Process as Product: Promoting Practices and Habits Through Feedback

Link to this slideshow with downloads

https://bit.ly/2UiLecx

More info

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Goals

- Determine mathematical practices you value
- Improve ability to give qualitative feedback
- Develop better problems for richer assessment
- Identify ways to look for and communicate patterns of growth

student.desmos.com

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Computer/mobile
Play along!

Why might we want to give feedback on more than just accuracy of answers?

Think/Write/Share

What mathematical practices
do you value?

Coaching students how to think mathematically provides vital scaffolding to access higher math and broadens our definition of what it means to do mathematics and who is mathematically powerful.

John Hattie, in Sutton, R., Hornsey, M.J., & Douglas, K.M. (Eds., 2011), *Feedback: The communication of praise, criticism, and advice*

Effective feedback needs to address one of three major questions asked by the teacher and/or by the student:

Where am I going? (What are the goals?)

The values you just outlined

How am I going? (What progress is being made towards the goals?)

The feedback you give students

Where to next? (What activities need to be undertaken to make better progress?)

What students do with your feedback

Brian’s Mathematical Values

- Mathematical Proficiency
  - Conceptual
  - Procedural
  - Mechanical
- Level of Participation
- Habits of Mind

Anna’s Mathematical Practices

- Explore, Organize, and Represent
- Generalize and Test
- Abstract and Symbolize
- Retrieve, Strategize, and Connect
- Prove
- Apply
- Communicate Clearly
- Estimate and Think Precisely
- Show a Growth Mindset
- Respect Community
- Reflect
- Develop Academic Habits

Common Core Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Looking at student work by subject area

Assessing Student Work

1) If a student in your class submitted this work, what feedback would you give this student?

2) How does this feedback align with the values you selected earlier?

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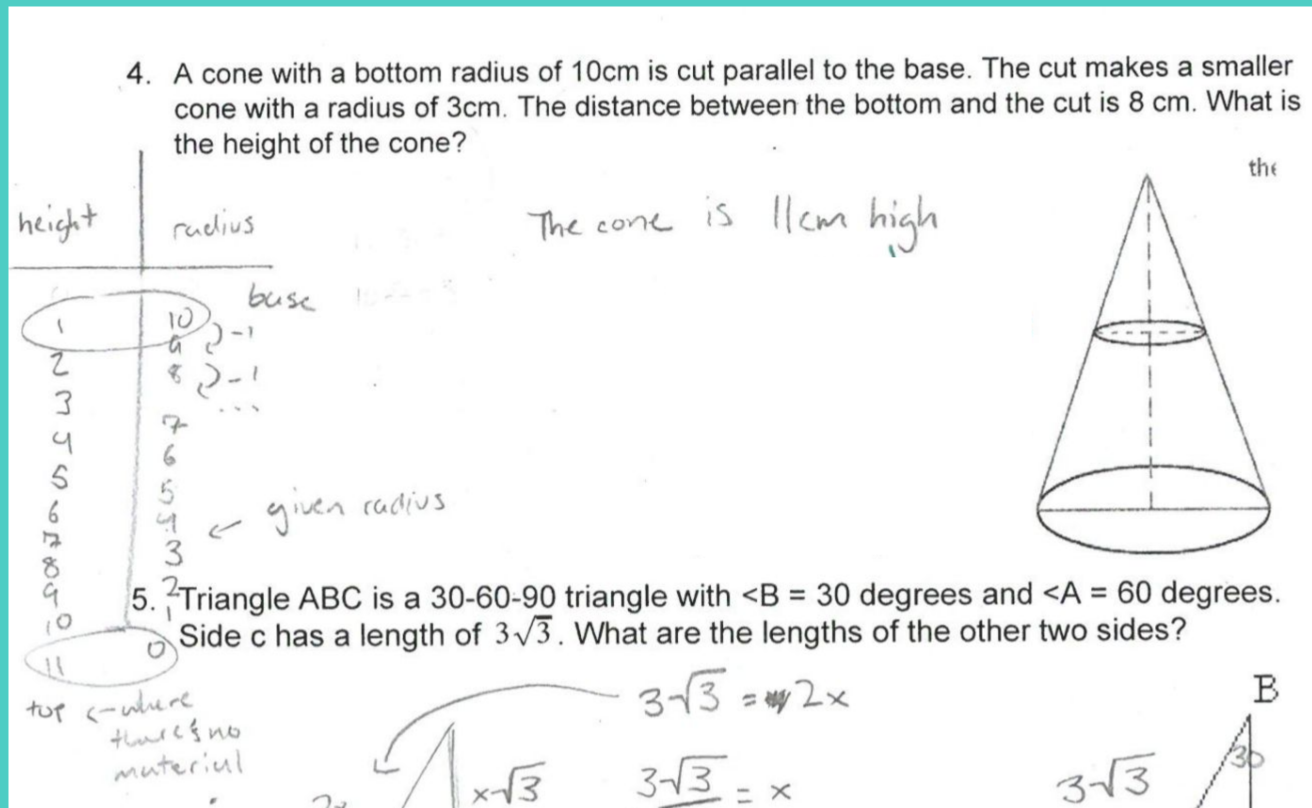
Student Work - Geometry



