Math Modeling: Our Favorite Ready-To-Use Problems That Encourage Communication and Collaboration

This handout can be found at http://tinyurl.com/ModelingNCTM2019

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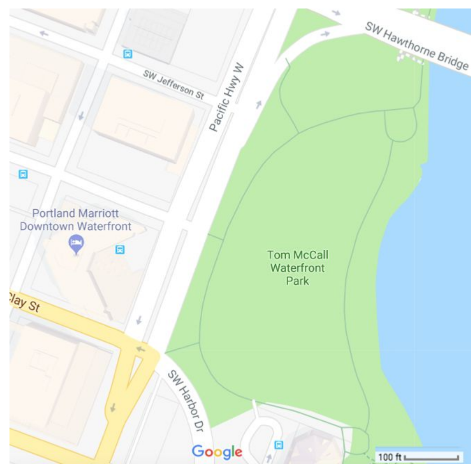
Activity #1: Population Estimation

Problem: Portland City Government is wondering how many people typically hang out at Waterfront Park on a sunny day. They take an aerial photo (below). They also look at a Google map of the area. Provide city officials with an estimate for the number of people in the park.



Activity #2: Driving for Gas

A friend tells you she buys her gas at a station several miles off your normal route where the prices are cheaper. Would it be more economical for you to drive the extra distance for the less expensive gas than to purchase gas along your route?

Activity #3: The Art Gallery Problem

An art gallery is holding a special exhibition of small watercolors. The exhibition will be held in a rectangular room that is 22 meters long and 20 meters wide with entrance and exit doors each 2 meters wide as shown below. Two security cameras are fixed in corners of the room, with the resulting video being watched by an attendant from a remote control room. The security cameras give at any instant a "scan beam" of 30°. They rotate backwards and forwards over the field of vision, taking 20 seconds to complete one cycle.

For the exhibition, 50 watercolors are to be shown. Each painting occupies approximately 1 meter of wall space, and must be separated from adjacent paintings by 1 meter of empty wall space and hang 2 meters away from connecting walls. For security reasons, paintings must be at least 2 meters from the entrances. The gallery also needs to add additional interior wall space in the form of portable walls. The portable walls are available in 5-meter sections. Watercolors are to be placed on both sides of these walls. To ensure adequate room for both patrons who are walking through and those stopped to view, parallel walls must be at least 5 meters apart throughout the gallery. To facilitate viewing, adjoining walls should not intersect in an acute angle.

The diagrams below illustrate the configurations of the gallery room for the last two exhibits. The present exhibitor has expressed some concern over the security of his exhibit and has asked the management to analyze the security system and rearrange the portable walls to optimize the security of the exhibit. Define a way to measure (quantify) the security of the exhibit for different wall configurations. Use this measure to determine which of the two previous exhibitions was the more secure. Finally, determine an optimum portable wall configuration for the watercolor exhibit based on your measure of security.

Figure 1: Exhibit Configuration: November 3-25, 2003

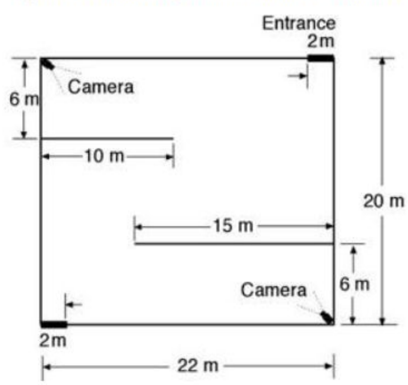
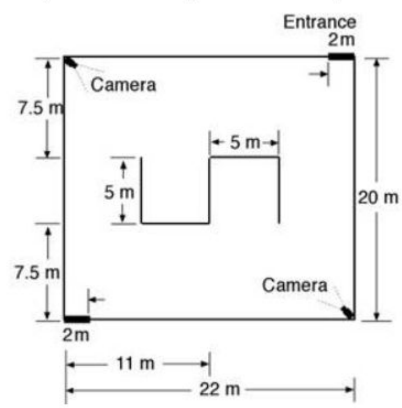


Figure 2: Exhibit Configuration: March 4-29, 2001



Activity #4: The Next Plague

A routine humanitarian mission on an island in Indonesia reported a small village where almost half of its 300 inhabitants are showing similar symptoms. In the past week, 15 of the "infected" have died. This village is known to trade with nearby villages and other islands. Your modeling team works for a major center of disease control in the capital of <u>your</u> country (or if you prefer, for the International World Health Organization).

