Routines for Developing Number Sense and Equation Sense with High School Students

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Context

- ➔ Seeing Structure
- ➔ Background of CATCH
- ➔ Math 10 Class

Seeing Structure

How do we think about structure in high school?

- ➔ Interpret parts of expressions (terms, factors, coefficients)
- ➔ Use properties of arithmetic to their advantage
- ➔ Develop connections between similar forms

$$x^2 + 5x + 6$$

$$(x-5)^2 + 5(x-5) + 6$$

Background

(CATCH) Creating Algebraic Teaching Communities for Hoosiers Grant

Math 10 Class offered at FJ Reitz High School

- ➔ At-risk students only
- ➔ Concurrent enrollment in Geometry (and some Algebra 1)
- ➔ Given autonomy (only teacher in building)
- ➔ Explicit instruction on growth mindset
- ➔ Embedded problem solving and numeracy experiences

Number Talks

- ➔ Norms
- ➔ Samples
- ➔ Why?

Norms of Number Talks

- ➔ “Rough Draft” thinking is welcome
- ➔ Everyone’s ideas are valued
- ➔ Everyone should be prepared to justify
- ➔ Respect others’ thinking
- ➔ Nothing in hands, Nothing on desk

Sample Number Talk

$$16 \cdot 25$$

Advanced Number Talk

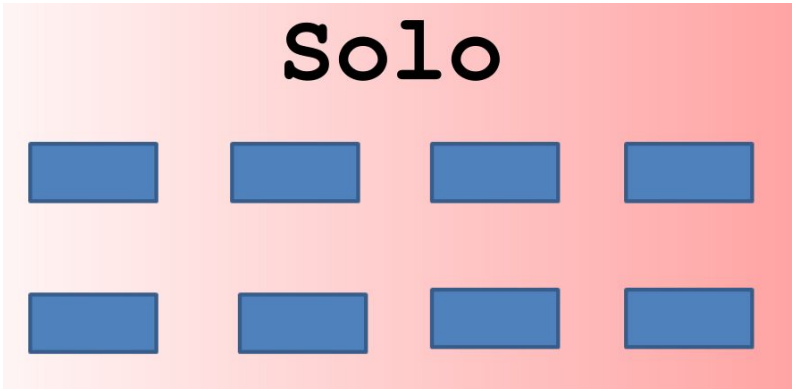
Harris (2011)

$$\frac{5}{6}\pi + \frac{1}{2}\pi$$

Why Number Talks in High School?

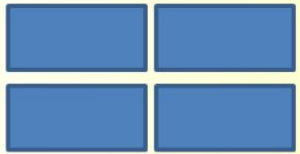
- ➔ Meet kids at their level
- ➔ Builds a culture that accepts struggle
- ➔ Engages in real mathematics
- ➔ Creates connections between arithmetic and algebra
- ➔ Lacking number sense is a barrier to upper-level mathematics

