Moving from Hands-on to Abstract: Teaching the Way Students Learn

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Why Use Manipulatives

- What the research says:
- Psychology
  - Piaget – children think more concretely when they are younger, and more abstractly as they age
  - Bruner – children progress through stages
    - Enactive—interacting with physical materials
    - Iconic – using pictures to visualize
    - Symbolic – using symbols to represent the concepts abstractly
  - Manipulatives serve as analogies or metaphors for difficult to understand concepts linking to prior knowledge. Manipulatives are “objects designed to represent explicitly and concretely mathematical ideas that are abstract” (Moyer, 2001: p. 176).

Role of Manipulatives

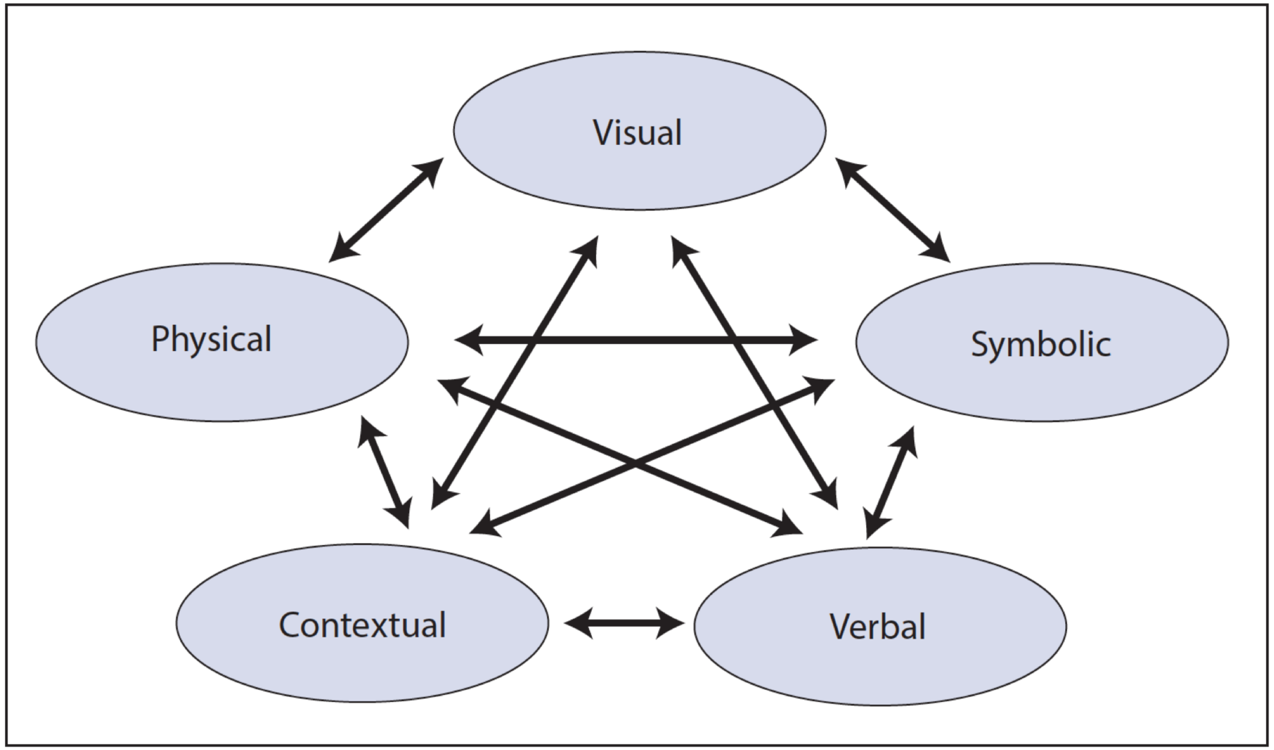
CCSS Mathematical Practices

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Principles to Actions

An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking (p. 78).

Translating Between Representations



(Principles to Actions, 2014, p. 25)

When and How to Use Manipulatives

We need to:

- have a specific purpose for using the manipulative, not just “for fun.”
- allow students to play at first (they will anyway)
- know when manipulatives aid understanding and when they distract
- represent concept, not necessarily detail
- Embed the use of manipulatives in a familiar or appealing scenario
- know how to use the materials to help students move from hands-on to abstract

Motivating and Planning the Lesson—Things to Consider

- What is our “hook” or “launch” before the use of manipulatives?
- What do we want students to know and be able to do after the lesson?
- How are we expecting the students to use the manipulatives? Are they familiar with this manipulative?
- When do we move to pictorial from hands-on, if we can? Is the lesson structured to support this transition?
- When do we move to abstract?

