Six.
Cline:	Alright, I'm a picture person and I want to go back and think about yesterday. Here's four aliens and one snickers bar [draws 4 dots and one line]. That's right here, isn't it [points to 4/1 on table]? And according to…ok, so that's this column right here, isn't it? [circles 4/1 with marker]. I simply added another one [draws 4 more dots and 1 line]. That's the second column….here, you be my column circler [hands marker to Ellie]. So, do I have two snickers and 4 or 8 aliens?

Students: Yes.

Cline: Alright, what if I did it again [draws four more dots and one line while Ellie circles 12/3 on table]. So, Michael, let's call Dr. Stephan something other than lazy. Let's call her efficient. If she wanted to go from here to here [4 dots 1 line to 12 dots, 3 lines], what would she have multiplied by?

Students: 3.

Cline: By 3? Because isn't one times 3, these three? Oh, gentlemen, you're missing out. And four times 3 is 12 and that's what we see in our table?

Students: Yes.

Cline: Would I keep doing that? Alright, here's our, what number group is this [draws four more dots and one more line]?

Students: Four. [Ellie circles fourth column]

Cline: What's this one [draws another set of four dots and 1 line]?

Students: Five [Ellie circles fifth column].

Cline: [draws one more set] Six. [Ellie circles the sixth column]. If I wanted you to count these dots, would you guys go 1, 2, 3, 4, 5, 6, 7, 8?

Students: No!

Cline: No. What would you probably do?

Students: [gesturing] count the rows and the dots.

Cline: So, how many's in the first row?

Students: 4.

Cline: How many rows?

Students: 6.

Cline: What's 6x4?

Students: 24.

Cline: Who else did 6x4?

Student: Natalie.

Cline: Yeah! And who else really did 6x4?

Students: Dr. Stephan

Cline: OK.