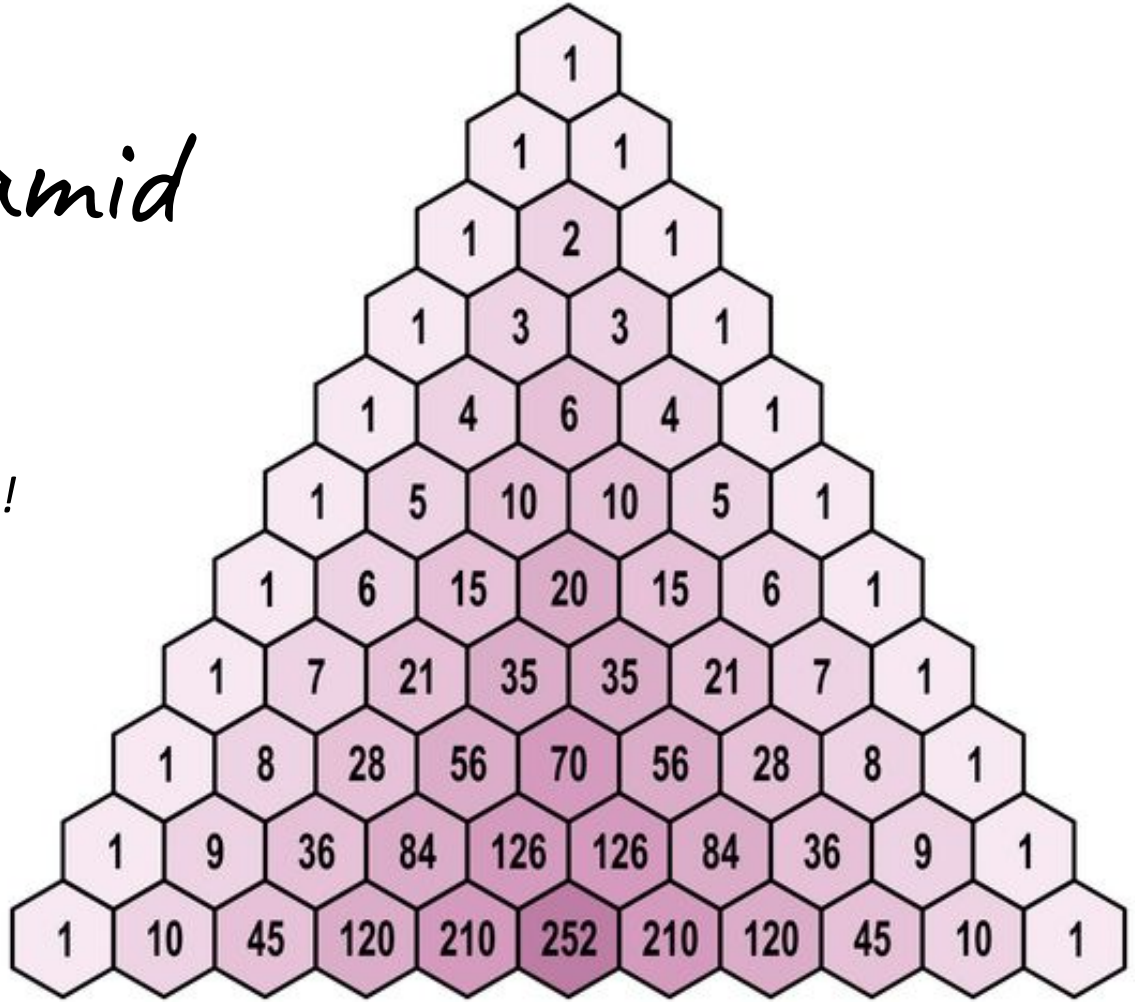
Pascal's Pyramid

Meet Pascal's Triangle's
equally interesting
yet often overlooked cousin!

Do & Think Now:

What would a 3-D
Pascal's Triangle look like?

