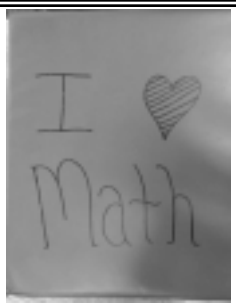
Connecting the Dots: Reimagining the Sequence of Topics for an Algebra Class

Daniel Brahier
Bowling Green State University
Saturday, April 6, 2019
Session 572: 8:00 – 9:00 a.m.
San Diego Convention Center, Room 20A

What comes to mind when your students think of Algebra?



The "End Product"?



Principles to Actions:
Ensuring Mathematical Success for All

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Ensuring Mathematical Success for All

2000 PSSM	2014 PtA
• **Equity**	• **Teaching and Learning**
• **Curriculum**	• **Access and Equity**
• **Teaching**	• **Curriculum**
• **Learning**	• **Tools and Technology**
• **Assessment**	• **Assessment**
• **Technology**	• **Professionalism**

5

Curriculum

An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world.

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First Day Problem 1

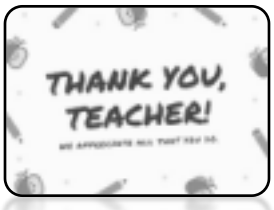
- Today is 040619. Using only the digits 0, 4, 0, 6, 1, and 9, along with operations, grouping symbols, exponents, and √, write the numbers 0-10.
- e.g., 5 can be written as $4 + 1 - (0+6+9) \times 0$
- Is it possible to write all of 0-10?
- What about 11-20?

First Day Mini Speech

- "When are we ever going to have to use this?"
- "The reason I call on you is not to see if you have the *right answer* – it's to see *what you're thinking* so I know how to help you better."

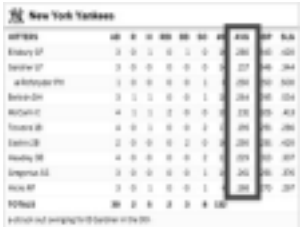
Parent Reaction

"I want to thank you …"



First Day Problem 2

The Batting Averages Problem



Batting Averages

Batting Average = ratio of "hits" to "times at bat" (e.g., 3 hits for 10 at bats is a 0.300 Average)

0.132

Hits a Single!

0.154

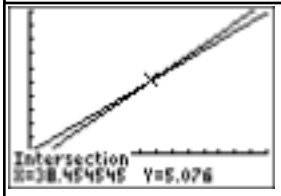
- How many hits does the player have, and how many times has he been "at bat" so far this season?
- Can you think of at least two different ways to come up with a solution?

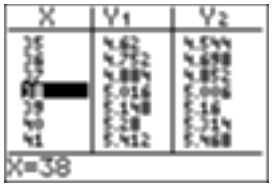
0.132 → 0.154

Simultaneous Equations

$$\frac{y}{x} = .132 \qquad \frac{y+1}{x+1} = .154$$

Graph/Table





Premise/Assumption

Technology is an inescapable fact of life in the world in which we live and should be embraced as a powerful tool for doing mathematics. Use of technology can assist students in visualizing and understanding important mathematical concepts and support students' mathematical reasoning and problem solving.

-*Principles to Actions*, page 83

An Old Joke

- How do you kill a blue elephant?
- With a blue elephant gun.
- How do you kill a white elephant?
- With a white elephant gun?
- No, you first strangle the elephant until it turns blue and then use the blue elephant gun you already have!

Anchor Problems

- Problems that we can refer back to all year long
- Take on a life of their own!

Homework – Day 1

- **Math Autobiography**
- 2 pages, double spaced (submitted as a Google Doc)
- "Tell me the story of your experiences in math classes, from as far back as you can remember to today. What did you like most? What scared you? Who were your favorite teachers? Why?"

