Fear No Fraction Workshop

NCTM Annul Conference, San Diego Convention Center

Saturday 4/6/19

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Agenda

8:00 Introductions & calibration of the workshop content

8:05 Workshop:

Foundations (35 minutes)

Multiple meanings of division (10 minutes)

Visualizing “Flip and Multiply” (10 minutes)

Creating division stories (5 minutes)

9:05 Questions

Summaries

Next steps

Workshop Content

1. Full disclosure: Rational numbers and their representations.

Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal for research in mathematics education*, 122-147.

Vamvakoussi, X., & Vosniadou, S. (2010). How many decimals are there between two fractions? Aspects of secondary school students' understanding of rational numbers and their notation. *Cognition and instruction*, *28*(2), 181-209.

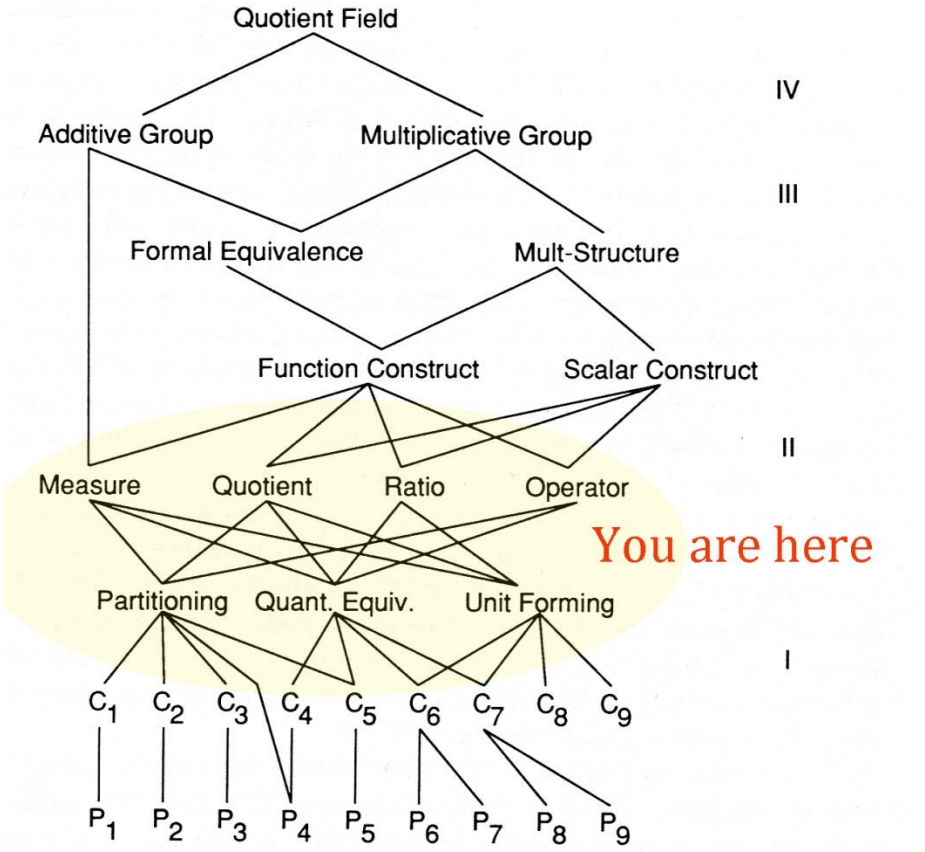
This workshop focuses on the fraction representation: $\frac{a}{b}$

