

Differentiating through Menus of Challenging Mathematical Tasks

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“How are you challenging _____ in math?”

“An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.”

NCTM, *Principles to Actions*, p. 11

Why is challenging students so challenging?

- Content
 - Management
- Student Priorities
- TIME

Goals for Mathematics Differentiation

- Students have curriculum-aligned opportunities for students to enrich/deepen/extend their understanding of content.
- Students are provided with problem solving experiences - engaging in a task for which the solution method is not known in advance.
- Students have some opportunity for choice and self-regulation.
- Students have an opportunity to work independently or collaboratively.
- Students have opportunities to develop the Standards for Mathematical Practice.

Math Menus

A math menu is “a collection of activities for students to do [that] may provide classwork for several days, a week, or for a longer period of time.”

Marilyn Burns, 1992, p. 37

Math Menus

- Curriculum-Aligned
- Problem Solving Tasks
- Challenging Puzzles
- Choice: Tasks & Pacing
- Choice: Independent or Collaborative
- Engagement: 8 Standards for Mathematical Practice
- Engagement: Productive Struggle

Not to be confused with...

- Actual Menus
- Varieties of options found on Pinterest and TPT

Math Menus

- Main Course Tasks
 - apply and deepen students' understanding of essential concepts
 - ask students to apply the mathematics they are currently working on in novel ways and/or authentic contexts
- Dessert Tasks
 - extend students' thinking and provide opportunities for them to make important mathematical connections
 - are significantly more challenging

Math Menus - Agenda

- Setting Up
- Selecting and Developing Tasks
 - Digging In
- More Differentiation Opportunities
- Questions

Math Menus - Setting Up

- Construction
 - Student Record Sheet
 - 6 Main Course Tasks
 - 6 Dessert Tasks
- Formatting
 - 1 page per task
 - NOT worksheets
 - NOT packets

Math Menu			
Grade 4, Unit 2			
Main Courses			
	Task	Task Complete	Teacher Initials
A	Story Problem Mashup 1	<input type="checkbox"/>	
B	Can You KenKen?	<input type="checkbox"/>	
C	An Epic Fundraiser	<input type="checkbox"/>	
D	Tricky Treats		
E	Wade King Fun Run		
F	Perimeters of Primes		

Desserts	
	Task
A	Story Problem Mashup 2
B	Wrestling with Remainders
C	Six Classes
D	Mia the Multiplier
E	Multiplication Square
F	So Many Squares

Six Classes

Unit 2 ADV-C

There are six classes of students in grades 4, 5, and 6 at Jefferson Intermediate School:

- Mrs. Balewa – 10 students
- Mr. Farooqi – 15 students
- Ms. Jamison – 17 students
- Ms. Roberts – 19 students
- Mrs. Chen – 21 students
- Mr. Osgood – 33 students