Write an expression for the total number of stars

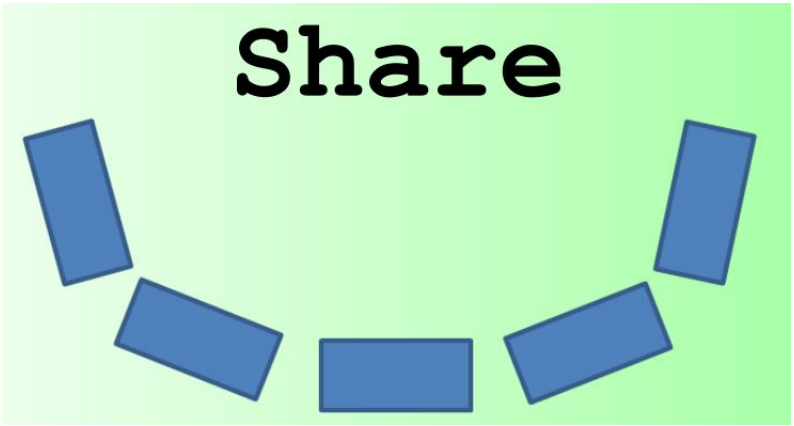


Write an expression for the total number of stars

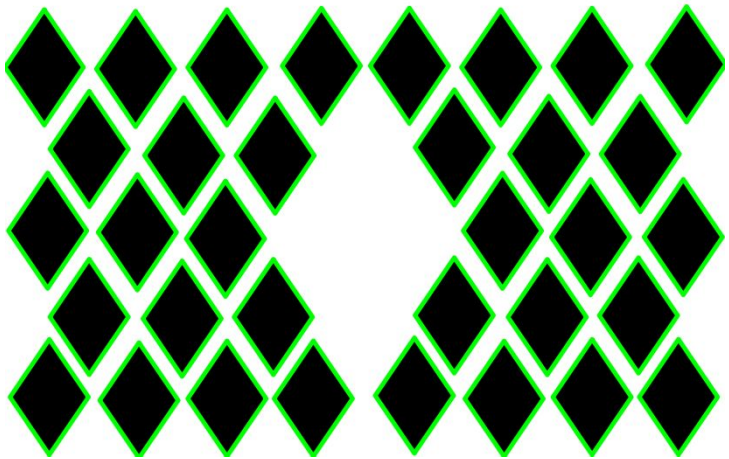
Show



Write an expression for the total number of stars



Getting Started with Number Talks



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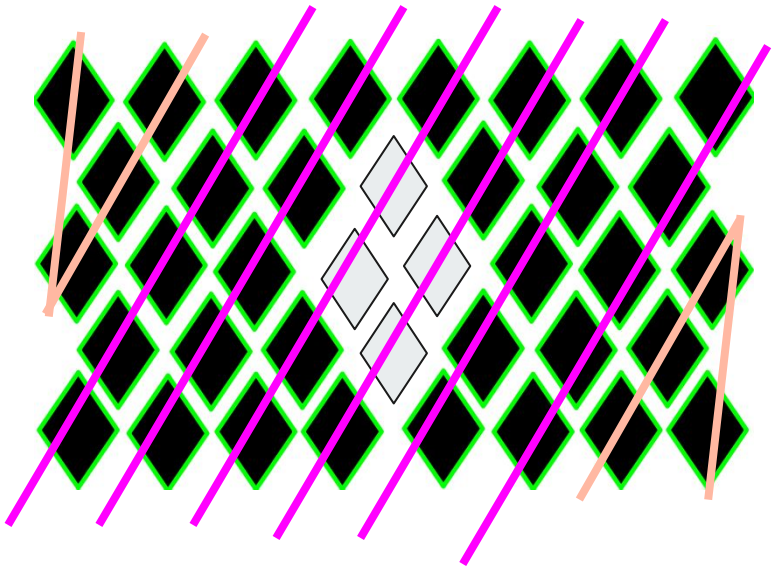
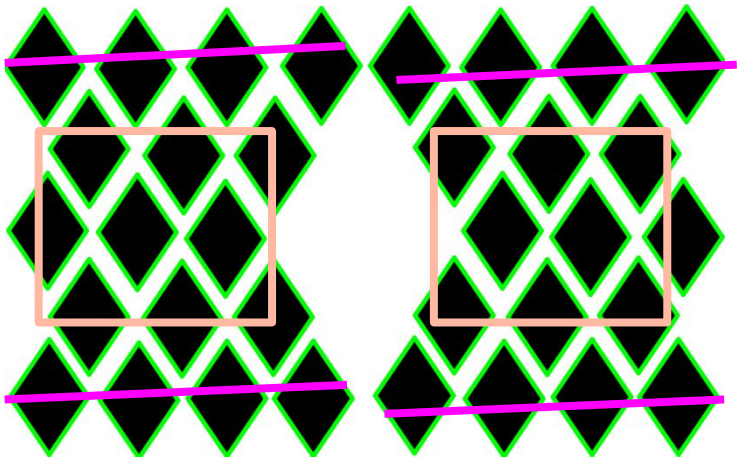