2. What is a fraction?

3. A warm up question:

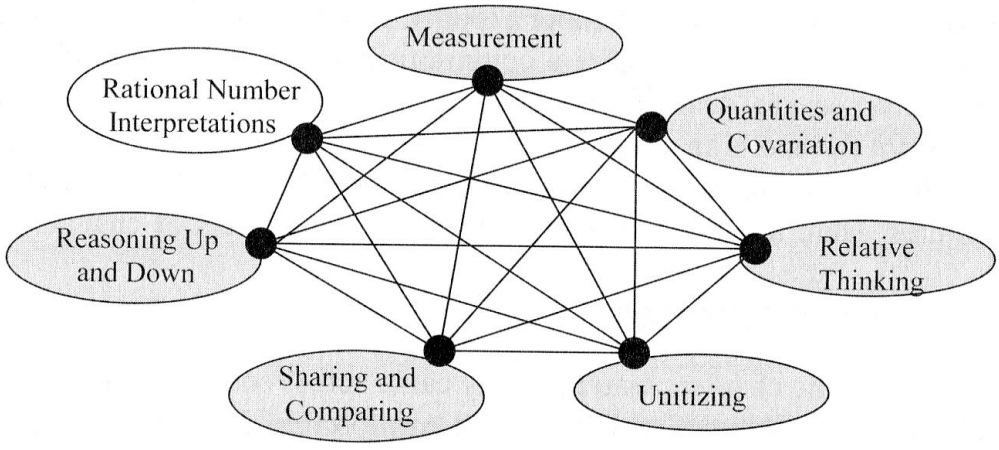
Ciosek, M., & Samborska, M. (2016). A false belief about fractions–What is its source?. *The Journal of Mathematical Behavior*, *42*, 20-32.

4. The first problem with fractions:



Kieren, T. E. (1993). Rational and fractional numbers: From quotient fields to recursive understanding. *Rational numbers: An integration of research*, 49-84.

5.



Lamon, S. J. (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. Routledge.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. *Second handbook of research on mathematics teaching and learning*, *1*, 629-667.

6. Discuss:

~ Fractions are bipartite (ontologically different) and multiplicative (not additive).

~ The problem with part-whole models.

~ The problem with repeated addition/subtraction.

Practice:

a. One load of laundry takes 1 ¼ cups of detergent. How many loads can you do with a box containing 80 cups?

b. Six people do a job in four days. How long will the job take with eight people?

c. Name and model the two types of proportion.

d. * From Lewis and Carroll: If 6 cats can catch 6 rats in 6 minutes, how many cats are needed to catch 100 rats in 50 minutes?

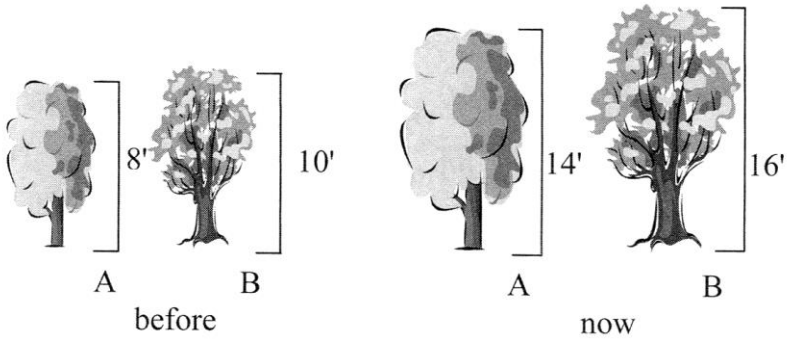
e. * What's the ratio of men to women in a community where 2/3 of the men are married to 3/4 of the women?

f. * A coffee shop uses two types of beans in their House Blend. One bean costs $8.00/lb and the other costs $14.00/lb. A 50-pound batch costs $10/lb. How many pounds of each bean are in the blend?

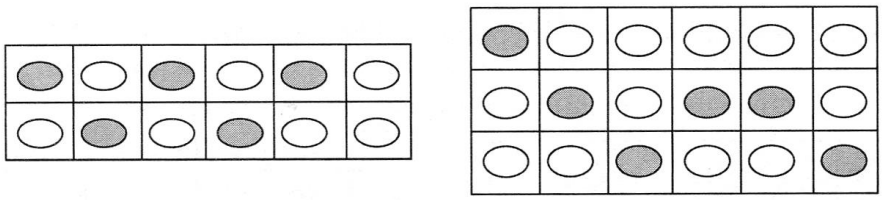
7. Draw 3/8 of a pizza.

8. Relative thinking is multiplicative; absolute thinking is additive.

a. Which tree grew more?

