Illustrative Mathematics

# Levels of Rigor in Proofs Using Transformations

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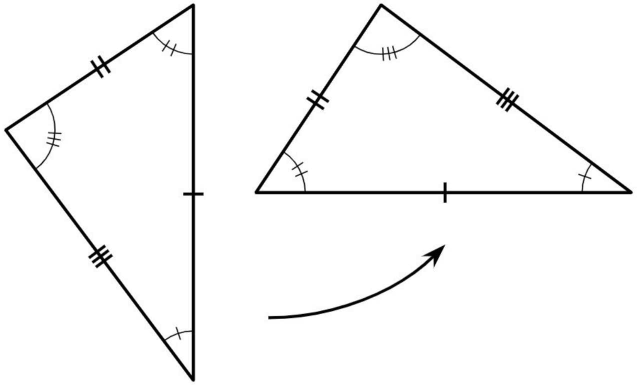
# Today's Goals

Understand:

- our definitions of transformations
- how we use transformations to prove congruence
- ways to support students in the process of attending to precision and developing rigorous arguments

# Rigid Transformations

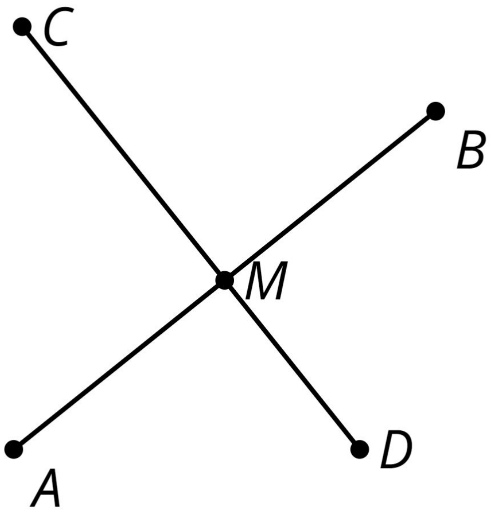
- A rigid transformation is a translation, reflection, rotation, or any sequence of the three.
- Rigid transformations take lines to lines, angles to angles of the same measure, and segments to segments of the same length.



# Math Talk

Segment *CD* is the perpendicular bisector of segment *AB*. Find each transformation mentally.

- A transformation that takes *A* to *B*.
- A transformation that takes *B* to *A*.
- A transformation that takes *C* to *D*.
- A transformation that takes *D* to *C*.

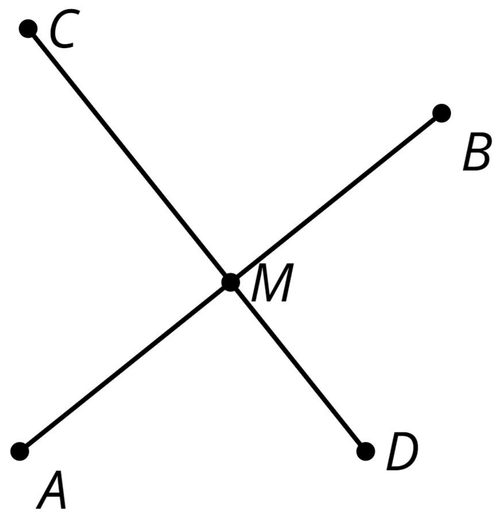


# The Reference Chart

- Students need a shared understanding of the foundations to write proofs.
  - Definitions
  - Assertions
  - Theorems
- Find the definition of congruent on your chart.

# Translation

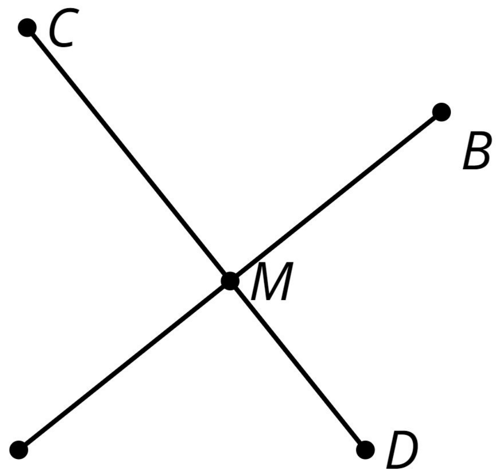
Translation is a rigid transformation that takes a point to another point so that the directed line segment from the original point to the image is parallel to the given line segment and has the same length and direction.




Translate *A* by the directed line segment from *A* to *B*.