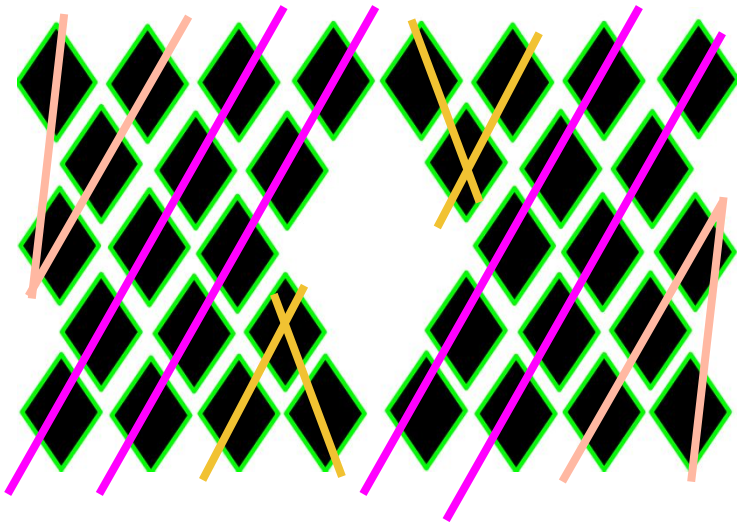
$$5(5) - 4 + 4(2)$$

Getting Started with Number Talks



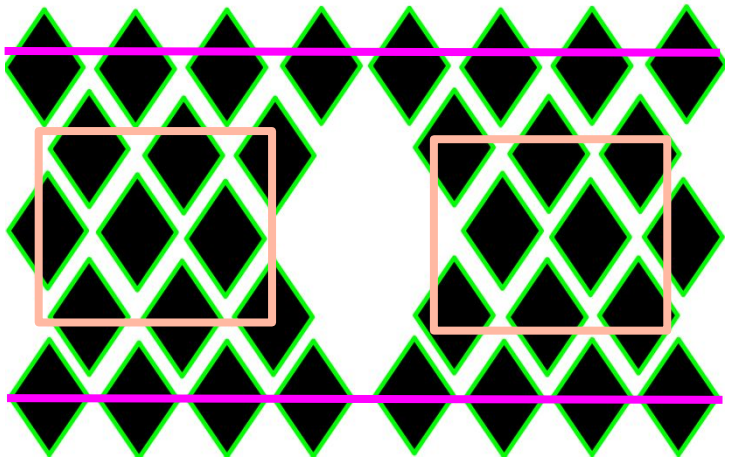
$4(4) + 2*3(3)$

Getting Started with Number Talks



$5(4) + 4(2) + 3(2)$

Getting Started with Number Talks



$8(2) + 2*3(3)$

Other Examples of Number Work

- ➔ Closer to one-half, one or two?
- ➔ Number line problems
- ➔ Greater than or Less than?

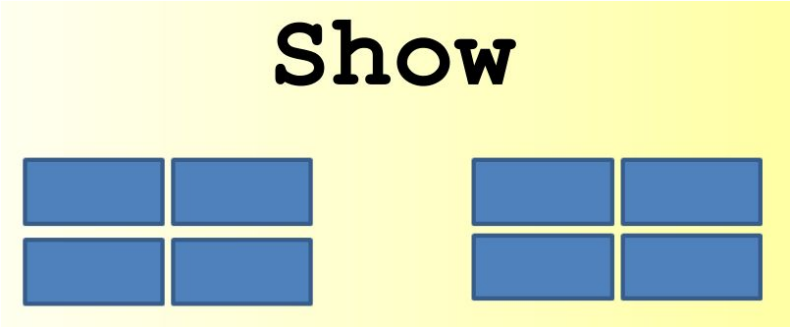
Closer to one-half, one, or two?