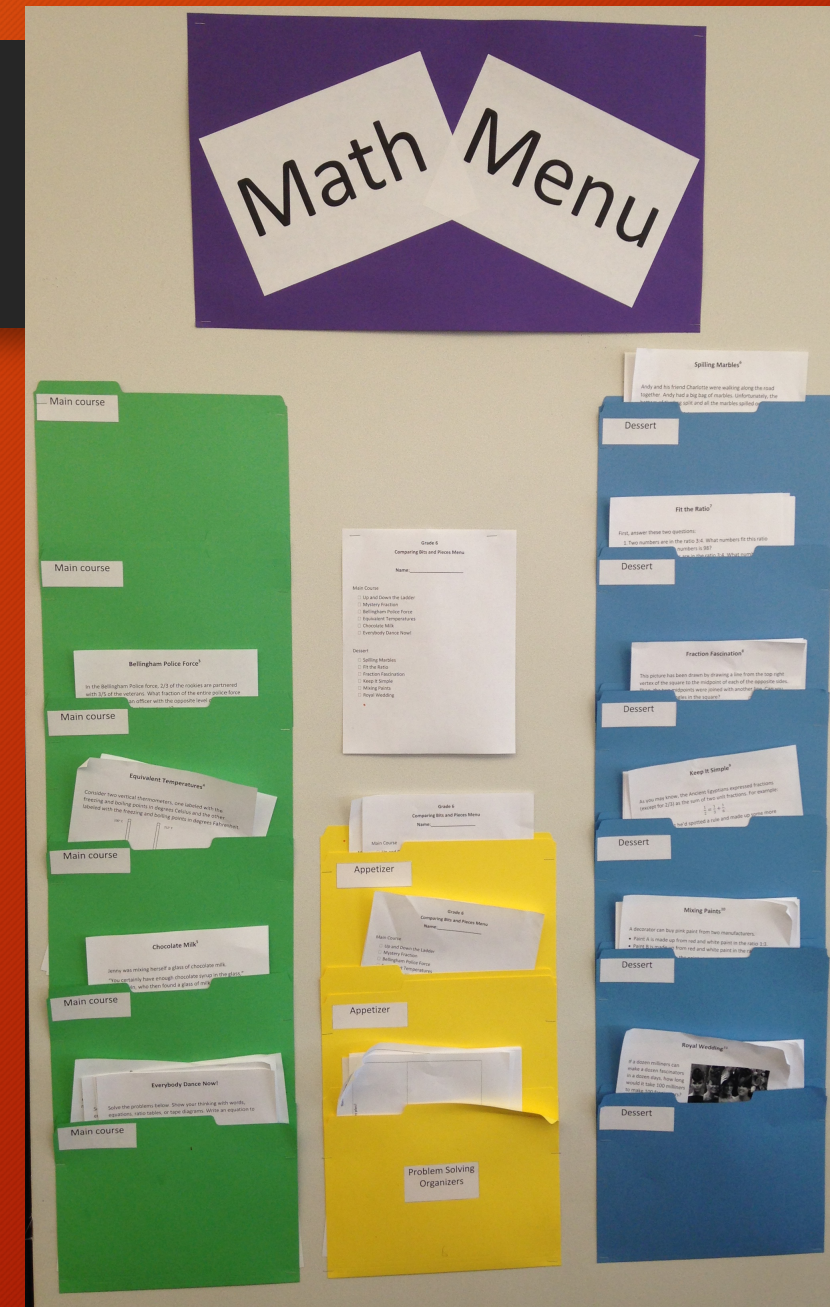
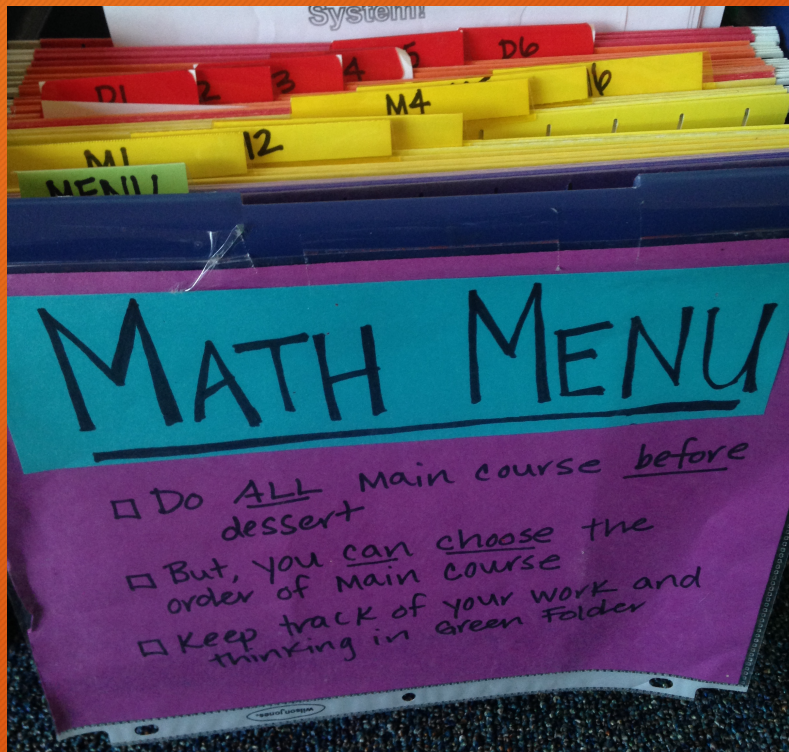
There is only one class of 4th graders, and there are twice as many 6th graders as there are 5th graders. What grade does each teacher teach?