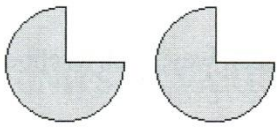


b. Which carton has more brown eggs?

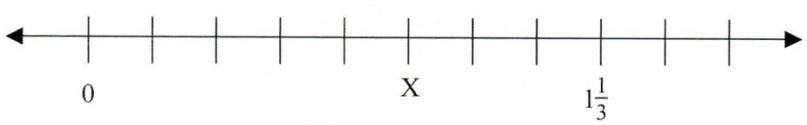


9. Core practice: Reasoning up and down.

a. How many lady bugs are on my tree if this picture shows 4/5 of them?

b.  is $\frac{3}{4}$ of something. How much is $1\frac{1}{2}$?

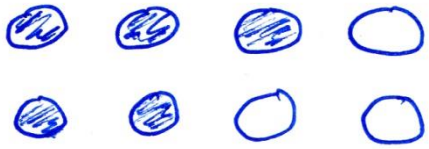
c. What number does X represent?



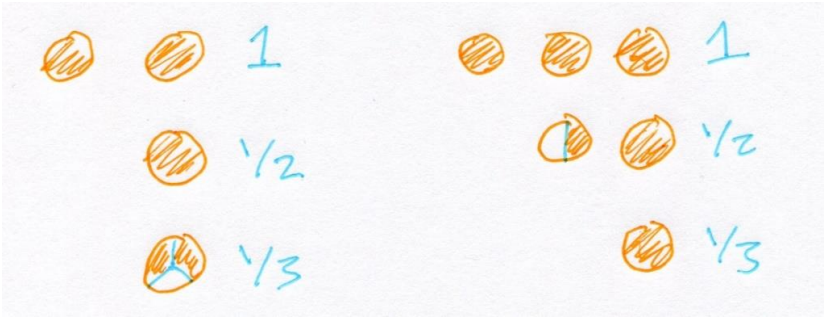
10. Core idea: Teach your students to identify the unit.

a. How much pizza do you have?

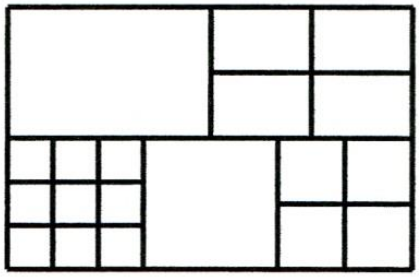
b. What fraction is represented here?

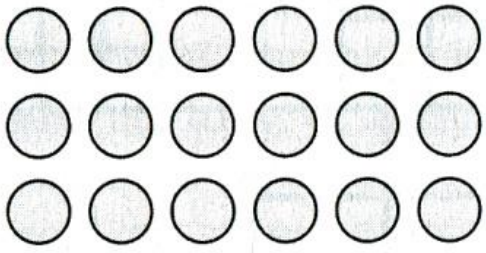


11. Unitizing.

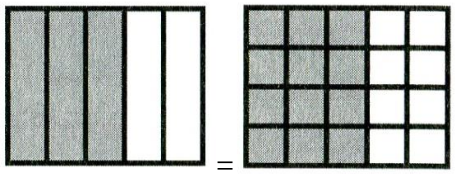
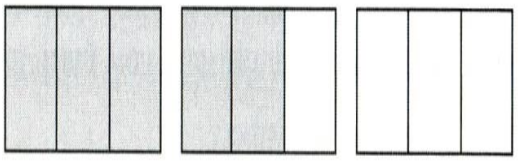


Unitizing game: Can You See?

a. 