Some Activities

- Tiling pools – Algebra
- Giant Rubik’s™ cube – Algebra and Geometry
- What is Average? – Statistics
- Sticks and Stones – Expected value

Tiling Pools

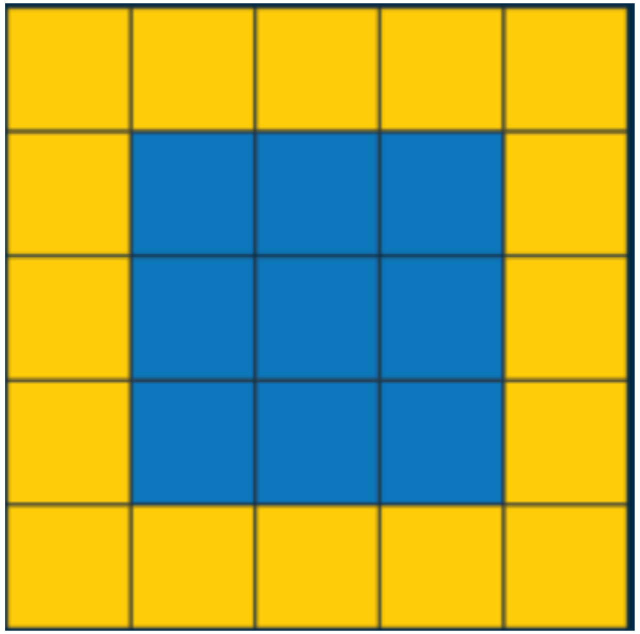
- Let's build a square pool with a border around it. Start with one square tile, how many square tiles does it take to make a border?

Tiling Pools

- Let’s build a square pool with a border around it. Next move to two by two square, how many square tiles does it take to make a border?

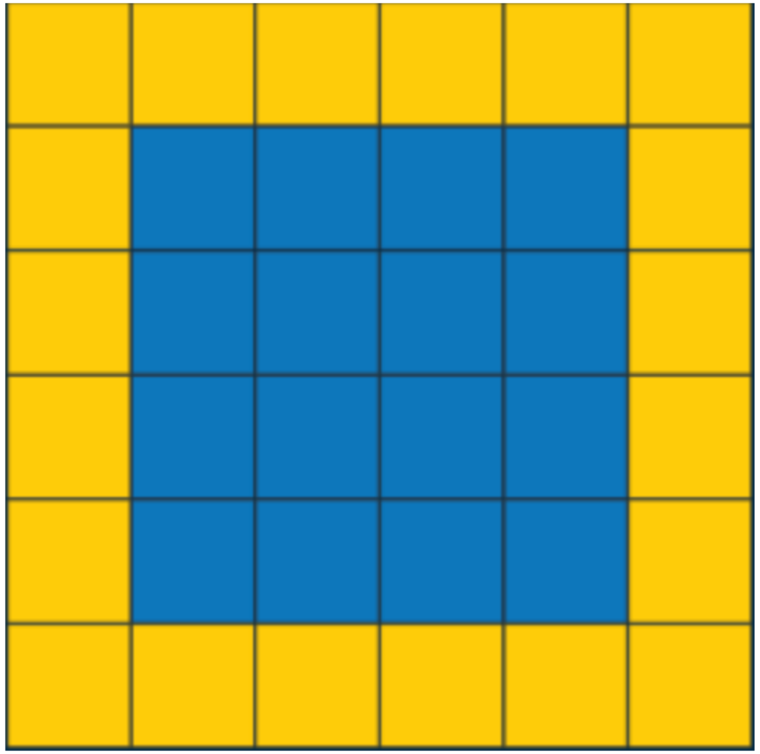
Tiling Pools

- Let’s build a square pool with a border around it. Next move to three by three square, how many square tiles does it take to make a border?



