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Flipped Classroom Designs for the Mathematics Classroom

National Council of Teachers of Mathematics Conference

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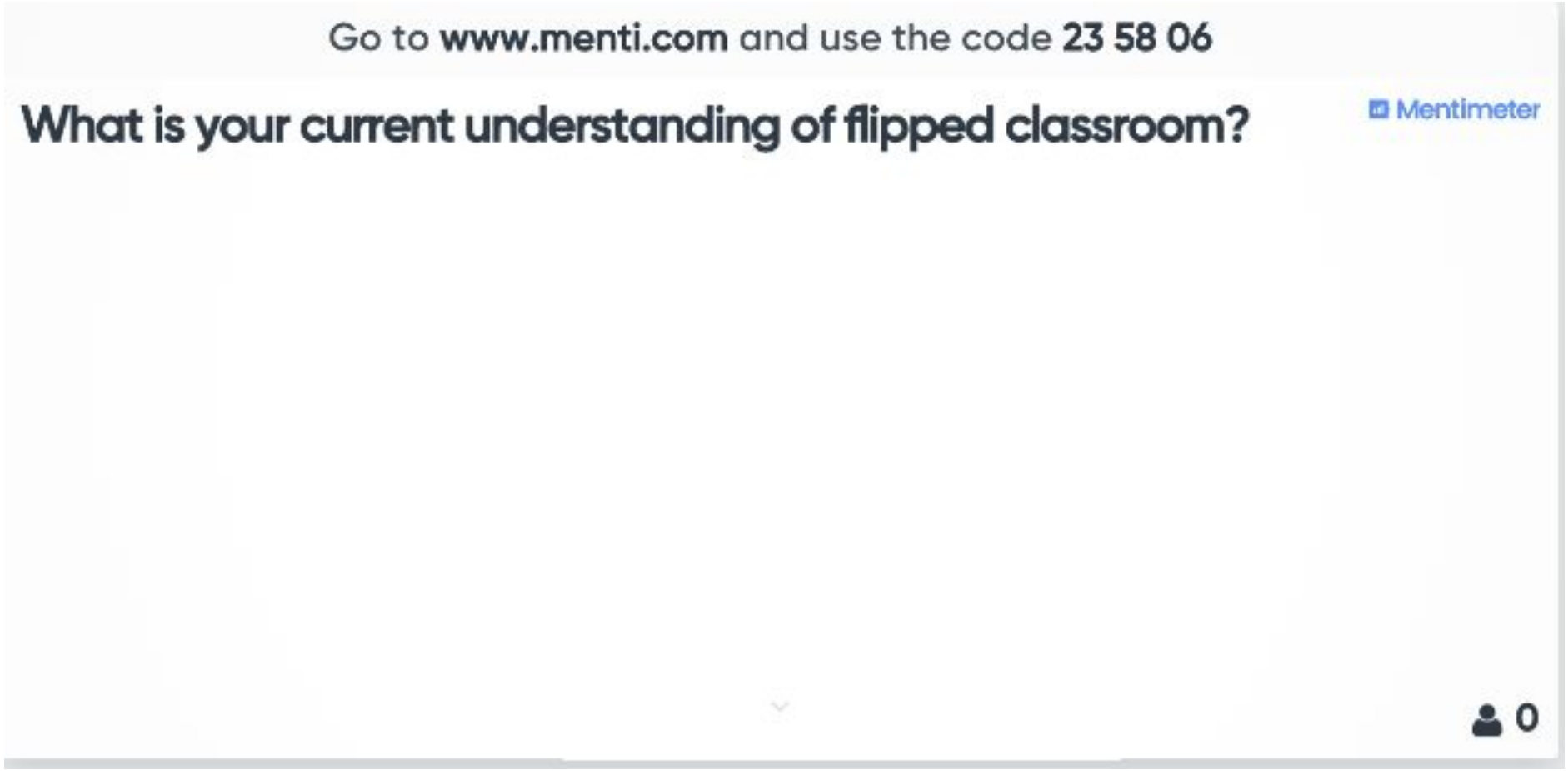
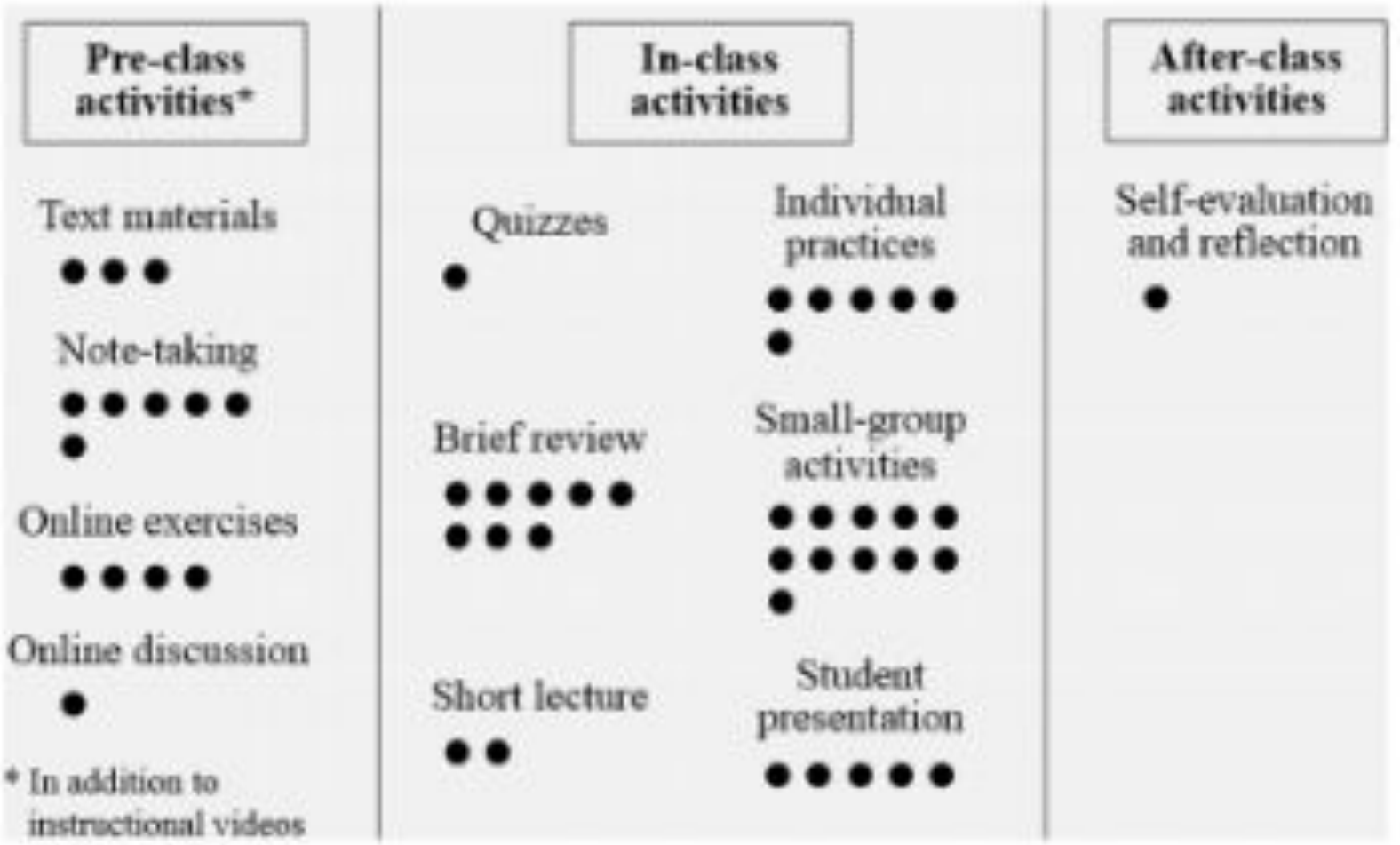
Presentation: http://bit.ly/2FR3PT7