NCTM 2019



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Oh … and I love math! *mathnunnies@gmail.com*

Pascal's Pyramid

Materials *(for a "7"-layer pyramid):*

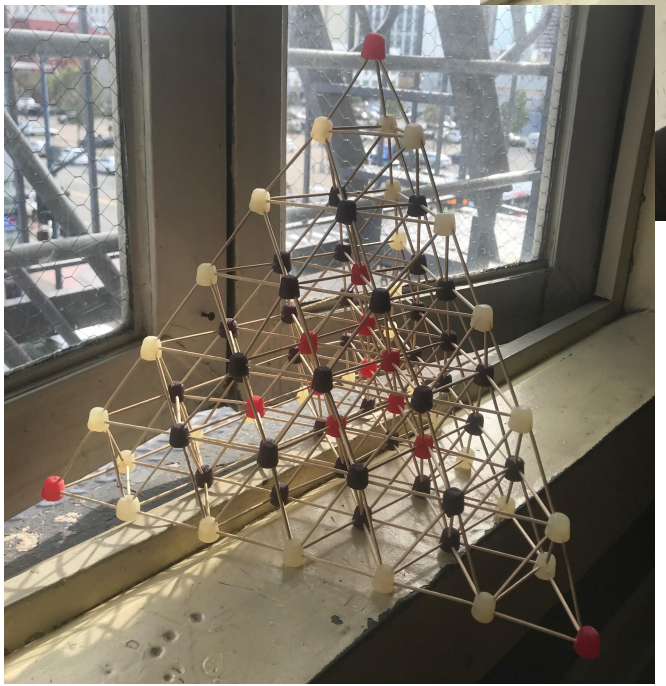
- ➔ *84 gumdrops*
- ➔ *336 toothpicks*

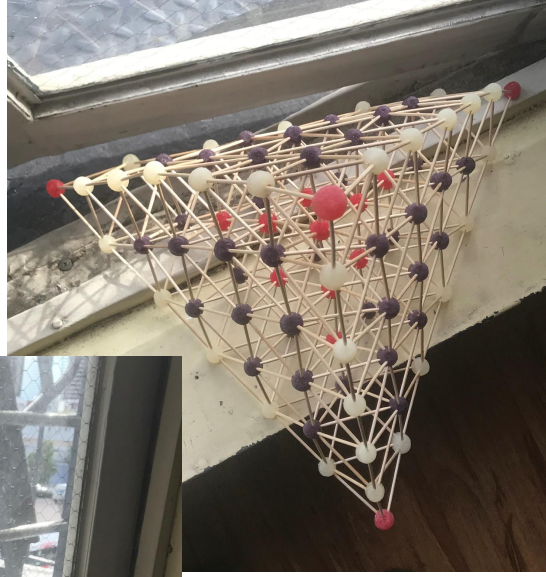
Assessments:

- ➔ *Construction*
- ➔ *20 Questions*

Math Practices:

- ➔ *Problem Solving & Persevering*
- ➔ *Reason Abstractly & Quantitatively*
- ➔ *Patterning & Attending to Precision*





The Amazon

Spice Drops Candy old fashioned gum drops jelly candy 7 pounds bag by SweetGourmet

★★★☆☆ 234 customer reviews | 16 answered questions




Note: Not eligible for Amazon Prime.

In Stock. Ships from and sold by SweetGourmet.

6 Sizes: 7Lb

1Lb	1.5 Lb	7Lb
$7.99 Prime	$10.03 Prime ($6.69 / pound)	$21.99

Get it as soon as April 12 - 18 when you choose **Standar** at checkout.

Deliver to Alison - Homer 99603

Qty: 1 Turn on 1-click ord

Add to Cart

Buy Now

Serve Clean Round Toothpicks, 5000 Count

by Serve Clean

★★★★½ 24 customer reviews | 6 answered questions

Amazon's Choice for "toothpicks bulk"

Price: $15.99 prime

Thank you for being a Prime member. Get $70 off instantly: Pay $0.00 upon approval for the Amazon Prime Rewards Visa Card.

