All solutions involve regrouping.

Proof by contradiction.

Let $a, b, c, \ldots, i$ represent the digits 1-9.

$$\begin{array}{r} a\,b\,c \\ +\; d\,e\,f \\ \hline g\,h\,i \end{array}$$

Then

(1) $a+b+c+d+e+f+g+h+i=45$

Assume there is no regrouping.

Then $a+d=g$, $b+e=h$ and $c+f=i$

Substituting into (1) gives

$$g+h+i+g+h+i=45$$

$$2g+2h+2i=45$$

$$2(g+h+i)=45$$

$$g+h+i=22.5$$

This is not possible since g, h, and i are whole numbers.

The sum of the digits of any solution add to 18.

Proof:

$$\begin{array}{r} abc \\ +\ \underline{def} \\ ghi \end{array}$$

$a+b+c+d+e+f+g+h+i=45$

Assume there is regrouping in the ones digit.

Then $c+f=10+i$, $b+e+1=h$, and $a+d=g$

Substitution gives $10+i+h-1+g+g+h+i=45$

$2g+2h+2i=36$ $\quad 2(g+h+i)=36$ $\quad g+h+i=18$ ✓

Similarly it can be shown that if there is regrouping only in the tens place, the sum of the digits is 18.

There is exactly one regrouping in each solution.

Proof:
$$\begin{array}{r} a\,b\,c \\ +\ d\,e\,f \\ \hline g\,h\,i \end{array}$$

Suppose there is regrouping in both the tens and ones digits.

Then

$c+f=10+i$ $\quad$ $b+e+1=10+h$ $\quad$ $a+d+1=g$

$\quad$ $b+e=10+h-1$ $\quad$ $a+d=g-1$

Since $a+b+c+d+e+f+g+h+i=45$

$10+i+9+h+g-1+g+h+i=45$

$2g+2h+2i=27$

$2(g+h+i)=27$ $\quad$ $g+h+i=13.5$

Therefore there is exactly one regrouping.