# Reflection

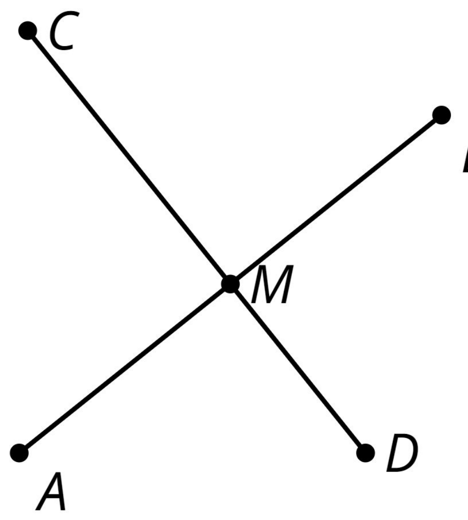
Reflection is a rigid transformation that takes a point to another point that is the same distance from the given line, is on the other side of the given line, and so that the segment from the original point to the image is perpendicular to the given line.




Reflect *A* across line *CD*.

# Rotation

Rotation is a rigid transformation that takes a point to another point on the circle through the original point with the given center. The two radii to the original point and the image make the given angle.




Rotate *A* counterclockwise by 180° using center *M*.

# What's the Segment?

Geometry
Unit 2: Congruent Triangles
Lesson 5: Points, Segments, and Zigzags

# A Conjecture

- If $AB$ is a segment in the plane and $CD$ is a segment in the plane, then $AB$ is congruent to $CD$.

# A Revised Conjecture

- If *AB* is a segment in the plane and *CD* is a segment in the plane with the same length as *AB*, then *AB* is congruent to *CD*.
- Prove the conjecture.

## Proving the Triangle Congruence Theorems
## Sentence Frames for Proofs

**Transformations:**

- Translate ______ from ______ to ______.
- Rotate ______ using ______ as the center by angle ______.
- Rotate ______ using ______ as the center so that ______ coincides with ______.
- Reflect ______ across ______.
- Reflect ______ across the perpendicular bisector of ______.
- Segments ______ and ______ are the same length so they are congruent. Therefore, there is a rigid motion that takes ______ to ______. Apply that rigid motion to ______.

**Justifications:**

- We know the image of ______ is congruent to ______ because rigid motions preserve measure.
- Points ______ and ______ coincide after translating because we defined our translation that way!
- Since points ______ and ______ are the same distance along the same ray from ______ they have to be in the same place.
- Rays ______ and ______ coincide after rotating because we defined our rotation that way!
- The image of ______ must be on ray ______ since both ______ and ______ are on the same side of ______ and make the same angle with it at ______.
- Points ______ and ______ coincide because they are both at the intersection of the same lines, and 2 distinct lines can only intersect in 1 place.
- ______ is the perpendicular bisector of the segment connecting ______ and ______, because the perpendicular bisector is determined by 2 points that are both equidistant from the endpoints of a segment.

**Conclusion statement:**

- We have shown that a rigid motion takes ______ to ______, ______ to ______, and ______ to ______, therefore triangle ______ is congruent to triangle ______.

# Levels of Rigor in Proofs

## Sample 1:

Two figures are congruent if there is a rigid motion that takes the first figure exactly to the second. First, translate segment *AB* by the directed line segment from *A* to *C*. Point *A′* will coincide with *C* because we defined our transformation that way! Then rotate the image, segment *A′B′*, by the angle *B′CD*, so that rays *A″B″* and *CD* coincide. Translation and rotation both preserve distance so segment *A″B″* is the same length as segment *AB*, which means segment *CB″* is the same length as segment *CD*. Since points *B″* and *D* are the same distance along the same ray from *C* they have to coincide. Therefore, there is a rigid motion that takes *AB* to *CD* and the segments must be congruent.

## Sample 2:

We need to find a rigid motion that works. Translate point *A* to point *C*. Rotate around that point so that the segments overlap. This will line up the image of point *B* with point *D* since the segments have the same length and our rigid motions don't change lengths. So, there is a rigid motion that takes *AB* to *CD*. The segments must be congruent.

# Invisible Triangles

Geometry
Unit 2: Congruent Triangles
Lesson 3: Congruent Triangles, Part 1

# Let's Play!

- Player 1 will be the transformer and takes the transformer card.
- Player 2 draws one triangle card, without showing it to anyone.
- Player 2 studies the diagram to figure out which sides and which angles correspond. Tell Player 1 what you have figured out.

# Let's Play!

Player 1:

- Take notes about what they tell you so that you know which parts of their triangles correspond.
- Think of a sequence of rigid motions that you could tell your partner to get them to take one of their triangles onto the other.
- Be specific in your language. The notes on your card can help with this.