Flipped Classroom



Flipped Classroom

- Lo & Hew, 2017



Today's Presentation

- We will explore design components of a flipped classroom
- We will consider alternatives to the traditional idea of a flipped classroom
- We will discuss implications and considerations for new designs

Flipped Classroom

- Most studies in higher education have compared performance of students in lecture vs. flipped versions of same course
  - (Day & Foley, 2006; Eichler & Peeples, 2016; Gross, Pietri, Anderson, Moyano-Camihort, & Graham, 2015; Maciejewski, 2015; Stone, 2012)
- Attention to design of flipped classroom learning environment has been historically neglected (O'Flaherty and Phillips, 2015)
  - Too much attention paid to pre-class activities
  - Lack of design frameworks available for instructors
- Naccarato and Karakok (2015) performed a qualitative study on flipped classroom in higher education
  - 19 faculty members at 14 institutions interviewed about reasons to flip
  - Common reasons include more student practice through exercises and application problems, increased collaborative learning opportunities, and more opportunities for higher-order thinking
- More recently, a two-semester study was completed in Calculus III (Wasserman, Quint, Norris, & Carr, 2017)
  - Students in flipped condition scored slightly higher on conceptual posttest items
  - Mixed perceptions from students with traditional students finding lecture more efficient

Design #1: Rotating Stations Setting

- Math 152 (Calculus II for Math/Physical Science Majors)
- 8-week summer course (MWF 6:00-8:45 PM) held in an "Active Learning Classroom" with an undergraduate Learning Assistant (LA)
- Mostly incoming sophomores and juniors
- Course content included volume/area, integration methods, sequences/series, polar, parametric, and the complex number system