Develop a mathematical model(s) that performs the following functions as well as how/when to best allocate these scarce resources and...

- Determines and classifies the type and severity of the spread of the disease
- Determines if an epidemic is contained or not
- Triggers appropriate measures (when to treat, when to transport victims, when to restrict movement, when to let a disease run its course, etc...) to contain a disease

Scaffolded activities:

The Flagpole Problem:

Your school's Buildings and Grounds Department needs to order a replacement flag for the flagpole behind the school. Standard flag sizing recommends that the length of the flag is at least 1/4 the height of the flagpole, and the width of the flag is approximately 2/3 the length of the flag. In addition, flag protocol requires that the flag should never touch the ground, even when flying at half-staff. Using only standard measuring tools, determine the height of the flagpole at your school, and use your results to give the minimum and maximum dimensions of the replacement flag.

The Clock Problem:

A. Your school needs to order replacement wall clocks to match existing wall clocks. Using a coin and standard measuring tools, and your knowledge of apparent size, estimate the diameter of the replacement wall clocks.

B. We have located an old purchase request which includes information about the diameter of the wall clock. Calculate the percent error in your estimate.

The London Olympics:

During the 2012 Summer Olympics, photographer Luke MacGregor (Reuters) took these photos of the moon behind the Olympic Rings.

Question: How far was the photographer from the bridge?

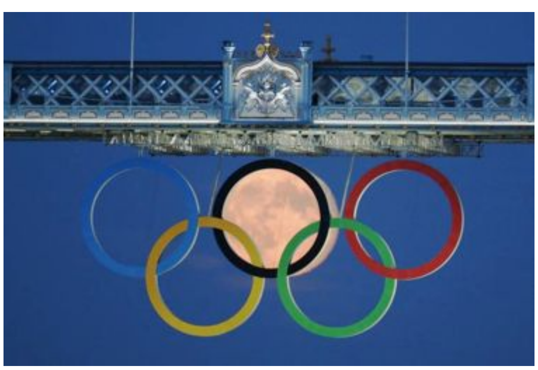


Image source: Reuters

NCSSM Summer Bridge Modeling Activities

Bridge Problem #1: Classifying Midges

In 1981, two new varieties of a tiny biting insect called a midge were discovered in the jungles of Brazil by biologists W. L. Grogan and W. W. Wirth. They dubbed one kind of midge an Apf midge and the other an Af midge. The biologists found that the Apf midge is a carrier of a debilitating disease that causes swelling of the brain when a human is bitten by an infected midge. Although the disease is rarely fatal, the disability caused by the swelling may be permanent. The other form of the midge, the Af, is quite harmless and a valuable pollinator. In an effort to distinguish the two varieties, the biologists took measurements on the midges they caught. The two measurements taken were wing length and antenna length, both measured in centimeters. The data are provided below.

Af midges

Wing length (cm)	1.72	1.64	1.74	1.7	1.82	1.82	1.9	1.82	2.08
Antenna length (cm)	1.24	1.38	1.36	1.4	1.38	1.48	1.38	1.54	1.56

Apf midges

Wing length (cm)	1.78	1.86	1.96	2.0	2.0	1.96
Antenna length (cm)	1.14	1.2	1.3	1.26	1.28	1.18

Is it possible to distinguish an Af midge from an Apf midge on the basis of wing and antenna lengths?
Source: "The Midge Problem," Everybody's Problems, Consortium, Number 55, Fall, 1995, COMAP, Inc., Lexington, MA.

Bridge Problem #2: Driving For Gas

Every driver recognizes the fluctuations in gas prices that happen almost on a weekly basis. Is it worth the drive across town for less expensive gas? There is almost always a station on your normal route that is convenient, but not necessarily inexpensive.

If you know the locations and the prices at all gasoline stations, at which station should you buy your gas? Does it matter if you think that you are buying gallons of gas or that you are buying miles of travel? Develop a model that can be used by owners of different cars that will tell them how far they should be willing to drive based on the specifications of their car. You will design the input and output of an App that drivers could use to pick the best gas station for them.

With your group you will turn in a poster showing your solution. It should have:

- the assumptions you have made in creating your model
- the mathematical model you used to determine which gas station is best
- the screens for an app based on your model. The first requests essential information, and the second is the app's response.