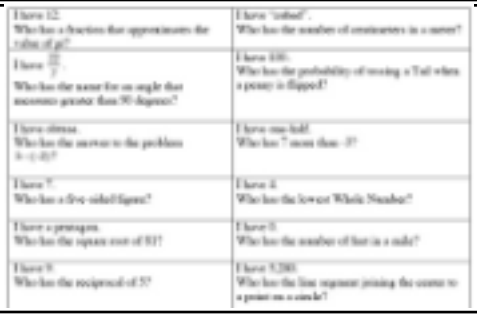
8th Grade Student Quote

There was a certain turning point where I finally realized, "wow I'm pretty good at math." When I finally realized that I was good at it, I also realized that maybe I'd just been hearing everyone all these years saying "I don't like math, it's horrible," and I just played along, but really I do enjoy math and the sensation you get when you figure out a new solution, or solve a really difficult problem. Math is hard work, I will say that, but for me I like the challenges it gives you.

First Week

- Assess what students know/remember
- Review and build on what they know
- Emphasize Order of Operations (key idea for the whole year!)
- Play "I Have … Who Has?" game

I Have … Who Has?



Test #1

- Primarily a "review" test
- Makes sure we are all on the same page
- Gives students **confidence** that they remember what they did in past years!

Contents of Test #1

- **Number Systems** (Whole, Integer, Rational, Real)
- Simplify/Evaluate **Expressions** (using Order of Operations)
- Writing/Solving **Proportions**
- Operations/Problems with **Scientific Notation**

Sample Item

Use scientific notation to rewrite and solve the following problem: The current national debt for the United States is about $18 trillion. There are approximately 300 million people living in the country. If we wanted to pay off the debt today, how much would it cost each person in the country?

$$\frac{1.8\times10^{13}}{3\times10^{8}} = 0.6\times10^{5} = \$60{,}000$$

Why Scientific Notation?

If you can do $\frac{1.8\times10^{13}}{3\times10^{8}} = 0.6\times10^{5} = \$60{,}000$

Then you can do $\frac{12x^5y^7}{6xy^4}$

Why Scientific Notation?

If you can do $(4.2\times10^{8})+(3\times10^{8}) = 7.2\times10^{8}$

Then you can do $3x^5 + 4x^5 = 7x^5$

And make sense of why the exponent stays the same!

Deciding Topic Order

1. What "new content" might my students find most interesting?
2. What topics have the greatest long-term impact in terms of connecting ideas later?
3. What visuals can I provide that will allow students to "see" related topics the rest of the year?

Unit 2

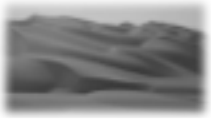
Functions

- Linear "real world" examples
- Terminology (increase/decrease, discrete/continuous, dependent/independent)
- Graphing lines using slope/intercept
- Finding equations of lines
- Parallel and perpendicular lines and slope
- Solving literal equations (e.g., $2x + 3y = 10$ for x)

Example

The temperature in the desert is 43° in the morning. If the temperature rises by an average of 4° per hour throughout the day, find the temperature y after x hours, later in the day.

- Is the function increasing or decreasing?
- Discrete or continuous?
- What is dependent and what is independent?



Unit 3

Linear Functions, continued

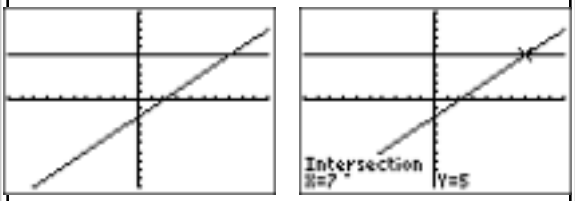
- Line of best fit
- Correlation
- Solving Linear Equations
  - Using a graph
  - Using algebra tiles to "balance" sides
  - By hand

Simple Linear Equations

Solve $x - 2 = 5$

$$\begin{cases} y_1 = x - 2 \\ y_2 = 5 \end{cases}$$

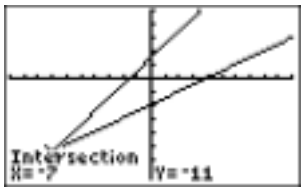
Graphing the equations …



What is the significance of the point (7, 5)?