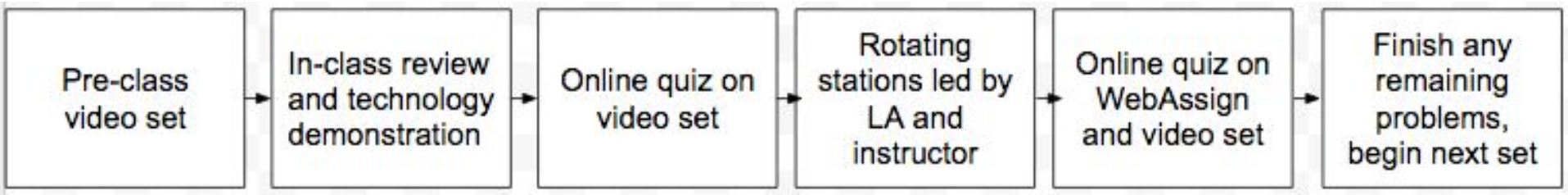
Problem Statement

- Math 152 has a high D/F/W rate
  - Dual challenges of difficult content and pacing
  - Students can come from Calculus 1 for Engineers or regular Calculus 1 into Math 152
- Coverage and Contact
  - Difficult to balance concepts, procedures, application, and rigor
  - Opportunities for interaction with students are limited

Purpose

- Class activities repurposed to fit a “flipped classroom” design
  - Content delivered outside of class via video
  - Class time is freed up for concept exploration and application
  - Learning assistant (LA) helps facilitate class activities and encourages discourse among students
  - More feedback opportunities for students

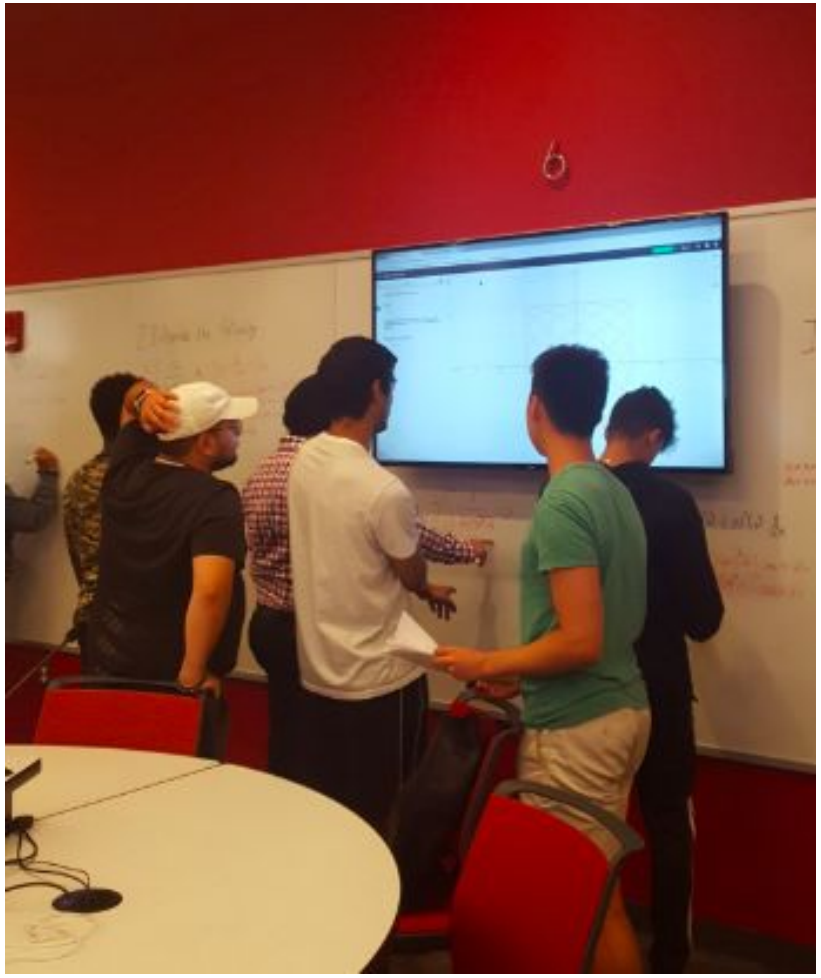
Class Flow


Pre-class video set
In-class review and technology demonstration
Online quiz on video set
Rotating stations led by LA and instructor
Online quiz on WebAssign and video set
Finish any remaining problems, begin next set

Rotating Stations

Each in-person class meeting had either two or three rotating stations, depending on whether there was a workshop for that evening.

- **Two stations**: 1 hour with the LA on the easier of the two topics and 1 hour with the instructor on the more difficult of the two topics. The instructor also worked with students on independent study as appropriate.
- **Three stations**: 40 minutes with the LA on the easier station, 40 minutes with instructor on the harder station, and 40 minutes at a "workshop" table that covered the previous class or two. The third station was spot checked by the instructor.