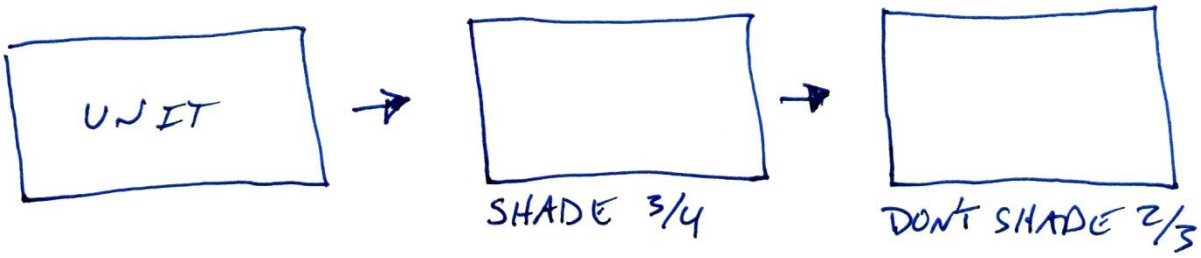
b. 

12. Area models.

a  b. 

c. Which is bigger, 3/8 or 4/9? What is their product?

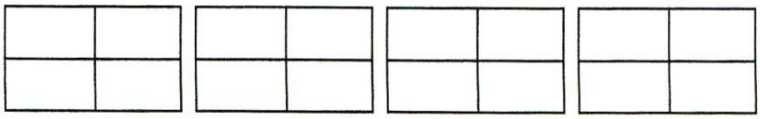
12. d. Use an area model to find $\frac{3}{4} \div \frac{2}{3}$



e. Use an area model to find $1\frac{3}{4} \bullet \frac{2}{5}$ and $1\frac{3}{4} \div \frac{2}{5}$

13. Partitioning (not the same as part-whole)

Six kids share these candy bars equally. How much does each kid get?



14. Measurement Discussion:

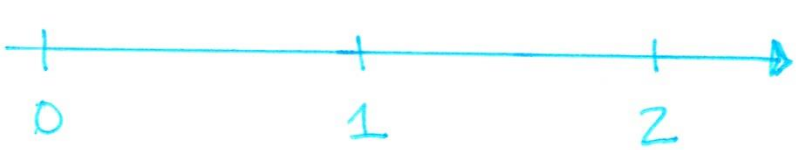
~ A conundrum: Fracturing reinforces the unit.
~ Measurement decentralizes the unit.
Unlimited iteration (easy to go past 1).
Nested units (coordination of levels of units).
~ Points on a number line represent distances.
~ $\frac{a}{b}$ means a intervals (iterations) of $\frac{1}{b}$.

~ Dynamic/density/proportional/compensatory/"Most powerful" (Lamon, 2008).

~ Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal for research in mathematics education*, 122-147.