Size: **5000 count**

1600 count	5000 count
$7.99 prime	$15.99 prime

Pascal's Pyramid Assessment - Part 1

Please work as a team to construct a neat, complete and timely product.

Neatness: + _____ /5 points

Completeness: + _____ /5 points

Timeliness: + _____ /5 points

Group Effort: + _____ /5 points

TOTAL: = _____ /20 points

Pascal's Pyramid Assessment - Part 2

Pascal's Pyramid Assessment – Part II Good luck to _____

Assume a perfectly built 7-layer pyramid for all questions unless otherwise stated.

A. GUMDROPS (Hint: Go layer by layer. Look for a pattern.)

1. How many gumdrops are in your pyramid? _____
2. How many gumdrops would you need for an 8-layer pyramid? _____
3. How many gumdrops would you need for a 10-layer pyramid?_____
4. How many gumdrops would you need for an n-layer pyramid?_____ *
5. How many gumdrops have 12 toothpicks in them?_____
6. How many gumdrops have 9 toothpicks in them?_____
7. How many gumdrops have 6 toothpicks in them?_____
8. How many gumdrops have 3 toothpicks in them?_____
9. How many gumdrops are located entirely in the middle? _____

B. PYRAMID

10. How many faces does your pyramid have? _____
11. How many edges does your pyramid have? _____
12. How many corners (vertices) does your pyramid have? _____
13. Are the answers for 10-12 the same for an 8-layer pyramid? _____

C. TOOTHPICKS (Hint: Try to do it a couple of ways. Discuss it with one another. Make sure you're not missing any!)

14. How many toothpicks are in your pyramid?_____
15. How many toothpicks would you need for a 10-layer pyramid?_____
16. How many toothpicks would you need for a n-layer pyramid?_____ *

D. TRIANGLES (Hint: Be systematic and orderly.)

17. Three gumdrops and three toothpicks form a small equilateral triangle. How many of these small equilateral triangles are in your pyramid? _____

E. SMALL PYRAMIDS

18. Four gumdrops and six toothpicks form a small pyramid (like the top two layers). How many small pyramids are in your 7-layer pyramid? _____

F. OTHER SHAPES

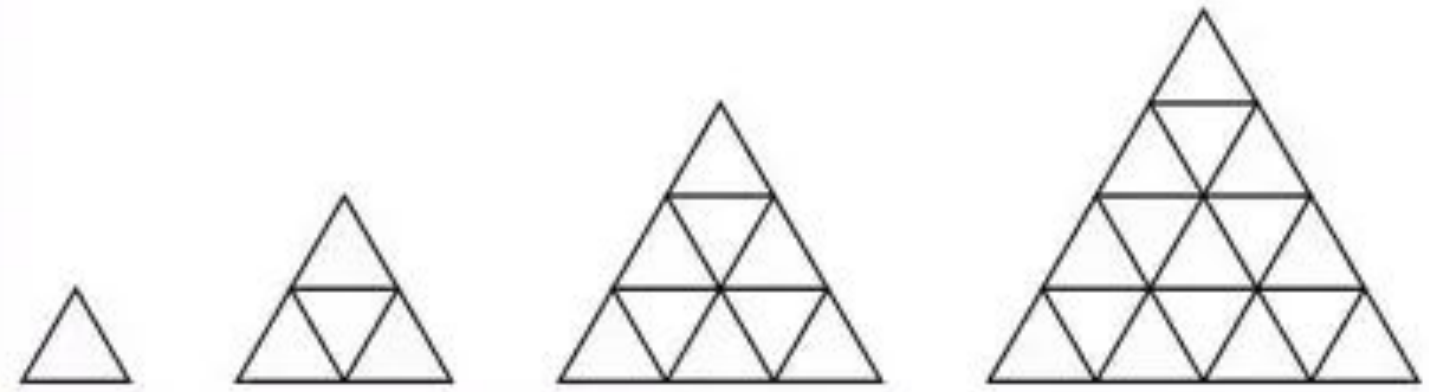
19. Name at least six other shapes (besides triangle and pyramids) that you see in your pyramid. Draw a sketch of each one.