$$\frac{5}{11} + \frac{1}{32}$$

$$2\sqrt{2} - 1$$

$$2 \cdot \pi - 5$$

$$(2\sqrt{2} - 1)(2 \cdot \pi - 5)$$

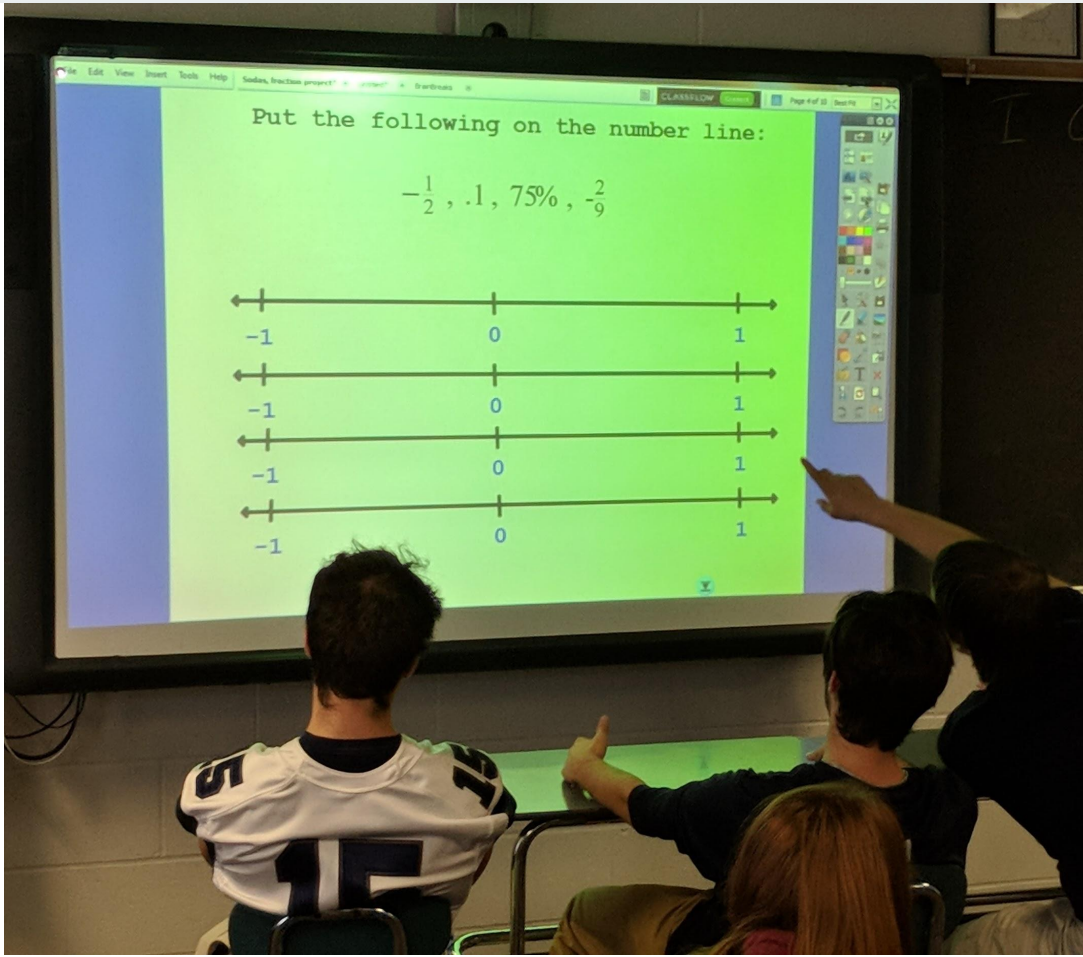


Closer to one-half, one, or two?

$\frac{5}{11} + \frac{1}{32}$	Closer to one-half
$2\sqrt{2} - 1$	Closer to two
$2 \cdot \pi - 5$	Closer to one
$(2\sqrt{2} - 1)(2 \cdot \pi - 5)$	Closer to two

Number Lines

$-\sqrt{2}$ $\sqrt{\frac{8}{9}}$ $\frac{\pi}{6}$



Greater Than or Less Than?

15.8 + 13.89	**Greater than 30? Less than 30?**
1/3 ÷ 7/8	**Greater than 1? Less than 1?**
-8.6 – 4.8	**Greater than -13? Less than -13?**
16 ÷ 1.9	**Greater than 8? Less than 8?**
$\cos 45^\circ - \cos 44^\circ$	**Greater than 0? Less than 0?**

Greater Than or Less Than?