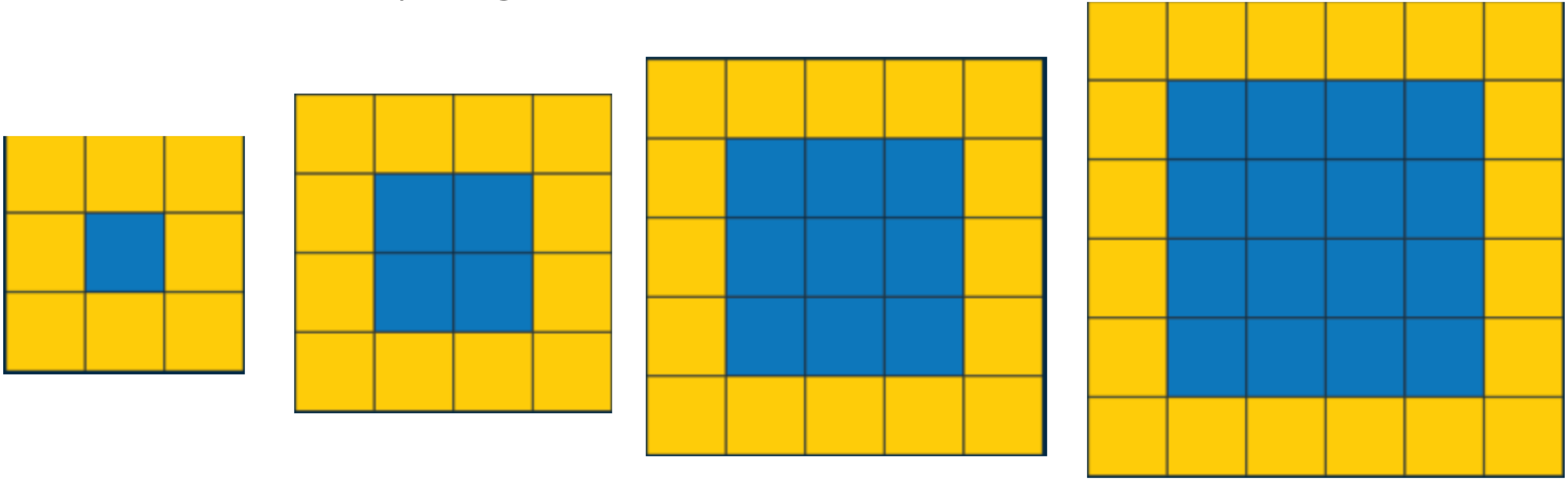
Tiling Pools

- Let's build a square pool with a border around it. Next move to four by four square, how many square tiles does it take to make a border?



Tiling Pools

- Can we see a pattern in the number of tiles needed to make the border as the pool grows?



Tiling Pools – Organizing the Information
What Patterns Do You See?

Side of Pool	Border
1	8
2	12
3	16
4	20

Tiling Pools – Organizing the Information
What Patterns Do You See?

Side of Pool	Border	
1	8	
		4
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		4
3	16	
		4
4	20	

The number of border tiles increases by four each step.

Tiling Pools – Analysis Question

Side of Pool	Border	
1	8	
		4
2	12	
		4
3	16	
		4
4	20	

How many tiles would it take to border a pool with side length zero?

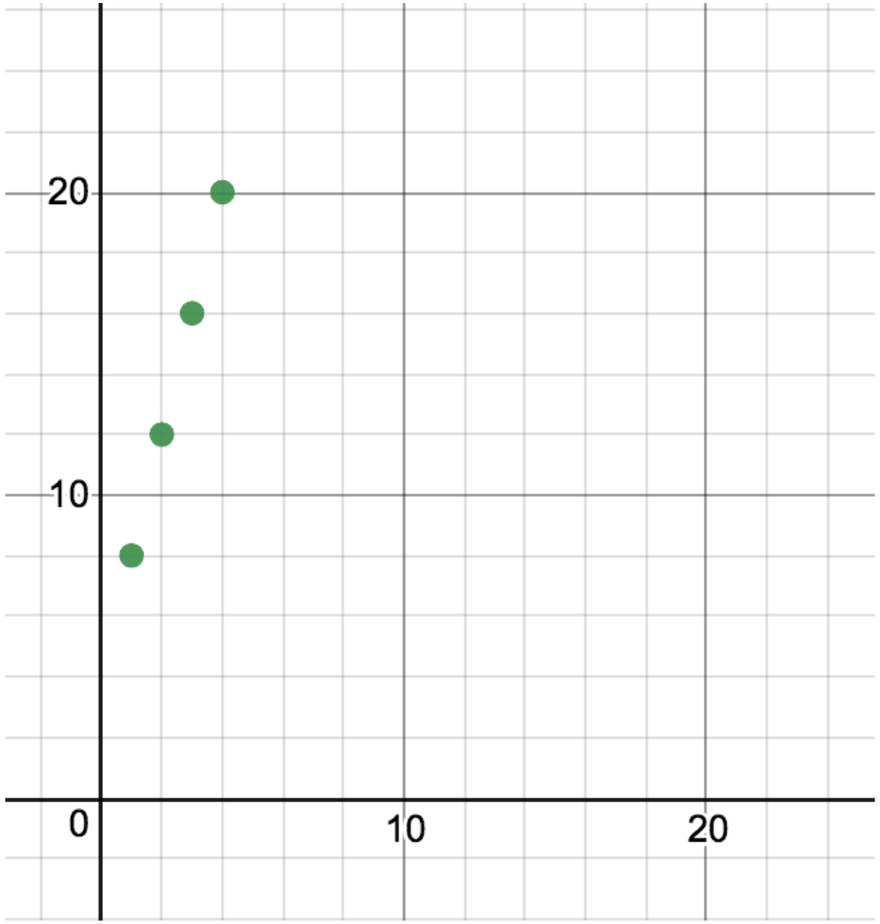
Tiling Pools – Can You Describe the Pattern With a Formula?