Math Menus - Setting Up

- *Main Courses* can be completed in any order.
- All *Main Courses* are successfully completed before moving on to *Desserts*.*
- *Desserts* can be completed in any order.
- There are no time limits or initial expectations for efficiency.
- Students choose to work independently, collaboratively, or both.

Math Menus - Setting Up

ALL students should have independent, physical access to the tasks:



Math Menus - Selecting and Developing Tasks

Cognitive Demand Framework

- Low Cognitive Demand Tasks
 - Memorization
 - Procedures without connections (to understanding, meaning, or concepts)
- High Cognitive Demand Tasks
 - Procedures with connections (to understanding, meaning, or concepts)
 - Doing mathematics

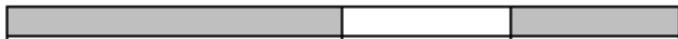
Math Menus - Selecting and Developing Tasks

What Fraction is White?

Unit 4 EXC-D

The strips below each equal one whole. Find the fraction of each strip that is white.

1.



2.



3.



4.



5.



Stamps on History

Unit 4 EXC-B

The table below provides information about 12 famous women who have left their stamps on history. Use the information to determine each woman's age when she died, and how many years passed between the year she died and when she was commemorated on a United States postal stamp. Challenge yourself to use multiple subtraction strategies!

Famous Woman	Year of Birth	Year of Death	Age at Death*	Year Honored by U.S. Postal Stamp	Years Passed from Death to U.S. Postal Stamp
Pocahontas	1595	1617		1907	
Martha Washington	1731	1802		1902	
Betsy Ross	1752	1836		1952	
Sojourner Truth	1797	1883		1986	
Susan B. Anthony	1820	1906		1935	
Harriet Tubman	1820	1913		1978	
Elizabeth Blackwell	1821	1910		1973	
Clara Barton	1821	1912		1948	
Helen Keller	1880	1968		1980	
Georgia O'Keeffe	1887	1986		1996	
Amelia Earhart	1897	1937		1963	
Rosa Parks	1913	2005		2013	

* Calculated age at death may be inaccurate. If the woman died before her birthday in that year, she would be one year younger than calculated.

Elevenes 2

Unit 4 ADV-A

Use the standard multiplication algorithm to calculate the following products.

$$132 \times 11$$

$$253 \times 11$$

$$421 \times 11$$

$$271 \times 11$$

$$636 \times 11$$

What do you notice about the products? Use your observations to predict the products of 354×11 , 545×11 , and 724×11 . What do you think will happen with 584×11 ? Check your predictions!

Can you use your observations to solve the following problems?

$$\underline{\quad} \times 11 = 3432$$

$$\underline{\quad} \times 11 = 9922$$

$$\underline{\quad} \times 11 = 7887$$

Math Menus – Selecting and Developing Tasks

The Answer Is...

Unit 4 EXC-F

Let's turn things around again. In this task, you are given the answer. Write story problems that result in the following answers. Make sure that your question uses reasonable measurements!

The answer is 158 grams.

1. Write a story problem involving addition of two or more numbers.
2. Write a story problem involving subtraction of two numbers.

The answer is 726 milliliters.

3. Write a story problem involving addition of two or more numbers.
4. Write a story problem involving subtraction of two numbers.

The answer is 56 kilometers.

5. Write a story problem that needs two steps in order to solve it. For example, you may need to add and then multiply in order to solve the problem.