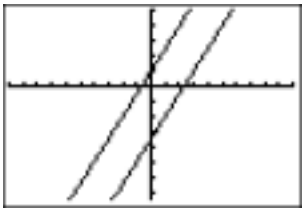
Example #2

$$2x + 3 = x - 4$$



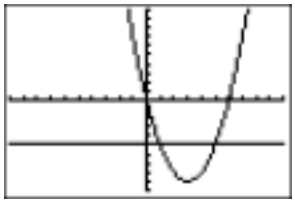
Example #3

$$3x + 2 = 3x - 7$$



More complex "pay off" …

$$x^2 - 6x = -5$$



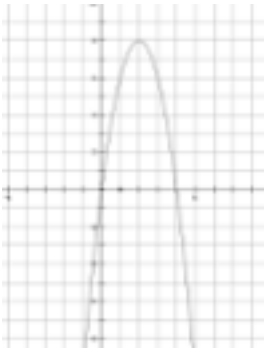
Unit 4

Function Notation and Quadratics

- Function Notation
- Solving equations with absolute value, square roots, and squares
- Quadratic Functions
  - Equations and terminology
  - Graphs
  - Families of curves

$$y = \frac{1}{2}(x-3)^2 - 5$$

Rain Gutter Problem



$y = x\,(8 - 2x)$

Helping Students Make Connections

Unit 5

Other Functions

- Absolute Value Function
  - Finding absolute value
  - Graphing/Transforming "V" shapes
- Rational Functions and Graphs
- Piecewise Functions and Graphs
- Nonlinear Systems (e.g., use a graph to find the intersection of a linear-quadratic system)

Timing

Unit 5 is completed by the first week of **November**

Unit 6

Still More Functions

- Exponential Functions
  - Writing equations
  - *The Brahier family buys a new dishwasher for $625. Experts say appliances depreciate at an average rate of 12% per year. Approximately how much will the dishwasher be worth in 5 years?*
  - Drawing graphs
- Step Functions

M & M Anchor

Unit 7

Solving Systems of Linear Equations

- Graphing Method
- Substitution Method
- Elimination Method
- Use **all 3** methods
- Word Problems using systems

Test Problem

Two new health clubs have just opened in your area. Sydney's Club offers an annual (yearly) membership for $75 and then charges you $2.25 for each visit for a workout. Ryan's Club charges an annual fee of only $25 and gives you 12 free visits per year, but each additional workout visit (after your first 12) costs $4.00.

a) Write a pair of simultaneous equations to represent this problem.

b) Find the intersection point of your two equations on your calculator and then explain the meaning of the point. Under what conditions might one choose one of the health clubs over the other?

Unit 8

Manipulating and Using Polynomials

- "Traditional" Word Problems
  - "Find two numbers that …"
  - Consecutive Integers
  - Perimeter/Geometry Problems
- Polynomials – add/subtract/multiply/divide
- Multiplying Polynomials
  - Algebra Tiles
  - Using "charts"
  - Mentally – tested on this

Timing

Unit 8 is completed by the end of **January**

Unit 9

"All Things Quadratic"

- Graphing (review)
- Factoring (Completely)
  - Factoring to identify zeros of the function
- Solving Quadratic Equations
  - Taking a square root
  - Factoring
  - Completing the Square
  - Quadratic Formula
    - Discriminant and what it tells us

Unit 10

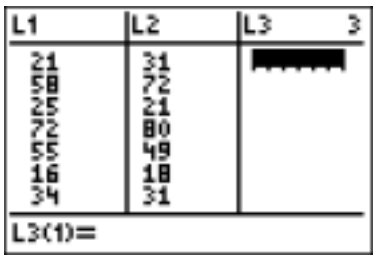
Inequalities

- Graphing 1-D inequalities on a number line
- Solving a linear inequality and graphing it
- Union and Intersection of inequalities
- Graphing 2-D inequalities in the plane
- Systems of 2-D inequalities

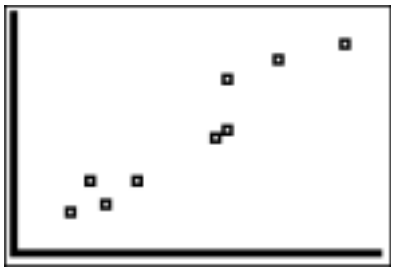
Age Guess Anchor

1. Crystal Bowersox
2. Bill Clinton
3. Kristen Stewart
4. Clint Eastwood
5. Barack Obama
6. Miley Cyrus
7. Alex Rodriguez
8. Drew Carey
9. Bob Barker