Bridge Problem #3: Assigning Elevators

In some buildings, all of the elevators can travel to all of the floors, while in others, the elevators are restricted to stopping only on certain floors. *Why? What might be the advantage of having elevators that travel only between certain floors?*

Suppose a building has 5 floors (1-5) that are occupied. The ground floor (0) is not used for business. Each floor has 60 people working on it. There are three elevators (A, B and C) available to take these employees to their offices in the morning. Everyone arrives at approximately the same time and enters the elevators on the ground floor. Each elevator holds 10 people and takes approximately 25 seconds to fill on the ground floor. The elevators then take 5 seconds between floors and 15 seconds on each floor on which it stops.

a) Suppose it is a holiday and only 5 people come to work today. Each person works on a different floor, and they all ride the same elevator. How long will it take for everyone to get to work?
b) Now suppose that 5 people come to work and these five people do not all work on different floors. How long will it take for everyone to get to work?

Why was question (b) harder to answer than question (a)? What assumptions will you need to make in order to simplify the problem?

The goal of this assignment is to get everyone to the floor on which they work as quickly as possible by specifying which elevators travel to which floors. In many modeling situations, the strategy one uses is to attempt to make the worst possible situation as good as it can be. This is known as solving the *worst case scenario.* In the context of the elevators in this building, the worst case scenario is that the elevator always stops on every floor to which it is assigned. So, if for example, you decide in problem # 4 below to assign an elevator to take workers to floors 1, 3, and 4, that elevator will always stops on those three floors.

In reality, this may not be true. That means that the actual solution is always equal to or probably better than the worst case solution. If you can make the worst case solution good enough, then you don’t have to worry about how it will perform in practice.

1. If all elevators go to all floors, how long will it take everyone to get to work?
2. Clearly state all of the simplifying assumptions you are making about this process.
3. If 80 people were late today, approximately when did the employees begin arriving?
4. Reassign the elevators to transport the employees to their offices as quickly as possible. What arrangement produces the shortest time? If this arrangement had been used today, would everyone have arrived on time?

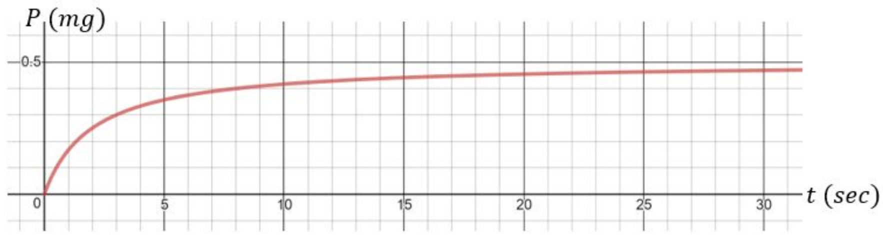
Extension: Suppose the number of workers on each floor is given below. How does this distribution of workers change your elevator assignments?

Floor	0	1	2	3	4	5
# of employees	0	50	80	70	40	60

Calculus Modeling Problem: Foraging for Pollen

Bees can carry around 15 mg of pollen, but flowers don't produce that much, so bees have to flit from flower to flower to collect enough pollen to fill their sacs before returning to the hive. In this problem you will explore the relationship between the availability of pollen and a bee's optimal foraging strategy.

Suppose a flower holds about 0.5mg of pollen, but not all of it is equally accessible to a bee. Let P(t) be the mass of pollen that a bee can collect if it spends time t on a single flower. We know that P(t) should be increasing and asymptotic to some horizontal limit that's around 0.5mg. The graph below shows a possible shape of such a function.



A bee might easily spend all day on a single flower trying to gather every last pollen grain, but at some point she decides it isn't worth the extra trouble, so she moves on to another flower nearby that hasn't had its pollen collected yet.

If flowers in her area are at a density such that it takes the bee around 4 seconds to fly from one flower to another, how long should the bee plan to spend collecting pollen on any single flower if her goal is to maximize the amount of pollen she can bring back to the hive during the course of her work day?

After you've addressed that question, generalize this problem as much as you can. What parameters in this model can you write as unspecified constants, rather than giving them actual numeric values? What is the bee's best strategy in terms of those parameters? Does the role of the parameters in her optimal strategy make sense?