Abundant Numbers

Unit 1 EXC-B

A factor is a whole number that divides evenly into another number. Factors of 24 can be found by finding all the pairs of numbers that multiply together to make 24:

$$1 \times 24$$

$$2 \times 12$$

$$3 \times 8$$

$$4 \times 6$$

If we leave out the original number, 24, and add all the other factors, the sum is 36:

$$1 + 2 + 3 + 4 + 6 + 8 + 12 = 36$$

We call 24 an *abundant number*, because 24 is less than the sum of its factors (36), not including itself.

1. Find five other abundant numbers.
2. What kinds of numbers can never be abundant numbers? Why not? Explain your reasoning.

ABC

Unit 5 ADV-F

In the multiplication below, some of the digits have been replaced by letters and others by asterisks. Where a digit has been replaced by a letter, the same letter is used each time, and different letters have replaced different digits. Reconstruct the original multiplication problem.

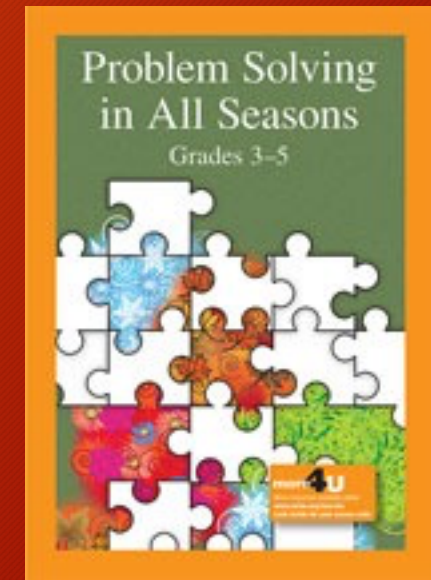
		A	B	C	
	×	B	A	C	
	*	*	*	*	
		*	*	A	0
*	*	*	B	0	0
*	*	*	*	*	*

Math Menus - Let's dig in!

Math Menus - Selecting and Developing Tasks

Our Favorite Resources


- *Teaching Children Mathematics* - Problem Solvers, Math by the Month
- *Mathematics Teaching in the Middle School* - Palette of Problems
- The Curriculum
 - Turning around or opening up problems
 - Problems in a new context
 - Working backwards
- Problem Solving Books



Math Menus - Selecting and Developing Tasks

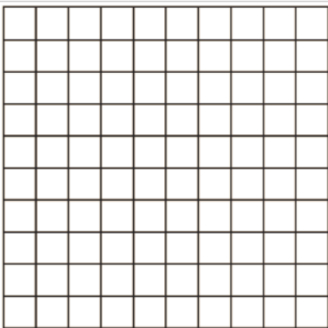
Our Favorite Resources

- www.youcubed.org
 - 5th grade
 - Division



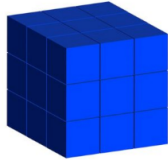
Broken Eggs

Number Sense 5 6
7 8 9 10
Multiplication



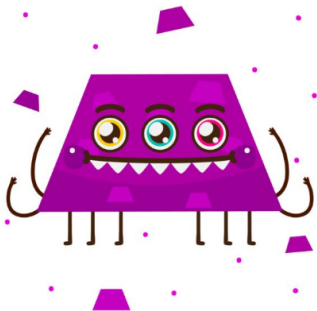
**How Many Rows?
How Many in Each Row?**

Number Sense 3 4
5 6 7 8 9
MP5 MP6 Area
Multiplication




Painting Youcubed

Generalization
Number Sense
Pattern Recognition
Shape & Space 5 6
7 8 9 10
MP1 MP2 MP3
MP4 MP5 MP6
MP7 MP8 3D Shapes
Geometry Modeling
Patterns Surface Area