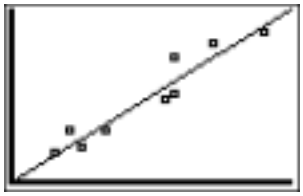
TI-84 Table



TI-84 Scatterplot

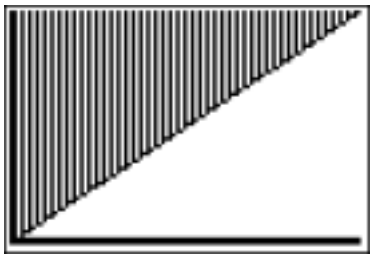


What if guesses were perfect?



What do we know about the guesses above the line $y = x$?

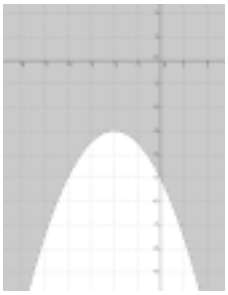
A million guesses and all celebrities were older than I thought …



Connecting Back to Quadratics

Graphing Inequalities

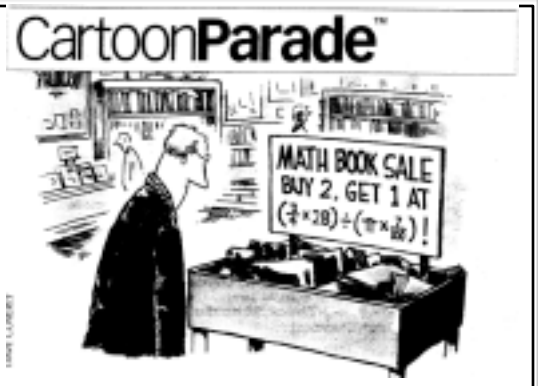
Graph: $y > -\frac{1}{2}(x+2)^2 - 3$



Unit 11

Irrational Numbers

- Number Systems, revisited
- Radicals (add, subtract, multiply, rationalize)
- Test – also includes 16 End-Of-Course exam items



Timing

Unit 11 is completed by the end of **March**

Unit 12

Data Analysis

- Distributions – symmetrical, asymmetrical
- Representing Data through Graphs
  - Dot Plots
  - Histograms
  - Scatterplots (revisited)
- Central Tendency
  - Mean, Median, Mode
- Five-Number Summaries
- Absolute and Standard Deviation

Unit 13

Right Triangles

- Terminology – postulate, theorem
- History – who were Pythagoras, Euclid, the Greeks, etc.?
- Pythagorean Theorem to solve problems
- Distance Formula (find distance using Pythagorean Theorem!)
- Right Triangle Trigonometry
  - Problems involving sine, cosine, tangent

Timing

Unit 13 is the end of the course (by June 1)

Other Features

- Real World/Applied problems in every unit
- Review Items on all tests
- EOC Released Items on tests (2 points each)
- Quarter Projects (usually 1 per quarter)
- Homework every night
- Quizzes (about 2 per week)
- EOC Review and computer lab practice leading up to the exam

Best Jobs (CareerCast, 2018)

1. Genetic Counselor
2. Mathematician
3. University Professor
4. Occupational Therapist
5. Statistician
6. Medical Services Manager
7. Data Scientist
8. Information Security Analyst
9. Operations Research Analyst
10. Actuary

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People's responses to the fact that I am a math major

Lowest Rated Job???

Lowest Rated Job???

Taxi Driver

Median Salary: $24,880

8 Teaching Practices

- Establish Mathematical **Goals** to Focus Learning
- Implement **Tasks** That Promote Reasoning and Problem Solving
- Use and Connect Mathematical **Representation**
- Facilitate Meaningful Mathematical **Discourse**
- Pose Purposeful **Questions**
- Build **Procedural** Fluency from **Conceptual** Understanding
- Support Productive **Struggle** in Learning Mathematics
- Elicit and Use Evidence of **Student Thinking**

Final Thought

Instead of saying "I *can't* do this **because**" …

say

"I *can* do this **until**" …

Steve Meiring, Retired, ODE

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