4. A cone with a bottom radius of 10cm is cut parallel to the base. The cut makes a smaller cone with a radius of 3cm. The distance between the bottom and the cut is 8 cm. What is the height of the cone?

height	radius
1	10
2	9
3	8
4	7
5	6
6	5
7	4
8	3
9	2
10	1
11	0

base
-1
-1
given radius
top ← where there's no material

The cone is 11cm high

5. Triangle ABC is a 30-60-90 triangle with <B = 30 degrees and <A = 60 degrees. Side c has a length of $3\sqrt{3}$. What are the lengths of the other two sides?

$3\sqrt{3} = 2x$

$\frac{3\sqrt{3}}{2} = x$

$x\sqrt{3}$

$2x$

$3\sqrt{3}$

B

30

How do you know that the radius decreases by one as the height increases by one?

What ... tudent?

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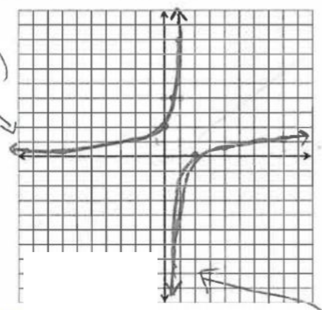
Student Work - Math 2 / Algebra II

5. Consider the function $f(x)=4(2)^x$.
   a. Graph $f(x)$ and state its domain and range.
   b. What is the inverse of $f(x)$?
   c. Graph the inverse on the same set of axes and give its domain and range. Make sure you label which function is which.

x	y
0	4
1	8
2	16
3	32
4	64



a. Range = All positive numbers Domain = All real numbers.
b. $f(x) = \log_2 4$
c. range = all real #)
Domain = all positi
#5

sho
scale

$f(x) = 4(2)^x,$

$f^{(x)-1} = \log_2 4$

6. Give an example of a function that is not one-to-one; explain how you know that it is not.

$y = 7x^2 + 6x + 5$ because it is quadratic.

wh

I liked how you made a table and worked through all portions of the problem.
On #6, you are correct in that the function is quadratic. However, your explanation needs more in terms of describing why it isn't one-to-one. How do you know that a quadratic is not one-to-one?

Great perseverance!
What about the quadratic makes it one-to-one? Can you provide more evidence.

Student Work - Precalculus

2. A triangle is made of sticks of length 18, 22, and 32 cm. What is the measure of its smallest angle? Round your answer to 2 decimal places.

$a = 18,\ b = 22,\ c = 32.$

$a^2 = b^2 + c^2 - 2bc \cdot \cos A^\circ$

$18^2 = 22^2 + 32^2 - 2 \times 22 \times 32 \cdot \cos A^\circ$

$1408 \cdot \cos A^\circ = 1184$

$\boxed{A = 32.76^\circ}$

$b^2 = a^2 + c^2 - 2ac \cdot \cos B^\circ$

$22^2 = 18^2 + 32^2 - 2 \times 18 \times 32 \cdot \cos B^\circ$

$\boxed{B = 41.41^\circ}$

$c^2 = a^2 + b^2 - 2ab \cdot \cos C^\circ$

$32^2 = 18^2 + 22^2 - 2 \times 18 \times 22 \times \cos C^\circ$

$792 \cos C^\circ = -216$

$\boxed{C = 105.83^\circ}$

A is the smallest angle in the triangle.

What feedback would you give this student?