Trap the Zoid

Modeling Shape & Space
4 5 6 MP1
MP4 MP7 Geometry




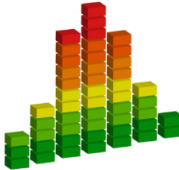
Patterns and Products

Number Sense 4 5
6 7 MP5 MP7



Border Problem





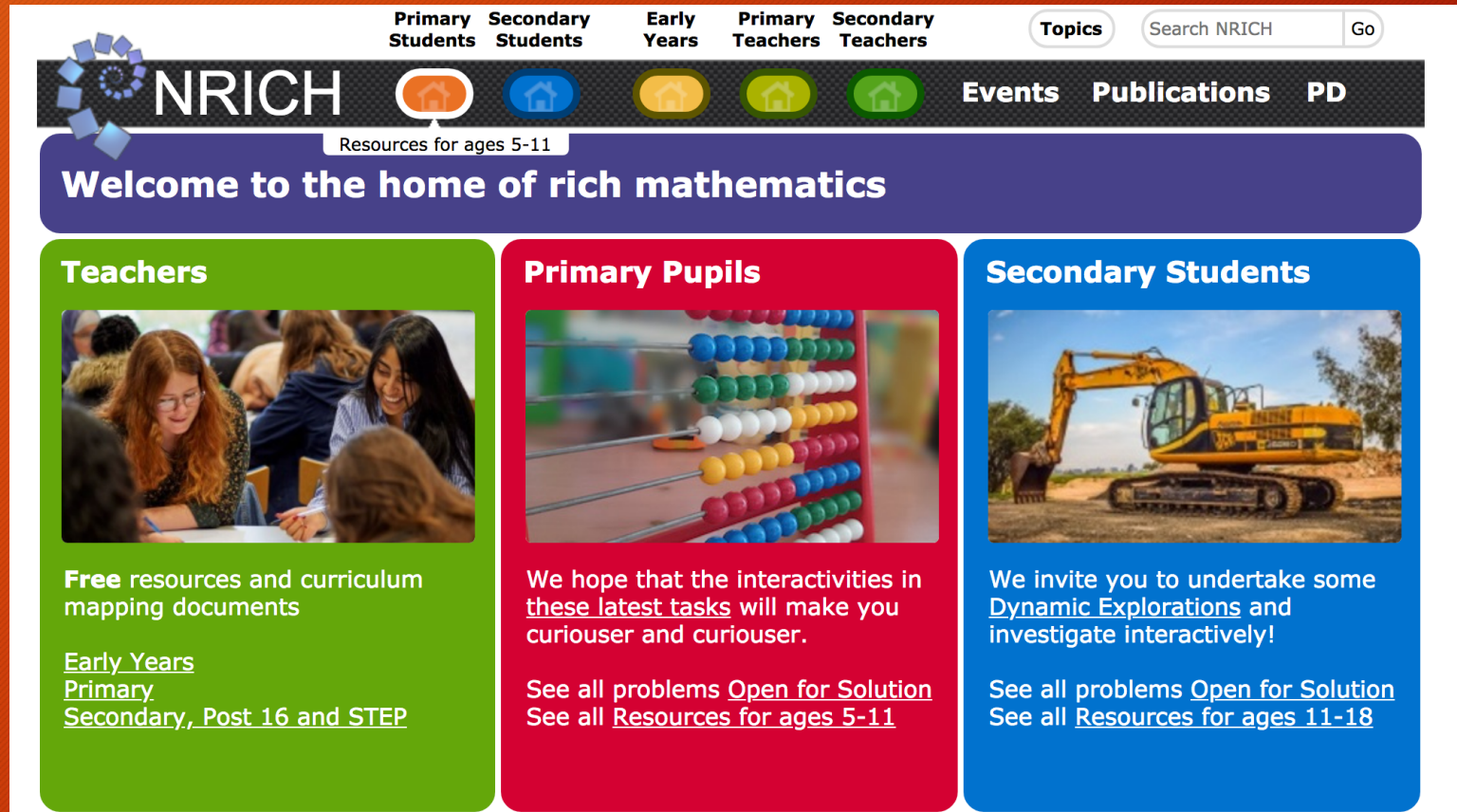
Squares Upon Squares

Generalization
Pattern Recognition

Math Menus - Selecting and Developing Tasks

Our Favorite Resources

- <https://nrich.maths.org/>
 - Division
 - Ages 11-14




The screenshot shows the NRICH website homepage. At the top, there is a navigation bar with links for Primary Students, Secondary Students, Early Years, Primary Teachers, and Secondary Teachers. A search bar labeled 'Search NRICH' and a 'Go' button are also present. Below the navigation bar, the NRICH logo is displayed, followed by the text 'Resources for ages 5-11'. A large purple banner reads 'Welcome to the home of rich mathematics'. The main content area is divided into three colored boxes: a green box for Teachers, a pink box for Primary Pupils, and a blue box for Secondary Students. Each box contains a representative image, a brief description of the resources, and links to specific content.