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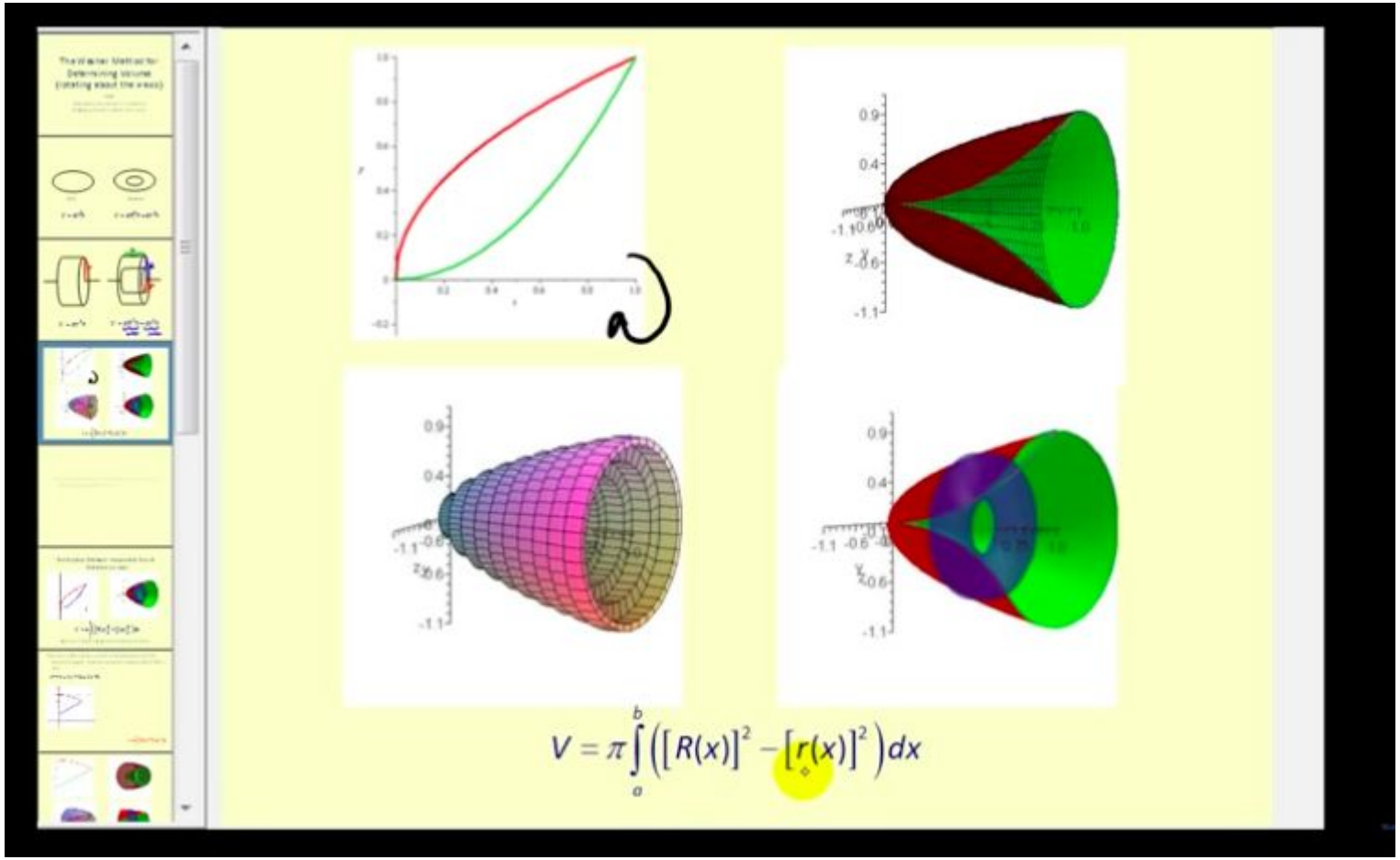
EXIT

Video Sets

Approximately 40-60 minutes of videos due before each class (6-8 videos, 6-7 minutes each)