15.8 + 13.89 — **Less than 30**

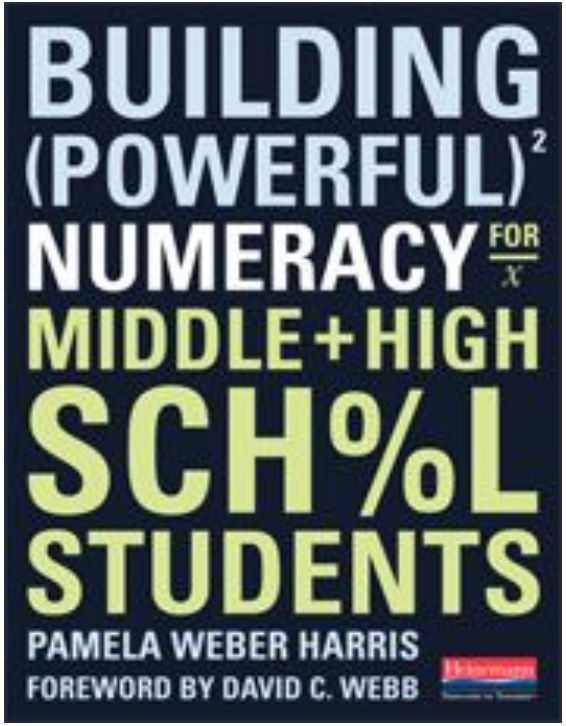
1/3 ÷ 7/8 — **Less than 1**

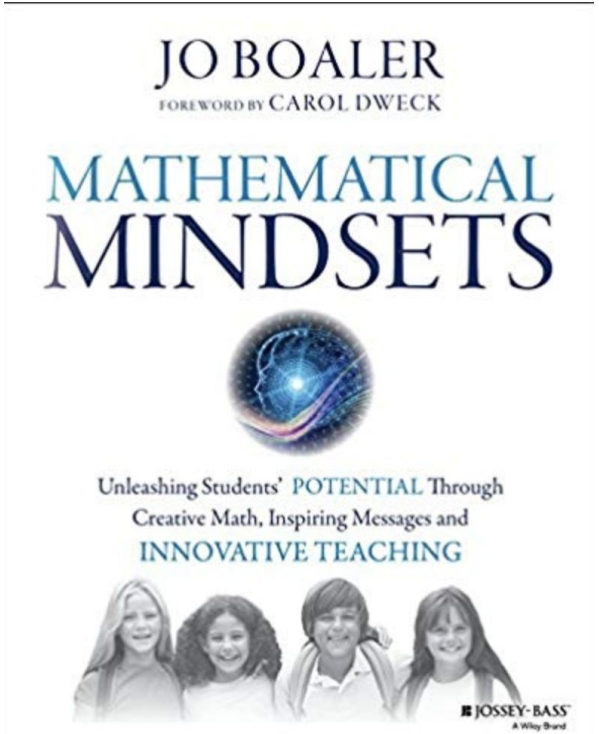
-8.6 - 4.8 — **Less than -13**

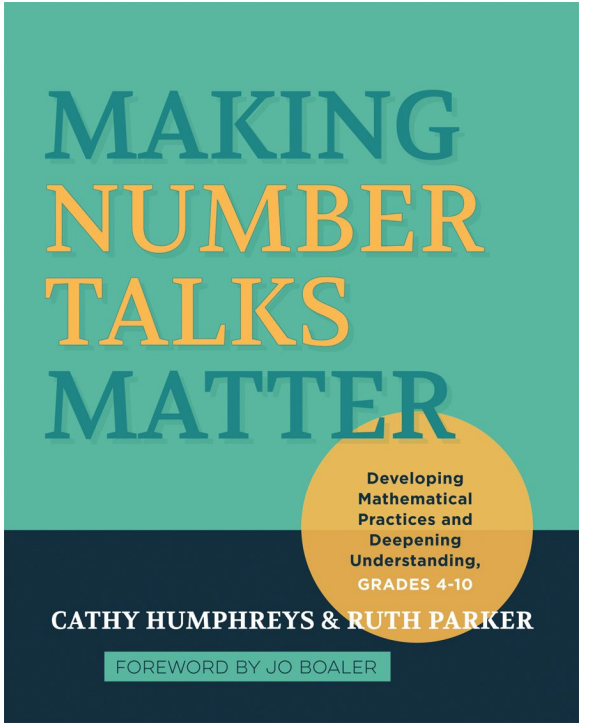
16 ÷ 1.9 — **Greater than 8**

$\cos 45^\circ - \cos 44^\circ$ — **Less than 0**

Three Influential Resources







Building Equation Sense through ‘Inspecting Equations”

- ➔ Before seeing algorithmic methods for solving equations, develop students’ intuitive notions about relationships between expressions.
- ➔ Focus on relationships between numbers, concepts of equality and meaning of the equals sign.
- ➔ Work was influenced by Balanced Math Framework (Madison, WI)

Inspecting Equations Routine

A short routine where students view an equation or series of equations, think about the relationships mentally, and discuss solution(s).

Equations should be carefully chosen to emphasize key ideas.

Remember it’s mental!