Primary Students **Secondary Students** **Early Years** **Primary Teachers** **Secondary Teachers** **Topics**

NRICH Resources for ages 5-11

Welcome to the home of rich mathematics


Teachers



Free resources and curriculum mapping documents

[Early Years](#)
[Primary](#)
[Secondary](#), [Post 16](#) and [STEP](#)


Primary Pupils



We hope that the interactivities in [these latest tasks](#) will make you curiouser and curiouser.

See all [problems Open for Solution](#)
See all [Resources for ages 5-11](#)

Secondary Students



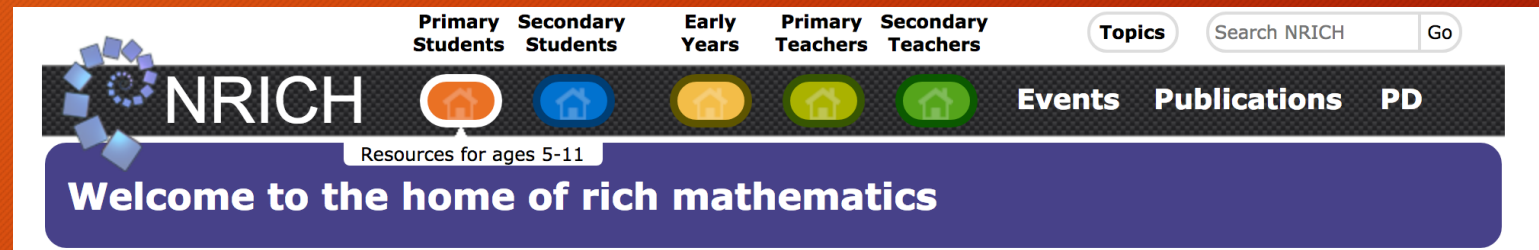
We invite you to undertake some [Dynamic Explorations](#) and investigate interactively!

See all [problems Open for Solution](#)
See all [Resources for ages 11-18](#)

Math Menus - Selecting and Developing Tasks

Our Favorite Resources

- <https://nrich.maths.org/>
 - Division
 - Ages 11-14



3388
Age 11 to 14 ★★

Using some or all of the operations of addition, subtraction, multiplication and division and using the digits 3, 3, 8 and 8 each once and only once make an expression equal to 24.



Skeleton
Age 11 to 14 ★★★

Amazing as it may seem the three fives remaining in the following 'skeleton' are sufficient to reconstruct the entire long division sum.



The Remainders Game
Age 7 to 14 ★★

A game that tests your understanding of remainders.

Math Menus - Selecting and Devising

Our Favorite Resources

- <https://nrich.maths.org/>
 - Division
 - Ages 11-14

Primary
Students



NRICH

Resources for ages 5-16

Welcome to the home

Teachers



Free resources and curriculum mapping documents

[Early Years](#)
[Primary](#)
[Secondary, Post 16 and STEP](#)

ABC

Unit 5 ADV-F

In the multiplication below, some of the digits have been replaced by letters and others by asterisks. Where a digit has been replaced by a letter, the same letter is used each time, and different letters have replaced different digits. Reconstruct the original multiplication problem.

$$\begin{array}{r} \text{A B C} \\ \times \text{B A C} \\ \hline * * * * \\ * * \text{A} 0 \\ * * * \text{B} 0 0 \\ \hline * * * * * * \end{array}$$