G. CHALLENGE PROBLEM

20. How many different triangles of all different sizes are in your pyramid?

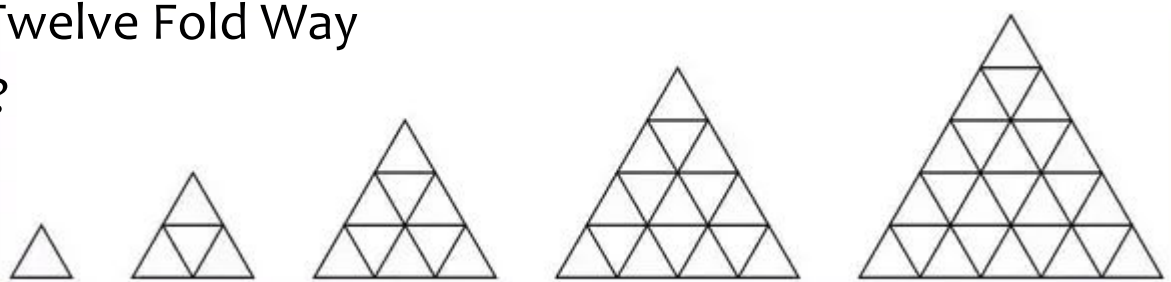
"5"-Layer (n=4)

Pascal's Pyramid - Construction



Where's the math?

- Pascal's Pyramid is to trinomial expansion as Pascal's Triangle is to binomial expansion.
- Counting, patterning, multiple solution paths.
- Finding recursive and explicit formulas for an n-layer pyramid.
- Vocabulary: pyramid, faces, edges, vertices, triangles, squares, tetrahedron, octahedron, trapezoid, equilateral, hexagon (concave and convex), figurative numbers.
- Discrete Math: The Twelve Fold Way
- How many triangles?



Trinomial Expansion

$$(x+y+z)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4 + 4y^3z + 6y^2z^2 + 4yz^3 + 1z^4 + 4xz^3 + 6x^2z^2 + 4x^3z + 12x^2yz + 12xy^2z + 12xyz^2$$

Layer 0: 1

Layer 1: 1 1

Layer 2: 1 2 2 1

Layer 3: 1 3 3 3 6 3 1 3 3 1

Layer 4: 1 4 4 6 12 6 4 12 12 4 1 4 6 4 1

Layer 5: 1 5 5 10 20 10 10 30 30 10 5 20 30 20 5 1 5 10 10 5 1

http://wfhsmathclub.blogspot.com/2012/05/pascals-pyramid.html

Counting, Patterning, Multiple Solution Paths

How did you figure the number of toothpicks?

Counting?

6 picks per small pyramid?

Multiply the number of gumdrops by 3, 6, 9 or 12 then divide by two?