Types of Number Sentences

- ➔ True/False Number Sentences
- ➔ Open Number Sentences

True/False

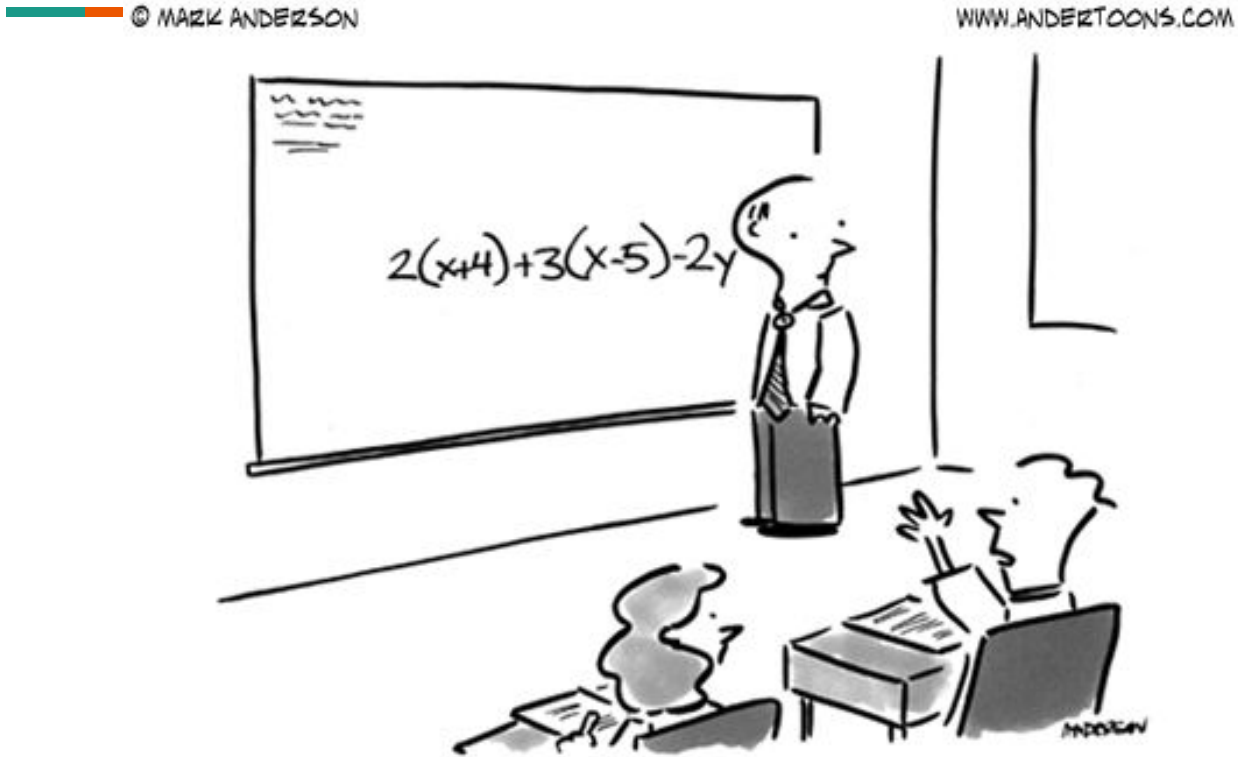
$$9+2 = 11+5 = 16$$

True/False

$$6 \times 7 = 2 \times 21$$

True/False

$$672 + 672 = 2 \cdot 789$$



"How come we never complicate equations?"