Math Menus - Selecting and

Our Favorite Resources

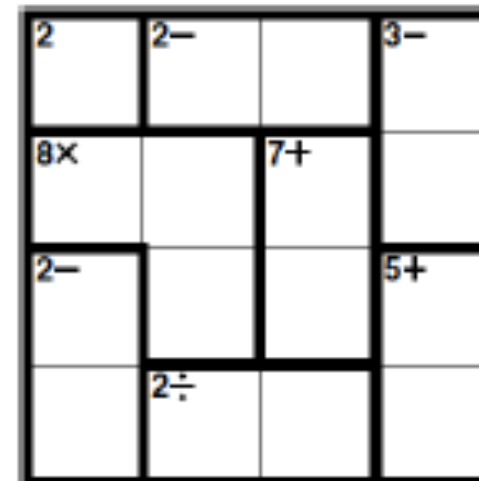
- Puzzles!
 - KenKen
- <https://www.kenkenpuzzle.com>

KenKen Again

Unit 5 DOC-A

Do you remember KenKens from a previous Math Menu? In the KenKen puzzles provided for this task, the numbers 1, 2, 3, and 4 only appear once in each row and each column. The bold, outlined sections indicate what numbers add, subtract, multiply, or divide to the number provided.

Complete this KenKen puzzle, and the two other puzzles on the sheet provided.



Math Menus - Selecting and

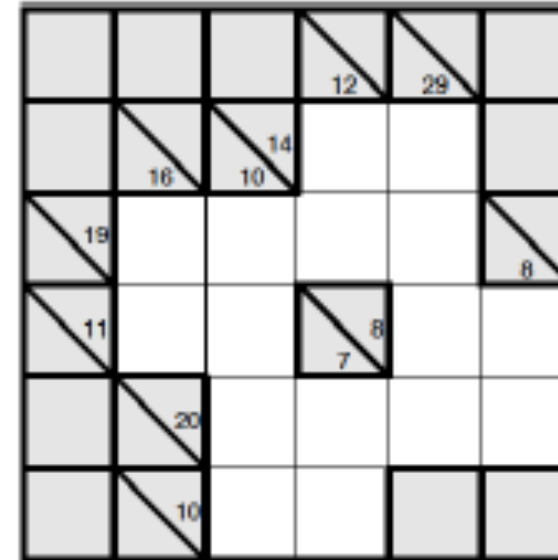
Our Favorite Resources

- Puzzles!
 - KenKen
 - Kakuro
- <https://www.kenkenpuzzle.com>
- <https://krazydad.com/>

Introducing... Kakuro Puzzles!

Unit 6 EXC-F

To solve the Kakuro puzzle below, fill in the white squares with the numbers 1 through 9, without repeating any numbers in any row or column. In the shaded squares, the numbers above a diagonal indicate the sum of the numbers in that row (going across). The numbers below a diagonal indicate the sum of the numbers in that column (going down). Solve the puzzle!



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Math Menus - Selecting and

Our Favorite Resources

- Puzzles!
 - KenKen
 - Kakuro
 - Futoshiki
- <https://www.kenkenpuzzle.com>
- <https://krazydad.com/>

Fascinating Futoshiki

Unit 7 ADV-E

To solve a Futoshiki puzzle, fill in the squares so that the numbers 1, 2, 3, and 4 are used once in each row and once in each column. The < and > signs show the relationship between two neighboring squares. Good luck!

	3		
	^	^	^
		v	
2			

Be sure to grab the supplement that goes with this task. It has an extra Futoshiki puzzle for you to do!

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Math Menus - Selecting and

Our Favorite Resources

- Puzzles!
 - KenKen
 - Kakuro
 - Futoshiki
 - Nonograms
- <https://www.kenkenpuzzle.com>
- <https://krazydad.com/>

Nonograms

Unit 7 ADN-F

A nonogram is a logic puzzle where you determine which squares in a grid are shaded in, and which squares are left blank. The numbers above each column indicate how many squares are shaded in each run in that column. The numbers to the left of each row indicate how many squares are shaded in each run in that row. There is always at least one white space between each run of shaded squares.