Finding Recursive and Explicit Formulas
Problems #4, #16, et. al., for n=6

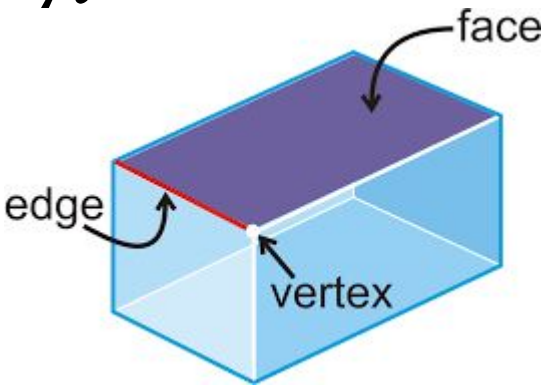
$$\frac{1}{6}n(n+1)(n+2)$$

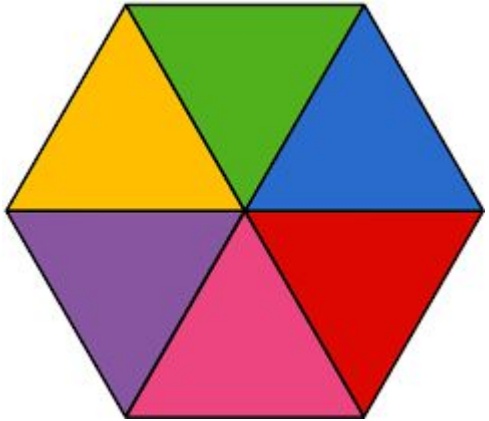
$$T_n = \sum_{k=1}^{n} \frac{k(k+1)}{2}$$

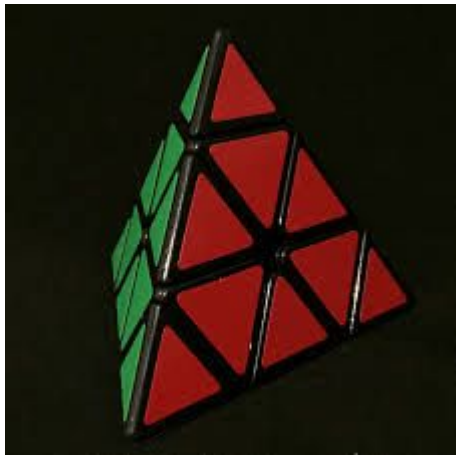
$$a_0 = 1$$

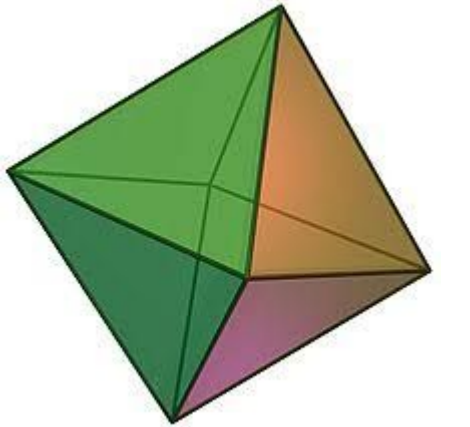
$$a_n = a_{n-1} + \frac{(n+1)(n+2)}{2}$$

Where's the vocabulary?







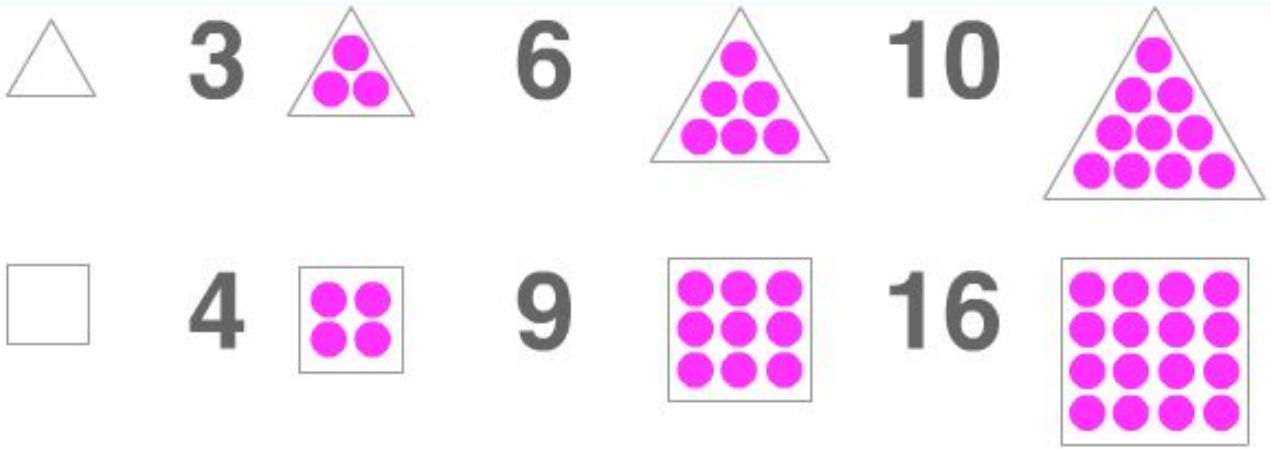


Figurate Numbers

figurate numbers

Numbers that can be represented by a geometrically shaped arrangement of equally spaced points.

examples



Discrete Mathematics

What is discrete mathematics?