Open Number Sentence

$$47 \cdot \frac{78}{x} = 47$$

Open Number Sentence

$$50 - ? = 25 + 60$$

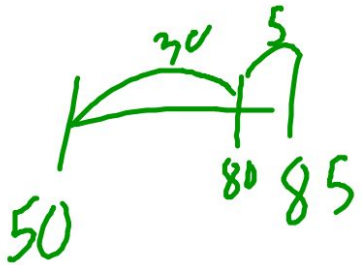
$50 + \_\_ = 25 + 60$

$60 + 25 = 85$

$85 - 50 = 35$

$60 - 50 = 10$

$10 + 25 = 35$



$50 + x = 25 + 60$

$50 + x = 85$

$-50 \quad -50$

$x = 35$

Open Number Sentence

$$g \div \frac{6}{7} = 1$$

$$\frac{g \div 6}{7} = 1$$

$$g \div 6/7 = 1$$

True/False

$$176 + 289 = 174 + 287$$

True/False

$$-1943 - 876 = -1941 - 878$$

Open Number Sentence

$$73 + 43 = 74 + d$$

Open Number Sentence

$$-93 + x = -92 + 67$$

True/False

$$.568 = .500 + .60 + .8$$

True/False

$$\frac{1}{2} \cdot 8 > \frac{1}{2} \cdot 5 + \frac{1}{2} \cdot 3$$

Open Number Sentence

$$65 \cdot 38 = 60 \cdot 38 + 5 \cdot g$$

Open Number Sentence

$$12 \cdot 4.1 = 8.2 + r$$

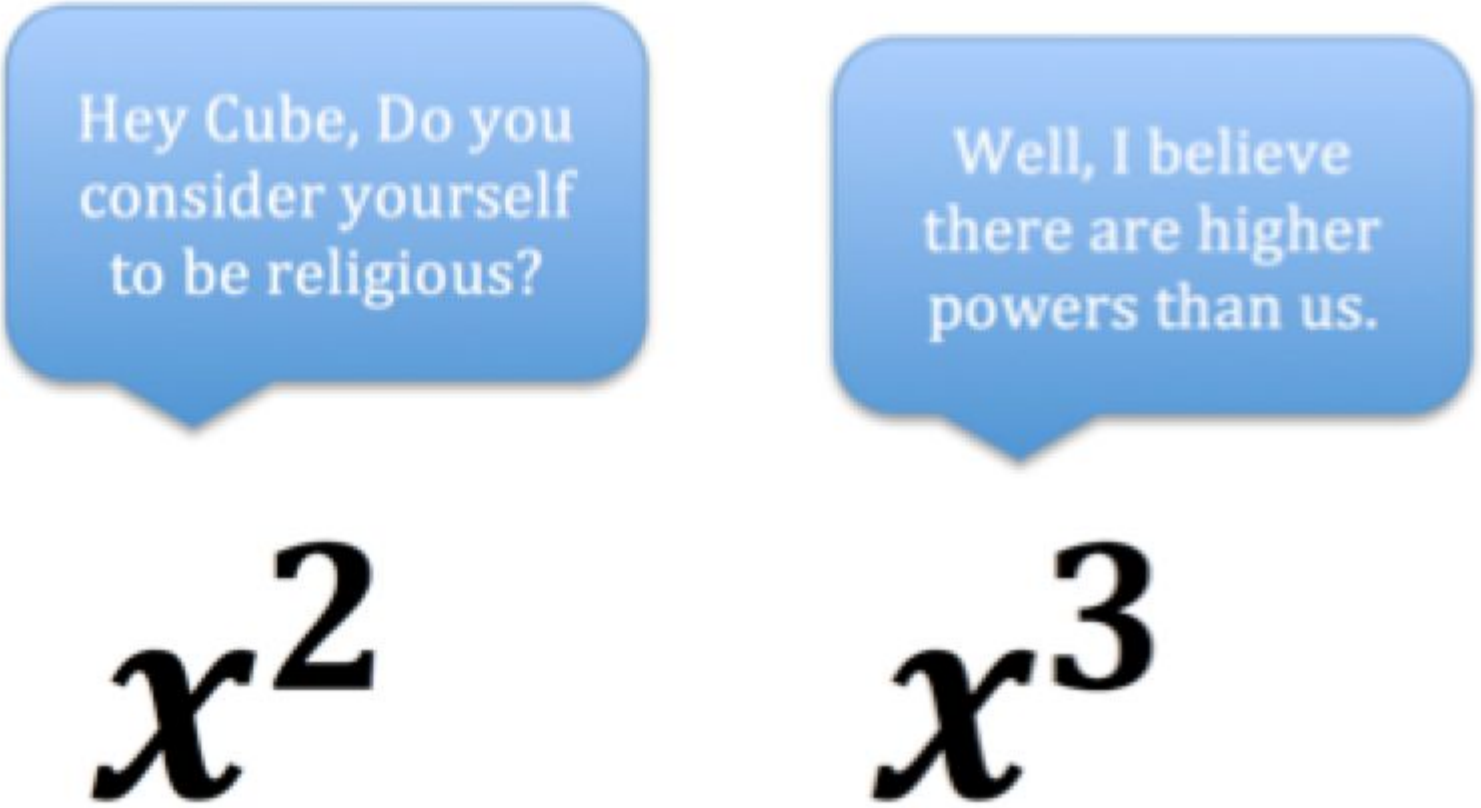
True/False Variation: Always True, Sometimes True, Never True?

$$5 - d \neq d - 5$$

"I find if you put that slash through the equal sign, the number of possible answers vastly increases."