The numbers 1 and 3 above this column indicate that there are two runs of shaded squares. The first run is 1 square, and the second run is 3 squares. At least one unshaded square separates the two runs.

The numbers 1 and 1 before this row indicate that there are two runs of shaded squares in the row. The first run is 1 square, and the second run is 1 square. At least one unshaded square separates the two runs.

	1	2			1
	1	2	4	2	3
2	2				
	3				
1	1				
2	1				
3	1				

Solve the three nonograms on the separate sheet!

Copyright © 2017 by Flexible Math Concepts
If you like these puzzles, <https://www.kennethcongratulations.com/> is a good source for more!

Math Menus - Selecting and

Our Favorite Resources

- Puzzles!
 - KenKen
 - Kakuro
 - Futoshiki
 - Nonograms
- <https://www.kenkenpuzzle.com>
- <https://krazydad.com/>
- No Sudoku?!?!
 - Search Sudokus on NRICH

Multiplication Equation Sudoku

Unit 4 ADV-E

Like the standard Sudoku, this variant has two basic rules: each column, each row, and each box (3×3 subgrid) must have the numbers 1 through 9; and no column, row, or box can have two squares with the same number.

The Sudoku below is a bit different, however. At the bottom and right side of the grid are numbers, each of which is the product of the squares in that row or column marked by asterisks.

For example, the first row of the puzzle has four squares with asterisks; the product of the four numbers in these squares is 840. Solve the puzzle!

			*		*	*		*	840
	*		*						27
	*							*	8
	*		*	*		*			60
		*		*					27
*				*					48
*							*		40
	*				*		*	*	120
		*					*		63
40	576	63	180	18	30	12	72	35	

Math Menus - Let's dig in!

Math Menus - Selecting and Developing Tasks

What to Avoid:

- Games that require more than one person
- Skills practice (that isn't more than that)
- Regurgitation tasks
 - Write a letter....
 - Make a video....
- Giving instructions on how to solve

Number Maze

Unit 1 ADV-A

Use multiplication and division to find the path through this maze. You may move horizontally, vertically, or diagonally. You may not pass through a square more than once. Watch out for dead ends!

Exit

4	60	6	10	20	3	4	12
1	4	8	2	18	5	54	16
18	16	63	3	9	27	6	9
42	9	2	21	24	6	1	9
7	6	3	8	11	4	8	81
15	5	3	10	80	2	6	12
3	48	30	6	5	7	4	5
2	6	8	25	2	14	8	3

Enter

Extra Challenge: Design your own multiplication and division number maze. Make sure there is only one route through your maze!

Math Menus - Selecting and Developing Tasks

What to Aim for:

- Balance
- Relevant Contexts
- New Contexts
- Variety
- Novelty
- Representation
- Assessment

Story Problem Mashup 2

Unit 2 ADV-A

Girls on the Run has been meeting for four weeks, and attendance has been increasing. Each girl is determined to finish a Girls on the Run 5K in November! For each of the problems below, write an equation, find a solution, and label your answer.

1. At the fourth Girls on the Run session, the girls altogether ran 54 kilometers. If each girl ran 2 kilometers, how many girls attended the fourth session?
2. At the second session, there were 15 girls in attendance. One third of these girls each ran 3 kilometers. The rest ran 2 kilometers. How many kilometers did the girls run altogether in the second session?
3. There are 14 elementary schools in Bellingham. There are 440 snack packages to be equally distributed to the 14 schools for their Girls on the Run programs. How many snack packages should each school get?
4. A box can hold 70 snack packages for easier delivery to the schools. How many boxes does the Girls on the Run staff need in order to transport all of the snack packages?
5. If 308 girls participate in the 5 kilometer event in November, and everyone finishes, how many kilometers will they have run in the event?

Math Menus - More Differentiation Opportunities

- Appetizers
- Working with Small Groups
- Using Tasks to Pre-teach Concepts
- Task Experts
- Writing About Mathematics

Questions?

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Thank you!

References

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National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM: Reston, VA.