Table 1

Types of struggle experienced by the student, adapted from Warshaur, 2014.

Kind of Struggle	Descriptors
Get started	Confusion regarding what task is asking
	Forgetting how to solve a type of problem
	Gesturing uncertainty and resignation
	No work written down
Carry out a process	Unable to progress on a problem due to inability to use or process a formulated representation, carry out an algorithm, or recall needed facts or formula
Uncertainty in explaining and sense-making	Difficulty in explaining or making sense of their work
	Express uncertainty
	Unclear reasons given for their choice of strategy
Express misconceptions and errors	Misconception related to mathematical content in problem
	Performing an arithmetic or technological error

Table 2

Types of teacher responses, adapted from Warshaur, 2014.

Teacher response	Descriptors	Dimensions
Telling	Supplying information	Cognitive demand lowered
	Directing students towards a strategy	Attended to student struggle
	Correcting an error	Removed struggle efficiently.
	Referring or referencing student to a simpler problem	Built on student thinking
		Suggested an explicit idea
Directed Guidance	Redirect student thinking	Cognitive demand
	Narrow down possibilities for action	Lowered or maintained from intended
	Direct an action	Attend to student struggle
	Break down problem into smaller parts	Assess cause and direct student
	Alter problem to an analogy	Build on student thinking:
		Used to build on with teacher ideas
Probing Guidance	Ask for reasons and justification	Cognitive demand
	Offer ideas based on students' thinking	Maintained
	Seek explanation that could get at an error or misconception	Attend to student struggle
	Ask for written work of students' thinking	Question, encourage student's self-reflection
		Build on Student Thinking
		Used as basis for guiding student
Affordance	Ask for detailed explanation	Cognitive demand
	Build on student thinking	Maintained or raised
	Press for justification and sense-making with group or individually	Attend to Student Struggle
	Afford time for students to work	Acknowledge, question, and allow student time
		Build on student thinking
		Clarify and highlight student ideas

Table 3
Outcome of Struggle, adapted from Warshaur, 2014

Outcome Type	Descriptors
Productive	• maintained the intended goals and cognitive demand of the task • supported students' thinking by acknowledging effort and mathematical understanding • enabled students to move forward in the task execution through student actions.
Productive at a lower level	• lowered somewhat in the cognitive demand of the intended task • the teacher rather than the students actively guided the students through the struggle • the students passively following a directed guidance.
Unproductive	• students continued to struggle without showing signs of making progress toward the goals of the task • reached a solution but to a task that had been transformed to a procedural one that significantly reduced the task's intended cognitive demand • if the students simply stopped trying.

Table 4
Impact the cognitive demand of a task, adapted from Stein & Smith, 1998.

Changes	Descriptors
Factors Associated with the Maintenance of High-Level Cognitive Demands	• Teacher uses scaffolding, questioning, comments, and feedback to press for student reasoning, explanation, justification, and conceptual connections. • Teachers supports students in monitoring their own progress and the modeling of high-level performance. • Teacher allows sufficient time for task.
Factors Associated with the Decline of High-Level Cognitive Demands	• Teacher emphasizes complete and correct answers rather than the meanings and understanding of the concepts. • Teacher provides their own thinking and reasoning at the expense of student reasoning. • Teacher reduces the complexity of the task by providing explicit procedures or proscribed routines. • Teacher accepts unclear or incorrect student explanations. • Teacher expectations are not clear or appropriate for high-level cognitive activities or does not maintain classroom environment suitable for high-level cognitive activities. • Teacher does not allow sufficient time for task or too much time is allowed, resulting in off-task behavior. • Teacher selects a task that is inappropriate for the group of students (e.g., students do not have prior knowledge needed or task expectations are not clear enough to put students in the right cognitive space).