- ➔ study of mathematical structures that are fundamentally discrete rather than continuous
- ➔ discrete = individually separate and distinct

Twelve Fold Way

The Twelve-Fold Way is a unified model for counting permutations, combinations and partitions.

Count the number of ways to place a collection X of $m \geq 1$ cubes into a collection Y of $n \geq 1$ cups, with the following options:

- ➔ *The **cubes** are either distinguishable (labeled) or indistinguishable (unlabeled).*
- ➔ *The **placement** in the **cups** is either unrestricted, injective (one-to-one) or surjective (onto).*

The Cubes

- Labeled (distinguishable)

- Unlabeled (indistinguishable)

The Cups

- Labeled

- Unlabeled

The Placements

Unrestricted Placement

- Any number of cubes can be placed in any of the cups.
- *Anything goes!*

Injective (one-to-one) Placement

- No sharing of cubes

Surjective (onto) Placement

- No empty cups

Twelve Fold Way

- 2 choices for cubes
- 2 choices for cups
- 3 placement choices

= 2 • 2 • 3 = **12 folds**

The First Fold

- Set X: m labeled cubes
- Set Y: n labeled cups
- Placement: unrestricted

How many possible ways are there for placement?

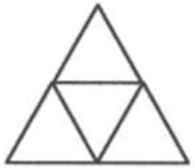
	1	**2**	**3**	**4**	**5**	**...**	***n***
1	1	2	3	4	5		n
2	1	4	9	16	25		n^2
3	1	8	27	64	125		n^3
4	1	16	81	256	1024		n^4
5	1	32	243	1024	3125		n^5
...							
m	1	2^m	3^m	4^m	5^m		n^m

How many triangles? (#20)

How many triangles are in each of the figures shown?

1.

2.



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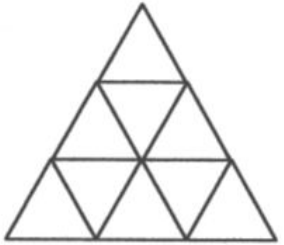

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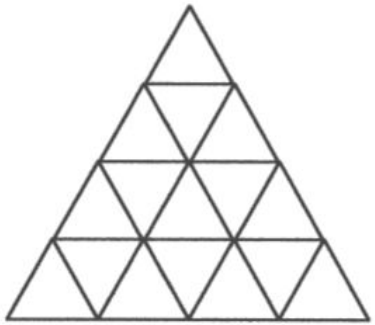

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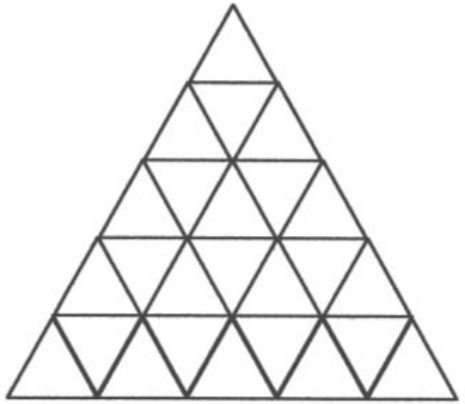

STEM lessons …

1. … focus on real-world issues and problems.
2. … are guided by the engineering design process.
3. … immerse students in hands-on inquiry and open-ended exploration.
4. … involve students in productive teamwork.
5. … lessons apply rigorous math and science content your students are learning.
6. … allow for multiple right answers and reframe failure as a necessary part of learning.

https://www.edweek.org/tm/articles/2014/06/17/ctq_jolly_stem.html

8TH ANNUAL STEM Forum & Expo

HOSTED BY NSTA National Science Teachers Association

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