2018 HiMCM Problem A: Roller Coaster

There are several Roller Coaster rating/ranking sites online that, while taking some objective measures into account, heavily rely on subjective input to determine the rating or ranking of a particular roller coaster (e.g., an "excitement" or "experience" score of an "expert" rider to measure "thrill").

In addressing this HiMCM problem, consider only roller coasters currently in operation. We have provided data for a subset of operating roller coasters whose height, speed, and/or drop are above the average of worldwide operating coasters. Therefore, we have not included family or kiddie coasters, nor have we included bobsled or mountain type coasters.

1. Create an objective quantitative algorithm or set of algorithms to develop a descriptive roller coaster rating/ranking system based only on roller coaster numerical and descriptive specification data (e.g., speed, duration of ride, steel or wood, drop).

2. Use your algorithm(s) to develop your "Top 10 Roller Coasters in the World" list. Compare and discuss the rating/ranking results and descriptions from your team's algorithm(s) with at least two other rating/ranking systems found online.

3. Describe the concept and design for a user-friendly app that uses your algorithm(s) to help a potential roller coaster rider find a roller coaster that she or he would want to ride. NOTE: You DO NOT need to program and/or write code for the app. You are developing the concept and design for the app only.

4. Write a one-page non-technical News Release describing your new algorithm, results, and app.

Attachment: **COMAP_RollerCoasterData_2018.xlsx**

Good Sources for Modeling Problems:

1. HiMCM past problems & its activity site, http://www.mathmodels.org/
2. M3 Challenge past problems: https://m3challenge.siam.org/archives/
3. GAIMME report: http://www.siam.org/reports/gaimme-full_color_for_online_viewing.pdf
4. Robert Kaplinsky tasks: https://robertkaplinsky.com/lessons/

Contests

1. The High School Mathematical Contest in Modeling (HiMCM) was designed to provide students with the opportunity to work as team members in a contest that will stimulate and improve their problem solving and writing skills. This competition takes place with teams consisting of up to four students working on a real-world problem during the 11 day contest period in November. The registration fee is $100 per team.

Tips for preparing a solution paper

- Conciseness and organization are extremely important. Key statements should present major ideas and results.
- Present a clarification or restatement of the problem as appropriate.
- Present a clear exposition of all variables, assumptions, and hypotheses.
- Present an analysis of the problem, motivating or justifying the modeling to be used.
- Include a design of the model. Discuss how the model could be tested.
- Discuss any apparent strengths or weaknesses to your model or approach.
- Incorporate lengthy derivations, computations, or illustrative examples in appendices. Summarize these in the main report. Results must be explicitly stated in the body of the report.

2. **MathWorks Math Modeling Challenge**

The Challenge takes place during a weekend in March and has no participation fees. High schools in the U.S. may enter up to two teams of three to five junior and/or senior students; homeschool and cyber school students may also participate. Winners receive scholarship prizes totaling $100,000.

Advice for coaches from past participants

1. The students who have had the most success in the Challenge are those who are not only good at math, but have an excellent work ethic, are enthusiastic, and are up to the challenge of working all day on a problem they may find extremely difficult.
2. Try to select students with a variety of key qualities, such as research capability, leadership skills, writing skills, and look for both logical and outside the box thinkers.
3. You may want to consider having the following roles represented on your team: a writer, a project manager, a mathematician, a researcher, and a well-rounded student.
4. Students' ability to work together, open-mindedness, resourcefulness, and strong math and problem solving skills will be important.
5. In preparation for the Challenge, it's a good idea to assign a scaled down version of the previous year's problem to your students. After they have completed that task, show them videos of the winning teams presenting their solution papers (in the online archives), and the winning solution papers. They tend to have more confidence in their ability to handle this competition after they see that the Challenge is "doable."
6. Have students critique the past winners' solution papers. Ask them to suggest what they would have done with the problem.
7. Students should familiarize themselves in advance with software or other technology that they could use for data analysis.
8. The open-ended nature of the problem means the students must perform research to fully understand and define the problem, identify the problems' important parameters, and learn to deal with uncertainty. The large scale of the problem means the students must work as an integrated team to prioritize their tasks and delegate responsibilities so they can complete, document, and write up their tasks in a comprehensive report within a very strict deadline.
9. Teach students other modeling techniques — like basic network theory — and include modeling projects in their calculus curriculum, if possible.