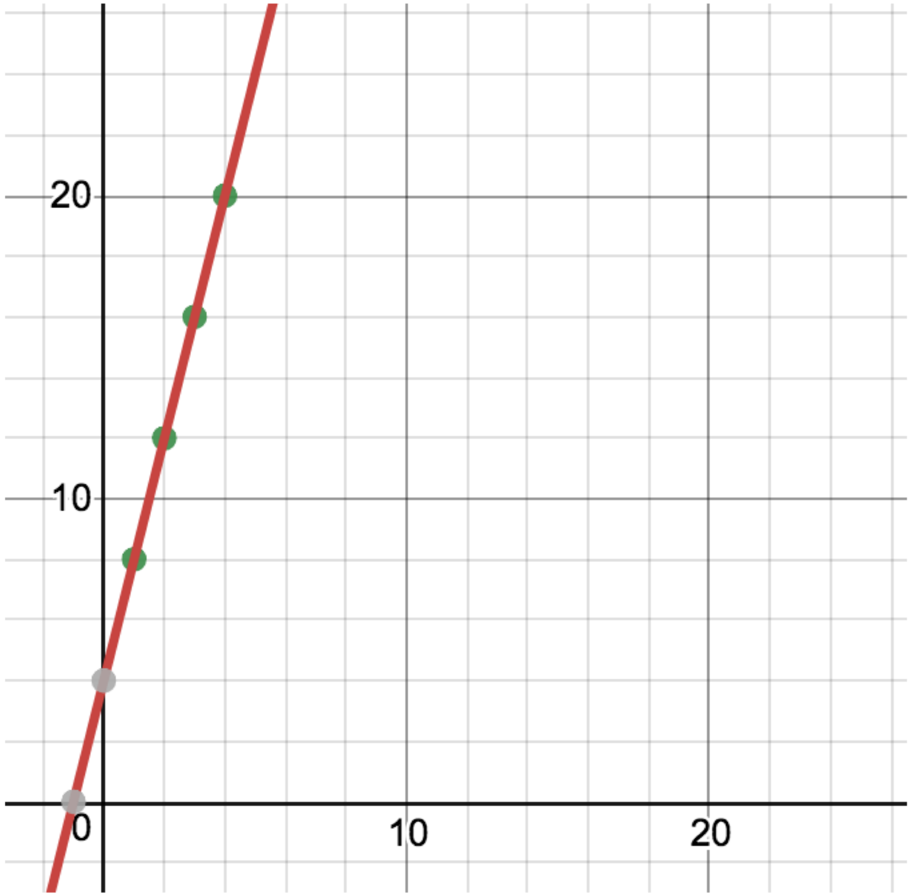
Side of Pool	Border	
1	8	
		4
2	12	
		4
3	16	
		4
4	20	

Next = Now + 4

$B = 4P + 4$

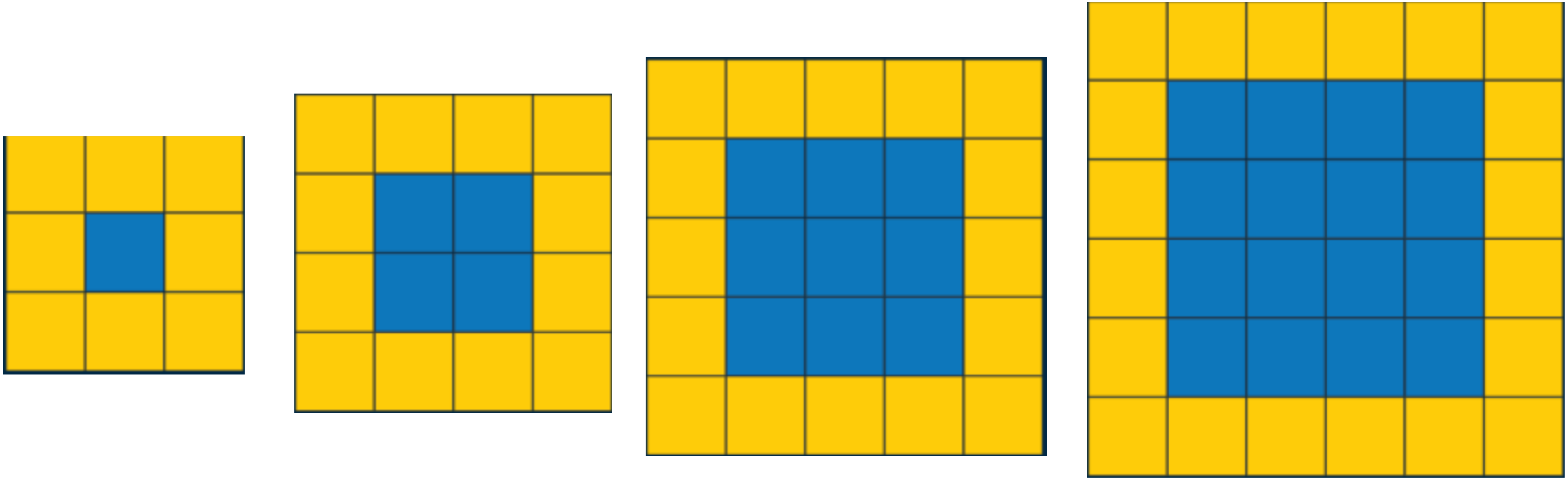
What do These Data Look Like as a Graph?