Always True, Sometimes True, Never True?

$$\sqrt{(-3x)^2} = -3x$$


Hey Cube, Do you consider yourself to be religious?
Well, I believe there are higher powers than us.
x^2
x^3

Open Number Sentence

$$x - 28 = 24 - x$$

Open Number Sentence

$$x(x+1)=56$$

Open Number Sentence

$$\frac{x+1}{x+1}=\frac{1}{2}$$

Open Number Sentence

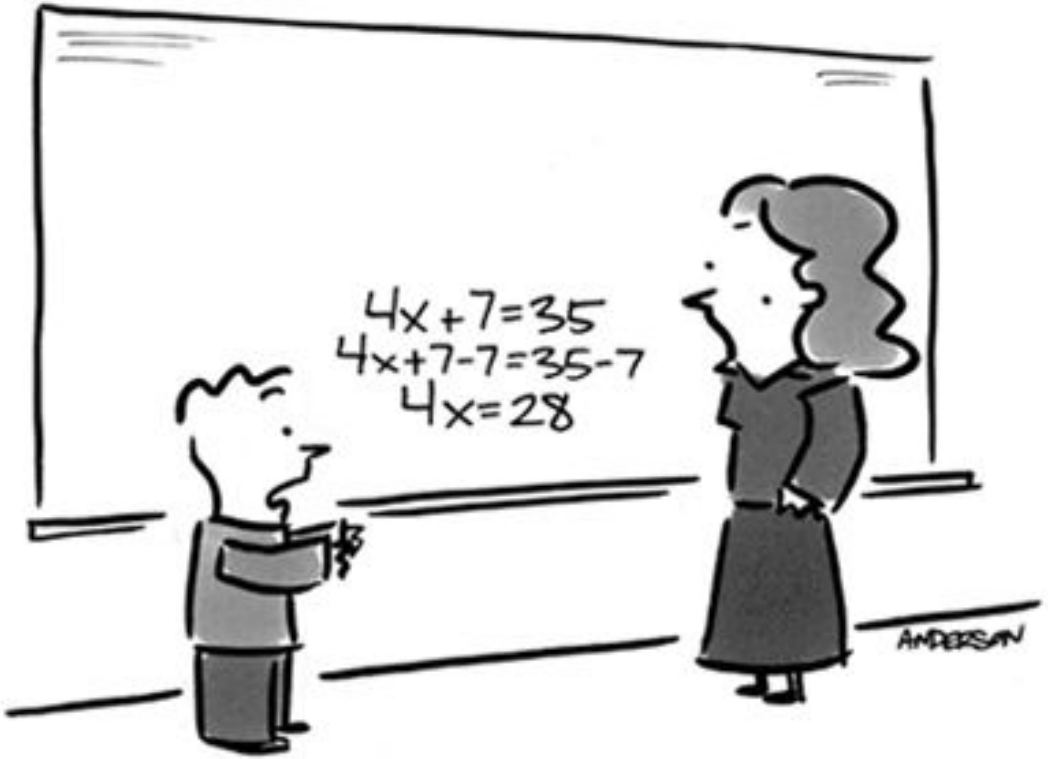
$$\frac{x+1}{x+2} = \frac{1}{2}$$

Open Number Sentence

$$\frac{x+1}{x+50} = \frac{1}{2}$$

Open Number Sentence

$$\frac{x+1}{x+10000}=\frac{1}{2}$$



"You *knew* X was 7 the *whole time*
and you *never said anything*?!"

Always True, Sometimes True, Never True?

$$5x \cdot 5y > 3x \cdot 7y$$

Open Number Sentence

$$9 \cdot 31 > 31x + 31y$$

Open Number Sentence

$$s + s + t = 17$$

True/False

$$\sqrt{53} - 1 < \sqrt{51} + 1$$

Discuss

What would be a ‘Inspecting Equations’ problem that would be appropriate for your students?

Impact on Students

“I do better in math class now that I took Math 10”

“I looked forward to doing number talks everyday of class”

“I found my voice in this class, and I am no longer shy about being wrong because like, I realise that I just think different and thats okay as well.”

“I wish it could be harder.”

“I like how close this class is.”

“I understand math better because of this class.”

Impact on Students

Students performed well on standardized testing for the year (65% pass rate)

Students have moved onto Algebra 2 without difficulty (many are at the top of the regular ed class)

Students still come back to ask math questions or show me math patterns they've noticed

Wanted to come to San Diego to tell you all about this

Thank You

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