In-Class Review

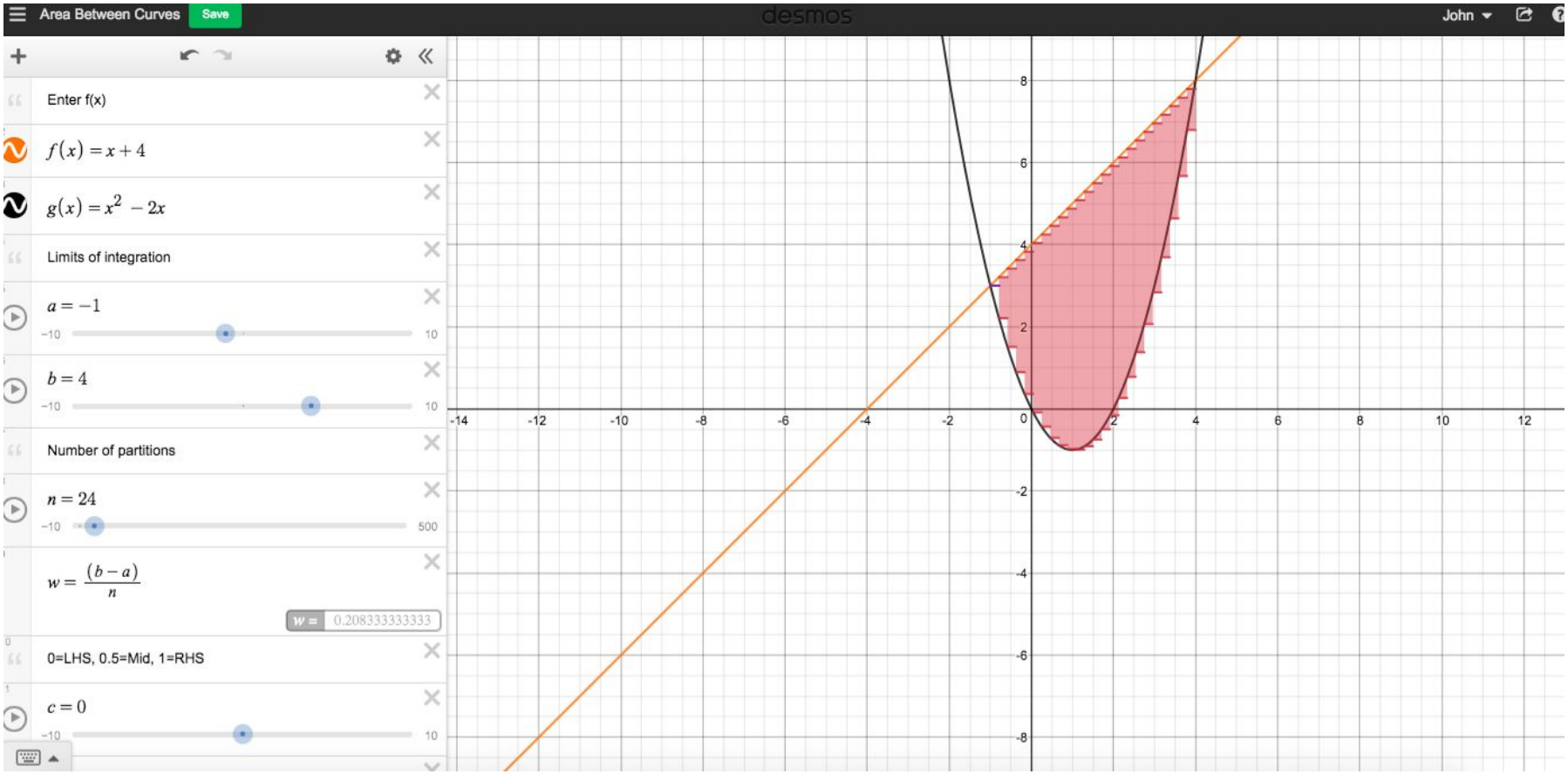
Partial Fractions Summary

The table below shows some common partial fraction decompositions. Before applying the method of partial fractions, be sure to do any long division if the degree of the numerator is greater than or equal to the degree of the denominator.

Form of Rational Function	Denominator	Form of Partial Fraction
$\frac{px+q}{(x-a)(x-b)}, a \neq b$	Distinct Linear Factors	$\frac{A}{x-a}+\frac{B}{x-b}$
$\frac{px+q}{(x-a)^2}$	Repeated Linear Factors	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	Distinct and Repeated Linear Factors	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	Linear Factor and an Irreducible Quadratic Factor	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$

In-Class Review

Desmos, GeoGebra, Maple



Stations

Class schedule:
6:00-6:10 Review and Q/A on Video Set
6:10-6:25 Quiz on Sakai
6:30-7:10 Stations: 7.2 OR 7.3 OR Workshop #2
7:10-7:50 Stations: 7.2 OR 7.3 OR Workshop #2
7:50-8:30 Stations: 7.2 OR 7.3 OR Workshop #2
8:30-8:45 Quiz on Sakai

***See Sakai study folder for a table on how to solve trigonometric integrals.**

Visual for trig substitution:
https://www.geogebra.org/m/nkBPcPRS

Applications:
Electrostatic integrals - solved using trigonometric substitution
Fluid pressure and force - determined using trigonometric substitution
Fourier Series derivation - determined using trigonometric integration strategies

Class schedule:
6:00-6:10 Key Concepts/Theory Review and Q/A on Video Set
6:10-6:25 Quiz on Sakai
6:30-7:30 6.2 Station OR 6.3 Station
7:30-8:30 6.2 Station OR 6.3 Station
8:30-8:45 Quiz on Sakai