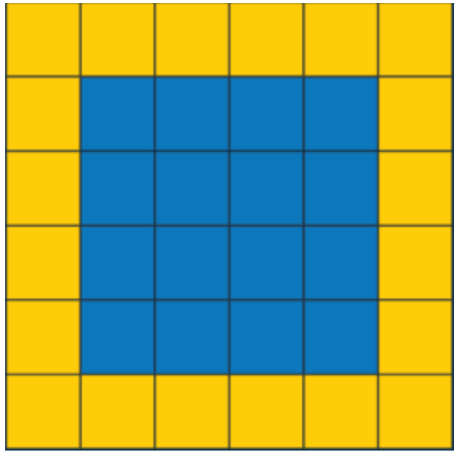


Tiling Pools

- Where do you see the four growing tiles?



Tiling pools



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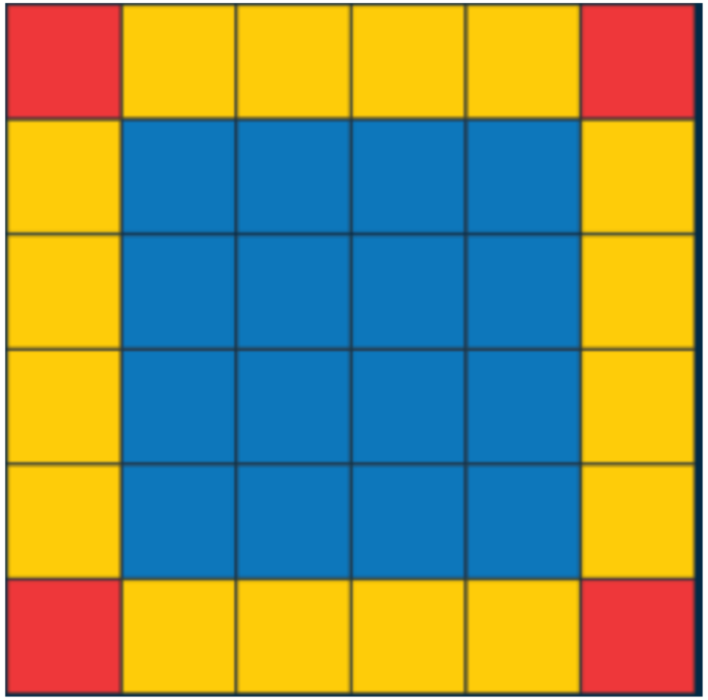
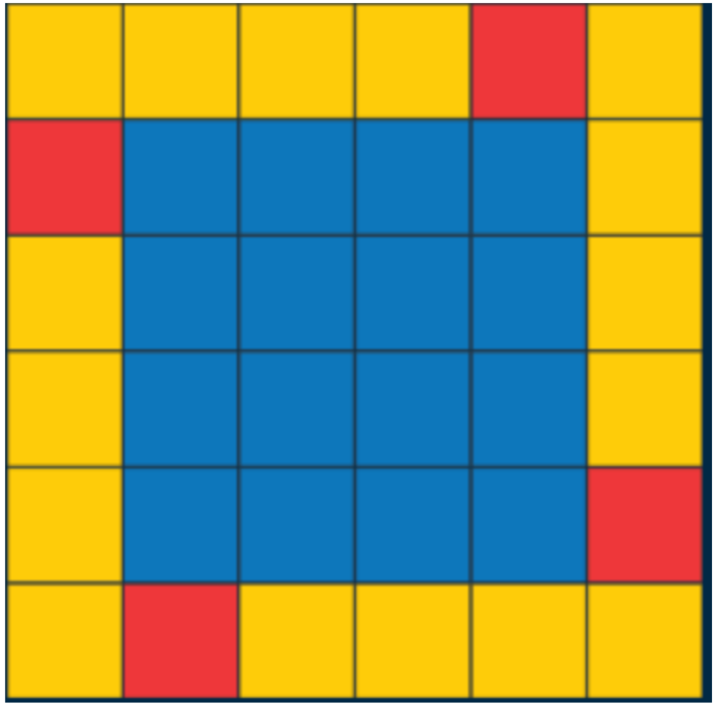
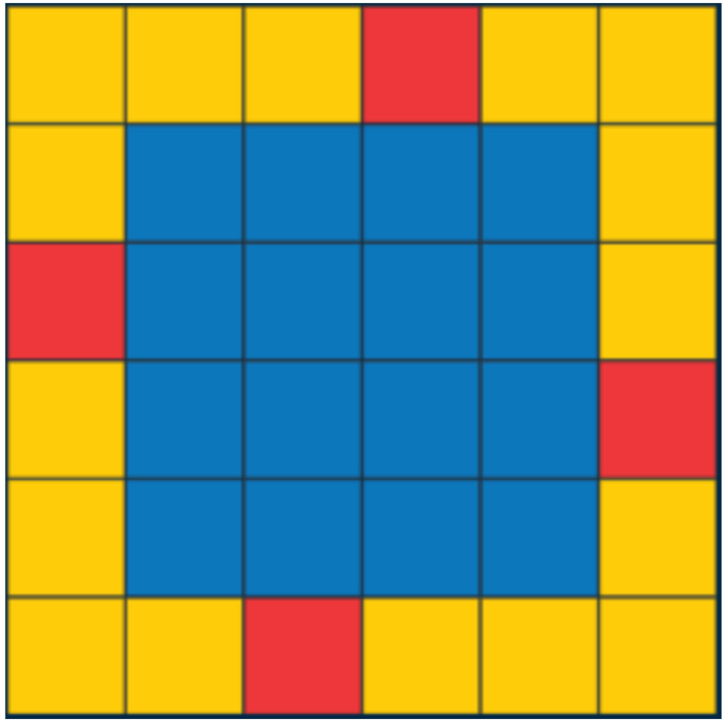
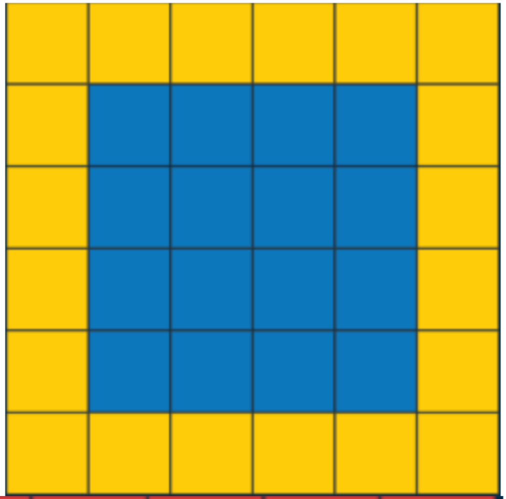


