ENGAGING IN PROBLEM SOLVING –

FOCUS ON STRATEGIES, COMMUNICATION, FUN WITH MATHEMATICS

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SESSION DESCRIPTION

- Collaborating on solving interesting yet challenging problems helps us to develop deeper mathematical understanding and improves our mathematical communication skills. We will work in small and large groups to develop solutions to a variety of problems and focus on different strategies used as well as associated mathematical content.

AGENDA

- The Walking Man
- The Census
- X and O
- Checkerboard Problem & Embedded Rectangles
- Friday the 13th
- Said the Spider to the Fly
- Walking Problem
- Crossing the River with Dogs
- The Camel Problem

PROBLEM SOLVING STRATEGIES

- Think with Algebra rates
- Eliminating Possibilities
- Working Backwards
- Look for Patterns
- Geometry, Visual Representation
- Logical Thinking
- Act it Out
- Organize Your Thinking
- Communication skills

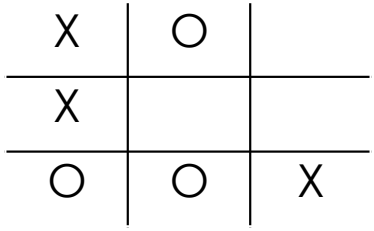
- The Walking Man – algebraic relationships, rates
- The Census – Eliminating Possibilities
- X and O – Working Backwards
- Checkerboard Problem & Embedded Rectangles – Look for patterns
- Friday the 13th – Look for Patterns
- Said the Spider to the Fly – Geometry, Visual Representation
- Walking Problem – Working Backwards and Counting
- Crossing the River with Dogs – logical thinking, act it out, organizational, communication skills
- The Camel Problem – logical thinking

THE WALKING MAN

- A man walked for 5 hours, first along a level road, then up a hill, and then he turned around and walked back to the starting point along the same path. He walks 4 mph on the level, 3 mph uphill, and 6 mph downhill. Find the distance he walked.
  - Jim Wilson's page – a problem from Lewis Carroll "A Tangled Tail"
  - http://jwilson.coe.uga.edu/emt668/EMT668.Folders.F97/Rapley/EMT725/others/tangle.html

THE CENSUS

- During the recent census, a man told the census-taker that he had three children. When asked their ages, he replied, "The product of their ages is 72. The sum of their ages is the same as my house number."
- The census-taker ran to the door and looked at the house number. "I still can't tell," she complained.
- The man replied, "Oh, that's right. I forgot to tell you that the oldest one likes chocolate pudding."
- The census-taker promptly wrote down the ages of the three children. How old are they?
  - Jim Wilson's Page
  - http://jwilson.coe.uga.edu/emt725/census/census.html

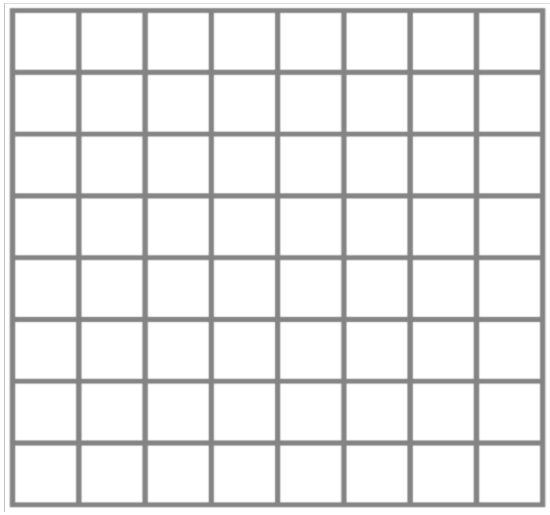


X AND O

- The game of tic-tac-toe is played in a 3x3 square with nine small squares.
  1. Each of two players in turn places his/her mark (X or O) in a square
  2. The player who first gets three marks in a horizontal, vertical, or diagonal line wins.
  3. A player will <u>ALWAYS</u> place his/her mark in a line that already contains
     - A) Two of his or her marks
     - B) or Two of his or her opponent's marks.
- Priority is given to A over B because it would lead to a victory.
- Knowing that 2 players <u>are following these rules</u>…who will win this game?
  - P.296
  - Crossing the River with Dogs: Problem Solving for College Students, Ken Johnson, Ted Herr, Judy Kysh, 2004