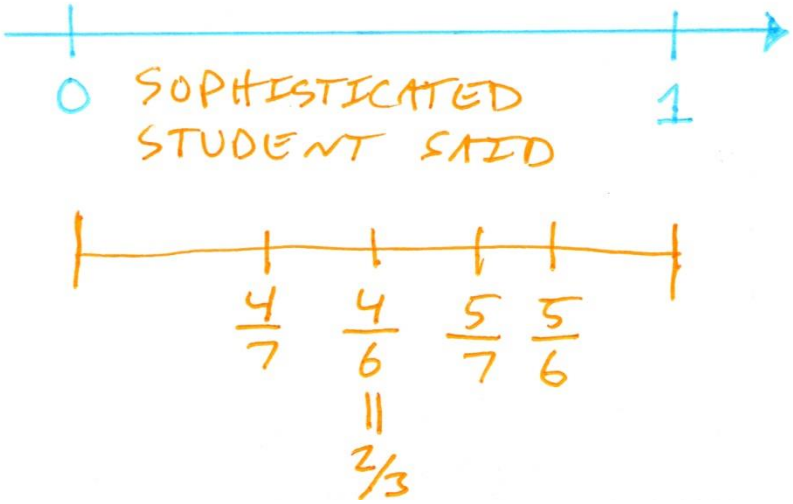
a. Locate $\frac{17}{24}$

b. Locate $\frac{7}{4}$



c. Find three fractions between $\frac{1}{2}$ and $\frac{1}{3}$

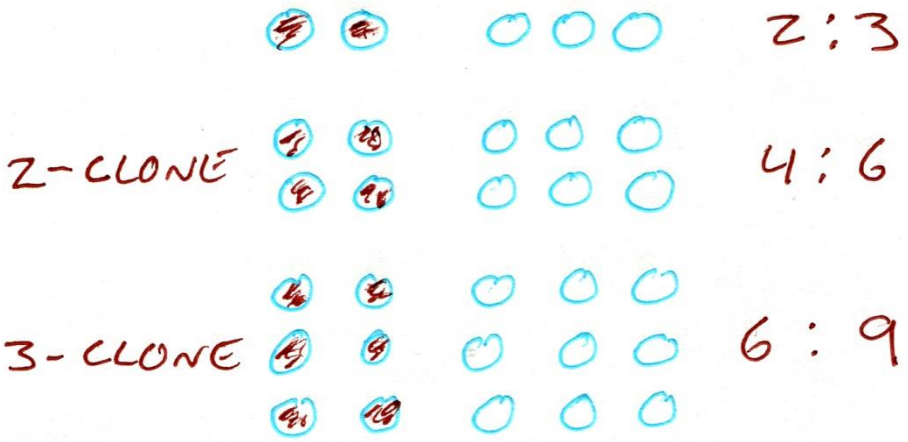
d. Which is larger, $\frac{2}{3}$ or $\frac{5}{7}$



15. Ratios:

~ A meaning that cannot be expressed with one number; a comparison.
~ Can be p-w, but usually p-p.
~ "Intensive quantity" is different from either piece; speed, density.
~ Cannot be directly measured.
~ Not always rational $\left(\text{e.g., } \pi = \frac{C}{D}\right)$

Clones:



a. Compare $\frac{3}{4}$ to $\frac{5}{6}$ by cloning.

b. Compare 2:3 to 5:6 by subtraction.

c. A poor fishing practice catches 9 pounds of bycatch for every 1 pound of shrimp.
- How much bycatch is caught in catching two pounds of shrimp?
- Express the amount of shrimp out of the total marine life caught as a fraction.

16. Measurement and partitive meanings of division.

Siebert, D. (2002). Connecting informal thinking and algorithms: The case of division of fractions. *Making sense of fractions, ratios, and proportions*, (NCTM 2002 Yearbook) 247-256.

Whole number division $8 \div 2 = 4$
- Measurement
- Partitive (sharing, unit rate)

Measurement and partitive division ask compensatory questions:

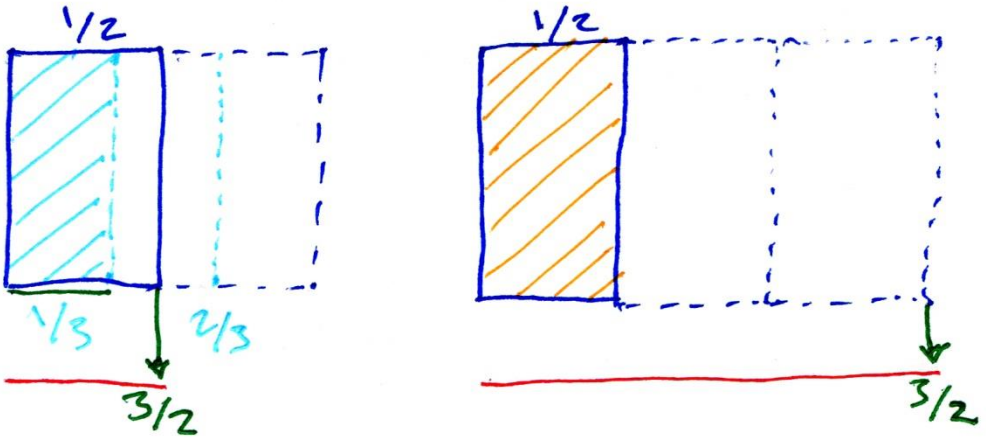
- How big is it?
- What did you measure it with?