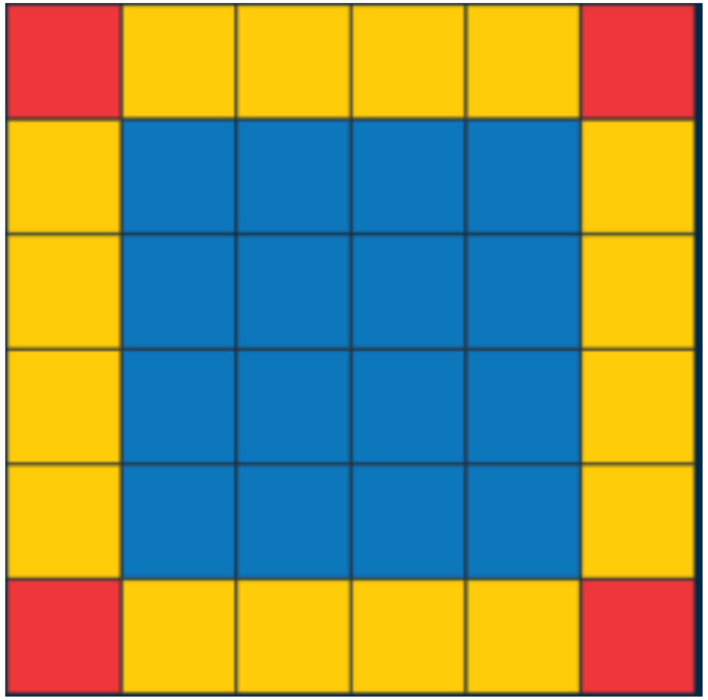




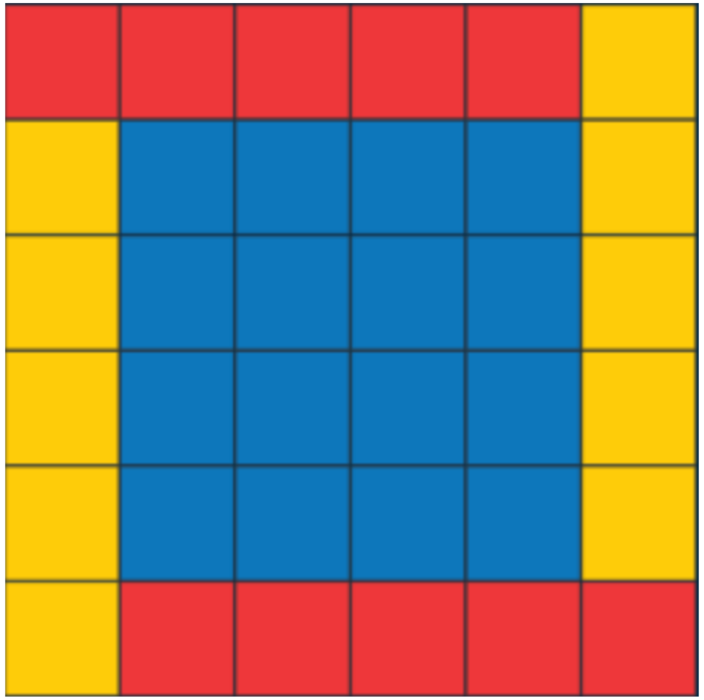
Tiling pools



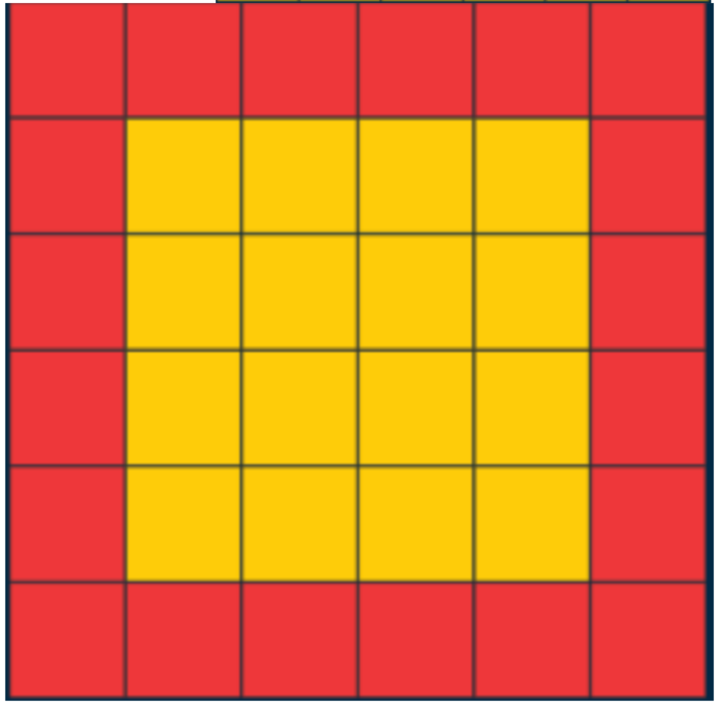
- How do you see the formula?



B=4n+4



B=4(n+1)



B=$(n+2)^2$ - n^2

Rubik's™ Cube Sculpture

- Prank sculpture Astor Place, East Village, NY:

http://www.alltooflat.com/pranks/cube/

What if We Made a HUGE Rubik's™ Cube Sculpture?

(photo: Meet Minneapolis)

Enchanted Highway (ND)

https://thehotflashpacker.com/enchanted-highway-regent-north-dakota/

The Envisioned Cube is So BIG, There Are Conditions

- We need to be able to transport it, so we are going to make it in pieces
- We need it to be strong, so we are going make it out of smaller cubes we can stack for structure
- We want to save money and only paint the sides we need
- We are going to have particular groups painting particular types of smaller cubes

Preparing to Paint

- Use the cubes you have to build a model of the Rubik's™ Cube (ignore colors for now)—how big is a normal Rubik's™ Cube?
- How many cubes do we need altogether?
- Are they all going to be painted on the same number of sides?
- How many on 1 side? 2 sides? 3 sides? Any painted on no sides? Make a table to organize your data.
- What if we make a bigger cube? Rubik's™ cube is 3 x 3 x 3. Have you seen others that are 4 x 4 x 4? Determine the the same info for a 4 x 4 x 4 cube.

Table for Painting Sides

Dimensions of large cube	Total number of smaller cubes	Number painted on 0 sides	Number painted on 1 side	Number painted on 2 sides	Number painted on 3 sides
3 x 3 x 3					
4 x 4 x 4					
5 x 5 x 5					
n x n x n					

What Do the Formulas Have to Do With the Shape of the Cube?

- How did you decide the number of cubes with three painted sides? Where are they?
- Two painted sides?
- One painted side?
- No painted sides?

Working in the Abstract

- What should be the sum of the differently painted cubes? How many cubes do we have in an n x n x n cube?
- How can you show that you found the correct formulas? Do the formulas add up to the number of cubes you should have?