Visuals for volume:
https://www.geogebra.org/m/BZWTCPfd - solids of revolution
https://www.geogebra.org/m/MEGkRFr3 - solids with cross sections

Visual for average value:
https://www.desmos.com/calculator/bkll9qv1pb

Applications to physical sciences:
Laminar flow

Stations

Workshops

Math 152, Workshop #1
Dr. John Kerrigan

1. **Problem statement** Suppose that f is a continuous function (defined for all x) and that the values of the following integrals are known:

$$\int_0^1 f(x)\,dx = 5\,;\quad \int_{-1}^1 f(x)\,dx = 3\,;\quad \int_0^2 f(x)\,dx = 8\,;\quad \int_0^4 f(x)\,dx = 11\,.$$

Evaluate these integrals:

a) $\int_0^2 f(2x)\,dx$ b) $\int_0^\pi \sin x f(\cos x)\,dx$ c) $\int_2^3 x f(8-x^2)\,dx$.

Hint Use substitutions, such as $u = \cos x$ in b).

2. **Problem statement** Suppose that the outdoor temperature (in ° F) on a particular day was approximated by the function

$$T(t) = 50 + 14\sin\left(\frac{\pi t}{12}\right),$$

where t is time (in hours) after 9 AM.

a) Find the *maximum* temperature $T_{\max}$, *minimum* temperature $T_{\min}$, and *average* temperature

$$T_{\text{aver}} = \frac{1}{12}\int_0^{12} T(t)\,dt$$

on that day during the period between 9 AM and 9 PM.

b) Show that $T_{\text{aver}} \neq \frac{1}{2}(T_{\min} + T_{\max})$. (This is the definition that the weather bureau uses for "average temperature".)

c) Show that if T is not given by the above formula, but rather $T(t)$ is a *linear* function of t, then $T_{\text{aver}} = \frac{1}{2}(T_{\min} + T_{\max})$. (Use either geometric reasoning or an integral.)

3. **Problem statement** Which has more area, the region in the first quadrant enclosed by the line $x + y = 1$ and the circle $x^2 + y^2 = 1$, or the region in the first quadrant enclosed by the line $x + y = 1$ and the curve $\sqrt{x} + \sqrt{y} = 1$? Justify your answer. Include a sketch of the regions discussed.

Takeaways

- Students commented on their appreciation of having two people to ask for help during class
- Students did more research during class (checked answers online, looked up identities, applications, etc.)
- Most students found the flipped classroom approach more favorable than the lecture approach
- Exam averages this summer were approximately 3-4% higher per exam with harder exams given

Future Considerations

- Embedded questions in videos to count as a participation grade
- Three stations felt overwhelming when content was dense
- Strategies needed to deter students from gaming WebAssign during class
- Role of the LA

Design #2: Flip w/ Productive Failure Setting

- Math 103 (Topics in Math for Liberal Arts)
- 16-week academic year course
- Hybrid (met once a week for 80 minutes vs. twice a week for 80 minutes)
- Non-math majors mostly majoring in education, political science, etc.
- Course content included voting theory, measuring power, growth and finance, apportionment, fair division, and fair distribution