# Let's Play!

Player 2:

- Listen carefully to the instructions from the transformer.
- Follow their instructions.
- Draw the intermediate steps.
- Let them know when they have lined up one, two, or all three vertices on your triangles.

# Reflections

- Would the same sequence of moves work for all the different triangles?
- Why might it be useful to have a sequence of moves that works for any pair of triangles with, say, two pairs of corresponding sides congruent and the pair of included angles between them congruent?

# Proving the Side-Angle-Side Triangle Congruence Theorem

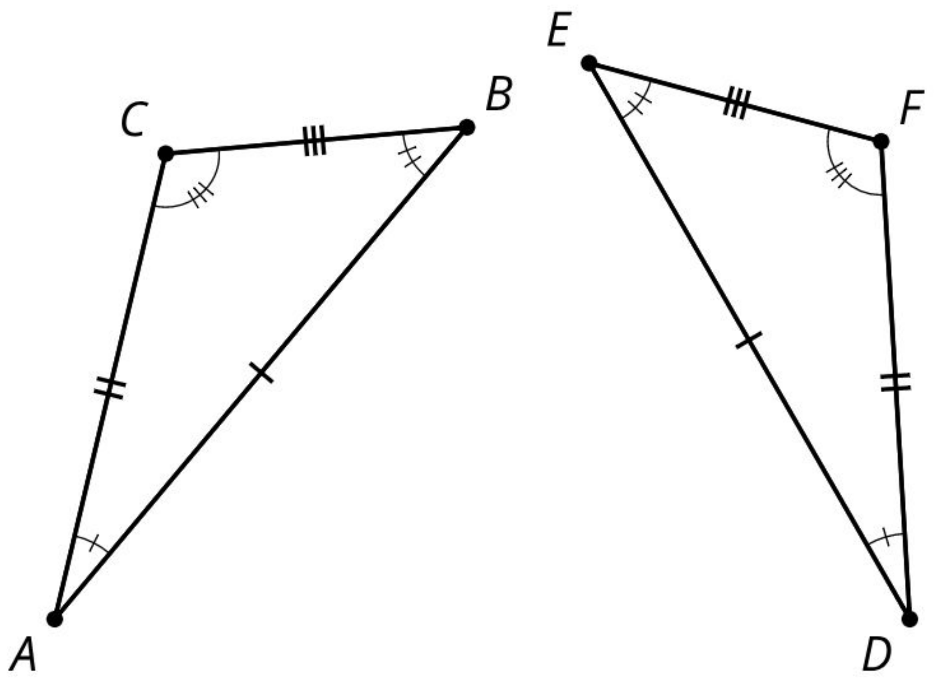
Geometry
Unit 2: Congruent Triangles
Lesson 6: Side-Angle-Side Triangle Congruence

# Warm-Up

Highlight each piece of given information that is used in the proof, and each line in the proof where that piece of information is used.

- $\overline{AB} \cong \overline{DE}$
- $\overline{AC} \cong \overline{DF}$
- $\overline{BC} \cong \overline{EF}$

- $\angle A \cong \angle D$
- $\angle B \cong \angle E$
- $\angle C \cong \angle F$



# Writing a Proof

- Work with your tablemates to do the activity.
- Use appropriate tools strategically.
- Be ready to discuss what aspects of this activity will be challenging for students.

1. Two triangles have 2 pairs of corresponding sides congruent, and the corresponding angles between those sides are congruent. Sketch 2 triangles that fit this description and label them $LMN$ and $PQR$, so that:
    - Segment $LM$ is congruent to segment $PQ$
    - Segment $LN$ is congruent to segment $PR$
    - Angle $L$ is congruent to angle $P$
2. Use a sequence of rigid motions to take $LMN$ onto $PQR$. For each step, explain how you know that one or more vertices will line up.
3. Look back at the congruent triangle proofs you've read and written. Do you have enough information here to use a proof that is like one you saw earlier? Use one of those proofs to guide you in writing a proof for this situation.

# Levels of Rigor in Proofs

- If we provide students with:
  - a reference chart
  - a template of sentence frames
  - a model proof

  what was the lift?

# So What?

After proving a theorem students might:

- Add a new sentence frame to the proof template.
- Add a new theorem to the reference chart.
- Use the theorem to prove new theorems.
  (properties of isosceles triangles is next)

**“Students should recognize the process of establishing valid mathematical statements as the central act of doing mathematics.”**

Catalyzing Change in
High School Mathematics

# Now What?

Write down (or tweet with #LearnWithIM) one thing you learned or are thinking about now.

- When do you invite students to improve the rigor of their statements?
- How do you build supports into the proof writing process?

# THANK YOU

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