Table for Painting Sides

Dimensions of large cube	Total number of smaller cubes	Number painted on 0 sides	Number painted on 1 side	Number painted on 2 sides	Number painted on 3 sides
3 x 3 x 3	27	1	6	12	8
4 x 4 x 4	64	8	24	24	8
5 x 5 x 5	125	27	54	36	8
n x n x n	n^3	$(n-2)^3$	$6(n-2)^2$	$12(n-2)$	8

Sum of the Differently Painted Cubes

= three sides + two sides + one side + no sides

$= \quad \mathbf{(n\text{-}2)^3 \quad + \quad 6(n\text{-}2)^2 \quad + \quad 12(n\text{-}2) \quad + \quad 8}$

$\mathbf{= n^3 - 6n^2 + 12n - 8 + 6n^2 - 24n + 24 + 12n - 24 + 8}$

$= \mathbf{n^3 - 0n^2 + 0n + 0}$

$= \mathbf{n^3}$

WE DID IT!!

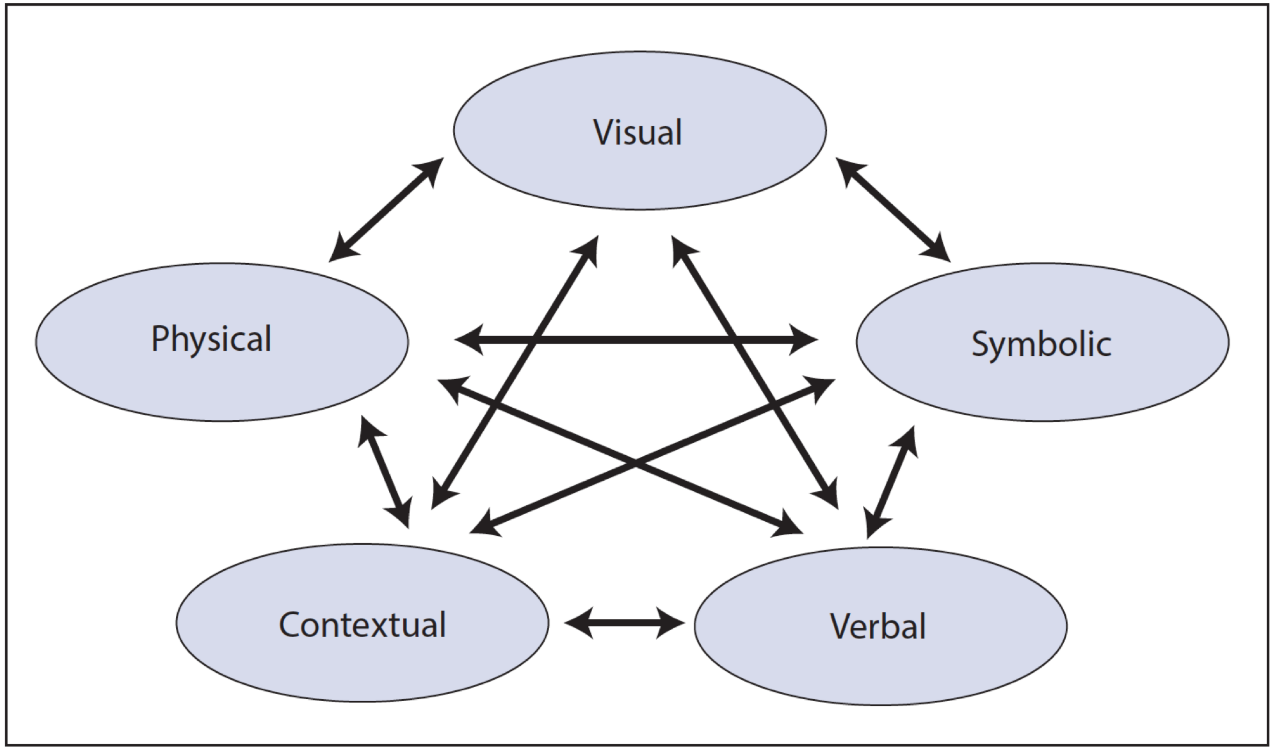
Clarifying an Abstract Concept With a Hands-on Activity

- Students sometimes know a mathematical process but don't know why they do it
- An activity with manipulatives can help clarify that process

What is “Average”?

- Kinesthetic activity
- Not algebra generating
- The real meaning of the mean

Translating between representations



(Principles to Actions, 2014, p. 25)

Summarizing Manipulatives

- Strong base in psychology
- National support in current mathematics education community
  - CCSS Mathematical Practices
  - Principles to Actions
  - AMTE Standards for Preparing Teachers of Mathematics
- Use manipulatives purposefully and intentionally to support student learning