CHECKERBOARD PROBLEM

- How many squares (of any size) are there on an 8x8 checkerboard?
- How about an n x n checkerboard?
  - Jim Wilson's page
  - http://jwilson.coe.uga.edu/emt725/7-11/checkerboard.html



EMBEDDED RECTANGLES

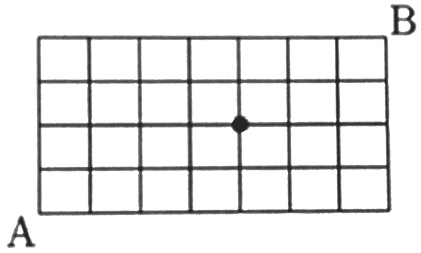
- This figure is a 2 row × 3 column rectangle. How many of each type of rectangle can you find?
- Draw a 3 row × 4 column rectangle. Find the number of each type of rectangle. Look for patterns
- Without drawing and counting: Predict the number of rectangles in a 6 row × 5 column rectangle
- Predict the number of rectangles in an "n row x m column" rectangle.
  - Jim Wilson's page
  - http://jwilson.coe.uga.edu/emt725/7-11/checkerboard.html

FRIDAY THE 13TH

- Prove that there is at least one Friday the 13th in some month during each year. Next, prove that there are at most three Friday the 13ths any given year.
  - Jim Wilson's Page
  - http://jwilson.coe.uga.edu/emt725/Fri13/Fri13.html

SAID THE SPIDER TO THE FLY

- A room is 30ft long (North and South walls), 12 feet wide (East and West walls), and 12 feet high. A spider on the centerline of the west wall of the room and 1 foot above the floor sees a fly asleep on the centerline of the east wall and 1 foot below the ceiling. The spider wants to get to the fly as quickly as possible. What is the shortest path for the spider to take to get the fly, and what is the length of the path (reminder: spiders do not have wings)?
  - P.460
  - Crossing the River with Dogs: Problem Solving for College Students, Ken Johnson, Ted Herr, Judy Kysh, 2004



THE PATHS PROBLEM

- Imagine that the 4 x 7 grid below represents the streets of a part of a city. You must walk 11 blocks to get from the lower left corner at point A to the upper right corner at point B.
- How many different 11 block walks are there?
- How many 11 block walks go through the terrible corner (marked with a bullet)?
- How many 11 block walks avoid the terrible corner?
- How many different 12 block walks are there from A to B?
  - P.95
  - Big Ideas in Mathematics for Future Middle Grades Teachers and Elementary Math Specialists: Big Ideas in Algebra, John Koker and Jennifer Szydlik, 2010

THE CAMEL PROBLEM

- A camel driver has 3000 bananas. The camel can carry up to 1000 bananas at a time. The driver wants to deliver bananas from one town to another town that is 1000 kilometers across the desert. However, his camel refuses to move unless fed a banana for every kilometer that it travels. What is the maximum number of bananas the driver can get across the desert to the other town? (Hint: The bananas will not spoil in the desert).
  - NCTM Math Forum
  - http://mathforum.org/dr.math/faq/faq.camel.html

QUESTIONS/COMMENTS?

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X and O

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- Knowing that 2 players are following these rules...who will win this game?

X	O	
X		
O	O	X

X and O

- What could the last play have been?
- If 'O' just played
- -Could it be top center?
- No. O would have played in the middle to block
- -Could it be bottom left?
- No. O would have played in the middle to win.
- -Could it be bottom middle?
- No. O would have played in the middle to block

X	O	
X		
O	O	X

X and O

- What could the last play have been?
- If 'X' just played
- -Could it be top left?
- No. X would have played in the middle to block
- -Could it be left middle?
- No. X would have played in the middle to win.
- -Could it be bottom right?
- Yes. X could have played here without violating any of the 'rules'

X	O	
X		
O	O	X