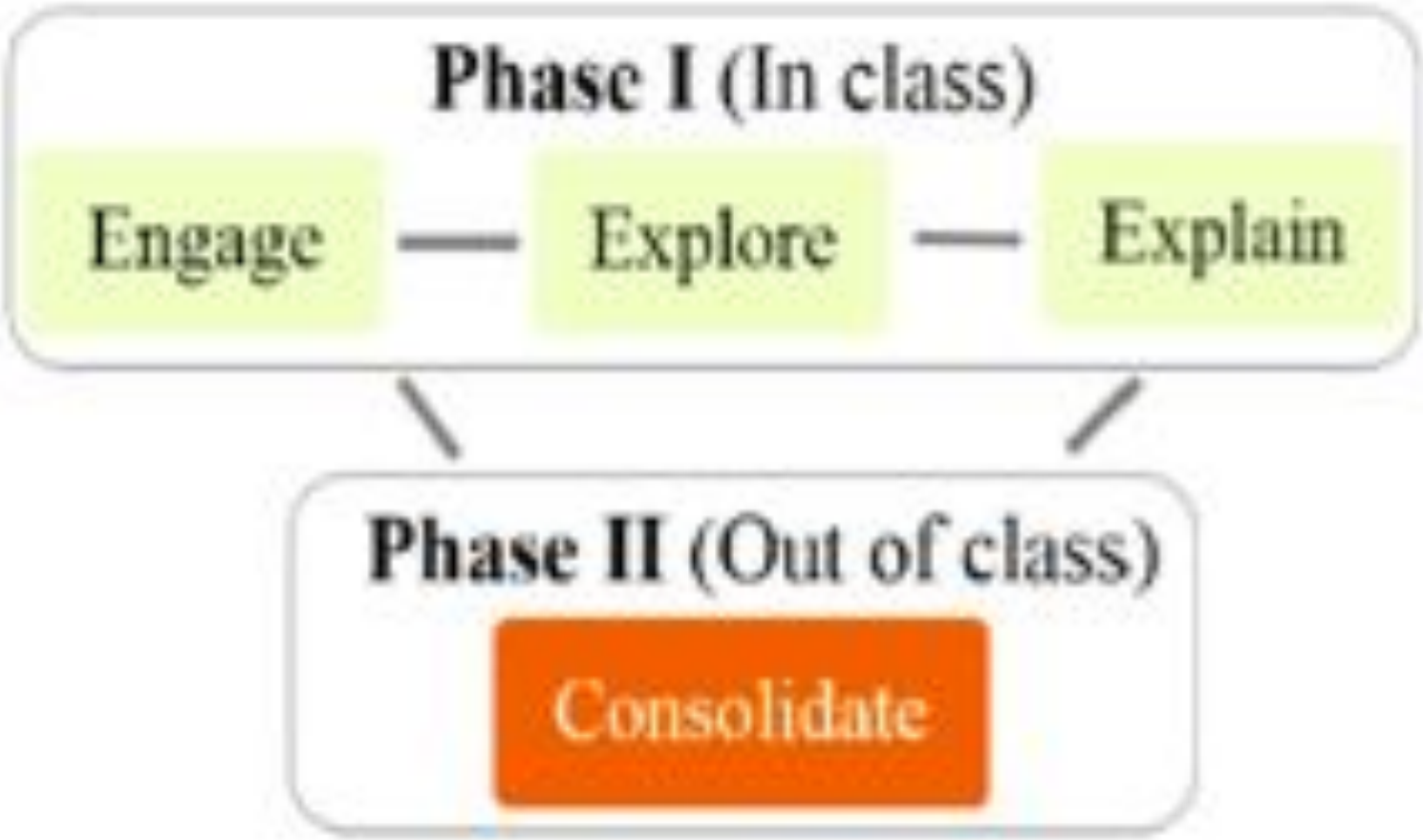
Problem Statement

- Students sometimes have trouble bridging at-home learning to in-class learning
  - Not sure how to begin problems
  - Not sure how to transfer knowledge from videos to class exercises
- Exams reflect this difficulty
  - When asked to apply concepts, students historically struggle in this area

Purpose

- Class activities repurposed to fit a “productive failure flipped classroom” design
  - At the end of each in-person class, students were given a problem (invention task) to solve that they had never received instruction on before
  - They work on this problem for 10 minutes or so in a group, ultimately “failing”
  - Subsequent video instruction at home allows for consolidation of content
  - “Productive” application of learned content/failure experience has been proven to help students apply concepts later on, especially on exams (Kapur, 2008; Kapur & Bielaczyc, 2012)

Productive Failure



Class Flow

Sequence of activities in the DIFC and PFFC learning environments

Learning Environment	End of in-person class #1 (15 min)	Outside of class preparation	Start of in-person class #2 (10 min)	Middle of in-person class #2 (55 min)	End of in-person class #2 (15 min)
DIFC	Additional practice problem on current topic	Work through a set of 13-15 videos	Quiz on videos	Solve 5-6 problems facilitated by teachers and peers	Additional practice problem on current topic
PFFC	Invention task on new topic	Work through a set of 13-15 videos	Quiz on videos	Solve 5-6 problems facilitated by teachers and peers	Invention task on new topic

Sample Invention Task

Invention Task #1

A new country has recently been founded. The country is split into six states, call them A, B, C, D, E, and F. The population of state A is 1,646,000 people, the population of state B is 6,936,000 people, the population of state C is 154,000 people, the population of state D is 2,091,000 people, the population of state E is 685,000 people, and the population of state F is 988,000 people. There are 250 seats available on a legislative body to govern the new country. How many seats should be assigned to each state so that each state would receive a fair representation? Show your work and justify why you think your method is correct.

Sample In-Class Problems

Suppose that Wendy knows Xavier's valuation of the house, and writes down $340,000 as her bid (even through the house is actually worth $300,000 to her). Xavier still writes down $350,000 as his bid. When they carry out the Method of Sealed Bids, how much is the total value of Wendy's share, and how does it compare to what she receives when she bids honestly as in HW1? How much is the total value of Xavier's share here, and is it a fair share? (Weingart & Seneres, 2013).