Whole number division vs. division with fractions:

$6 \div 3 = ?$ What question is this answering? (Measurement and partitive)

$6 \div \frac{1}{3} = ?$ What question is this answering? (Measurement and partitive)

a. Draw $\frac{1}{2} \div \frac{1}{3}$ for both measurement and partitive cases. How do they compare?



b. Now consider $\frac{1}{2} \div \frac{2}{3}$ for the partitive case:

- Reason down and up.
- Draw it

All division problems are scaling problems: $\frac{3}{4} \rightarrow \frac{0.75}{1}$

Goal: To determine how much of the dividend should be associated with 1 unit of the divisor

17. Flip and multiply. (Do we need to?)

a. Draw $1\frac{1}{2} \div \frac{3}{5}$ using measurement interpretation.

b. Draw $1\frac{1}{2} \div \frac{3}{5}$ using partitive interpretation.

Flip and multiply is an operator: First, divide by 3 to find the value of one part. Then, multiply that value by five to find the amount of one whole.

Algebraically, we do these operations to both dividend and divisor (they are proportional) to transform the divisor into 1.

$$\frac{1\frac{1}{2}}{\frac{3}{5}} \rightarrow \frac{1\frac{1}{2} \bullet \frac{5}{3}}{\frac{3}{5} \bullet \frac{5}{3}} \rightarrow \frac{1\frac{1}{2} \bullet \frac{5}{3}}{1}$$

18. Word problems**.**

The End

Questions/Summaries/Next steps

	Measurement	Sharing
Situations	Joel is walking around a circular path in a park that is 3/4 miles long. If he walks 2 1/2 miles before he rests, how many times around the path did he travel?	Joel is walking around a circular path in a park. If he can walk 2 1/2 miles in 3/4 of an hour, how far can he walk in an hour, assuming he walks at the same speed?
Guiding question for interpreting 2 1/2 ÷ 3/4	How many groups of 3/4 are in 2 1/2?	If 3/4 of a group gets 2 1/2, how much does a whole group get?
Meaning of reciprocal	The reciprocal 4/3 means there are 4/3 groups of 3/4 in 1.	The reciprocal 4/3 is the operator necessary to shrink 3/4 to 1/4 and then expand 1/4 to 1.
Reason for multiplying the dividend by the reciprocal of the divisor	There are 4/3 groups of 3/4 in 1. There are 2 1/2 times as many groups of 3/4 in 2 1/2 as there are in 1. Thus, there are 2 1/2 × 4/3 groups of 3/4 in 2 1/2.	Since we shrink/expand 3/4 by 4/3 to get 1 whole group, we have to shrink/expand 2 1/2 by 3/4 in order to find out how much the whole group gets.

Siebert (2002) summary

Rational Number Interpretations of $\frac{3}{4}$	**Meaning**	**Selected Classroom Activities**
Part–Whole Comparisons With Unitizing "3 parts out of 4 equal parts"	$\frac{3}{4}$ means three parts out of four equal parts of the unit, with equivalent fractions found by thinking of the parts in terms of larger or smaller chunks. $\frac{3}{4}$ pies $= \frac{12}{16}$ ($\frac{1}{4}$-pies) $= \frac{1\frac{1}{2}}{2}$ (pair of pies)	Unitizing to produce equivalent fractions and to compare fractions
Measure "3 ($\frac{1}{4}$-units)"	$\frac{3}{4}$ means a distance of 3 ($\frac{1}{4}$-units) from 0 on the number line or 3 ($\frac{1}{4}$-units) of a given area.	Successive partitioning of a number line; reading meters and gauges
Operator "$\frac{3}{4}$ of something"	$\frac{3}{4}$ gives a rule that tells how to operate on a unit (or on the result of a previous operation); multiply by 3 and divide your result by 4 or divide by 4 and multiply the result by 3. This results in multiple meanings for $\frac{3}{4}$: 3 ($\frac{1}{4}$-units), 1 ($\frac{3}{4}$-unit) and $\frac{1}{4}$ (3-unit)	Machines, paper folding, xeroxing, discounting, area models for multiplication and division
Quotient "3 divided by 4"	$\frac{3}{4}$ is the amount each person receives when 4 people share a 3-unit of something.	Partitioning
Ratios "3 to 4"	3:4 is a relationship in which there are 3 A's compared, in a multiplicative rather than an additive sense, to 4 B's.	Bi-color chip activities

Lamon (2008) summary