HW7. This problem pertains to the Alabama paradox.

a. Suppose that the Gesundheit Hospital described in problem HW1 above hires one more nurse, for a total of 401, but the number of patients on each shift remains the same. Find the Hamilton apportionment.

b. Does increasing the number of nurses from 400 to 401 create an instance of the Alabama paradox? Explain how you can tell.

c. Now suppose that the same hospital hires yet another nurse, for a total of 402, but the number of patients on each shift again remains the same. Find the Hamilton apportionment.

d. Does increasing the number of nurses from 401 to 402 create an instance of the Alabama paradox? Explain how you can tell. (Weingart & Seneres, 2013).

How Did Productive Failure Help?

- Students generated more solution proposals during class time in the productive failure section
- Students on average needed less instructor feedback in the productive failure group
- Video watching frequencies and exam scores were not different
- In follow-up interviews, productive failure students described learning in terms of their peers, while students in the traditional flipped classroom described learning in terms of their instructor more often

Considerations/Implications

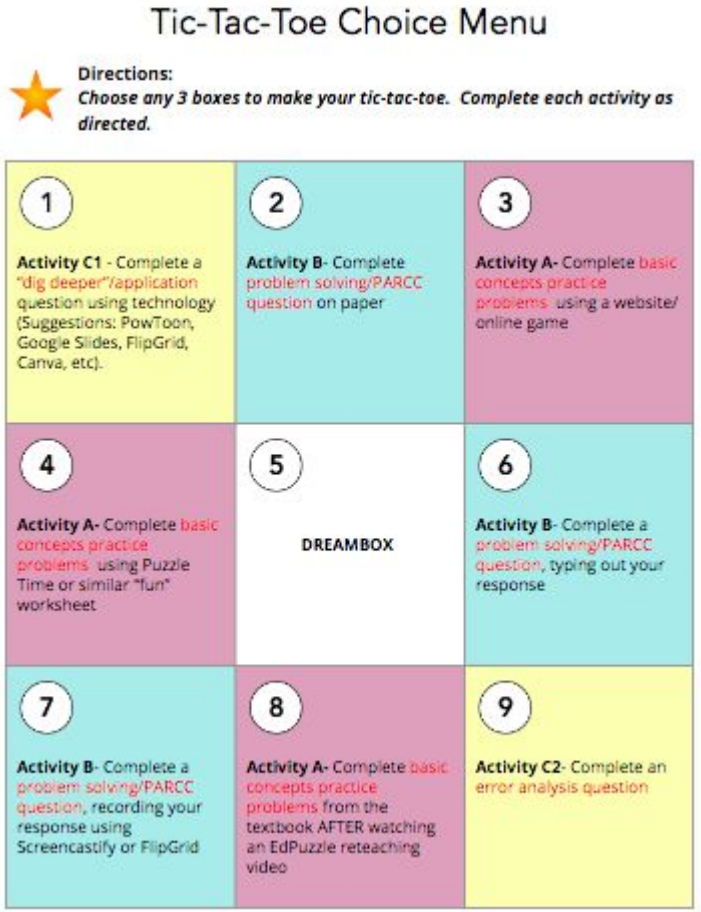
- Flipping isn't just inverting lecture and homework
- Ideas like rotating stations and productive failure support more meaningful collaboration and problem solving during class time
- The instructor's role in both situations is one of a facilitator
- In both settings, tracking video watching and embedding questions into the videos is encouraged to ensure students are working through the videos.
- In the rotating station model, one future iteration could include an "optional playlist" of pre-requisite skills, followed by a "mandatory playlist" of core content.

NCTM Standards

- Make sense of problems and persevere in solving them.
- Model with mathematics.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Look for and express regularity in repeated reasoning.
- Attend to precision.
- Look for and make use of structure.
- Model with mathematics.

Future Directions for K-12

- Combining flipped classroom with personalized learning
  - Mandatory and optional videos
  - Choice boards



Tic-Tac-Toe Choice Menu

Directions:
Choose any 3 boxes to make your tic-tac-toe. Complete each activity as directed.

1 **Activity C1** - Complete a "dig deeper"/application question using technology (Suggestions: PowToon, Google Slides, FlipGrid, Canva, etc).	2 **Activity B**- Complete problem solving/PARCC question on paper	3 **Activity A**- Complete basic concepts practice problems using a website/ online game
4 **Activity A**- Complete basic concepts practice problems using Puzzle Time or similar "fun" worksheet	5 **DREAMBOX**	6 **Activity B**- Complete a problem solving/PARCC question, typing out your response
7 **Activity B**- Complete a problem solving/PARCC question, recording your response using Screencastify or FlipGrid	8 **Activity A**- Complete basic concepts practice problems from the textbook AFTER watching an EdPuzzle reteaching video	9 **Activity C2**- Complete an error analysis question

Flipping AP Classes

- Flipping AP Calculus and Statistics allows for deeper diving during class time
  - EdPuzzle videos at home
  - Khan Academy practice multiple choice to assess understanding (repeated attempts for mastery)
  - In-class demonstrations and applications including AP multiple choice and free response
  - Opportunities for PBL