Student Work - Calculus

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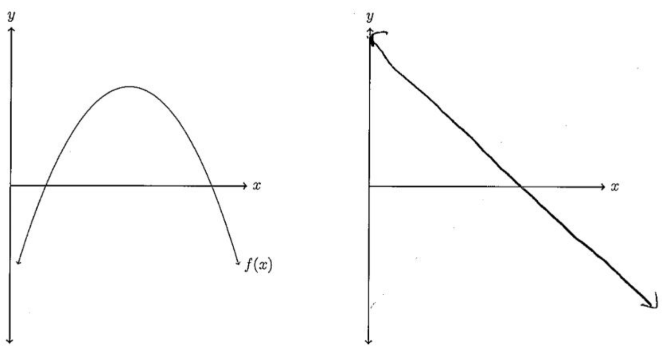
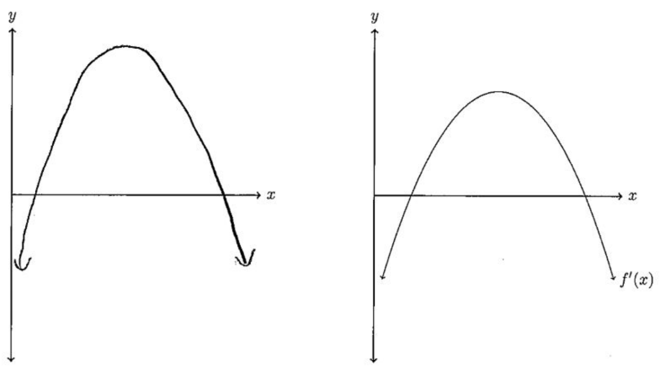
3. Below are two pairs of axes. Fill in the blank axes to appropriately match the given graph.

a. Given $f(x)$ below, sketch $f'(x)$ to the right.



b. Given $f'(x)$ below, sketch $f(x)$ to the left.



What feedback would you give this student?

Investigating a New Task

- Work on the task in groups (5 min)

- Select one or two mathematical values as a group

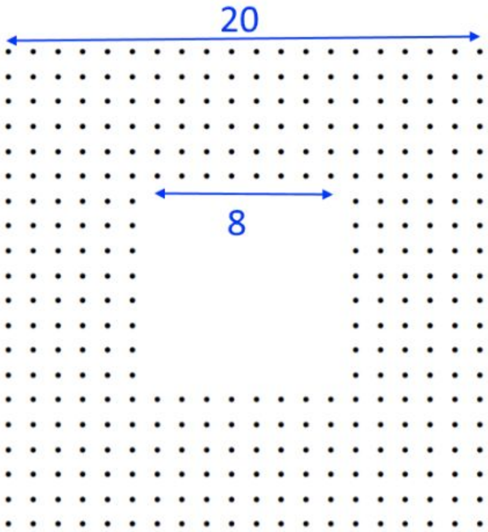
- Create a rubric to evaluate this problem based on those values (10 minutes)

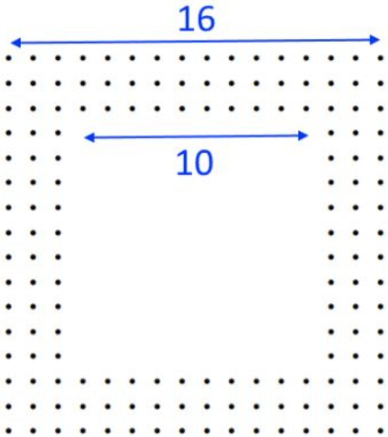
What do you notice? What do you wonder?

Napoleon squares

In Napoleonic battles, a hollow square was a popular formation designed to cope with enemy soldiers charging on horseback.

Below are diagrams showing two of these types of formations.

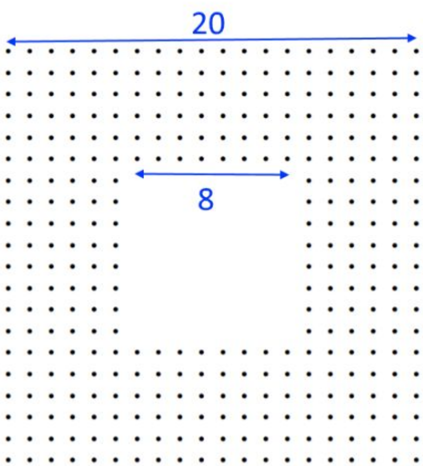


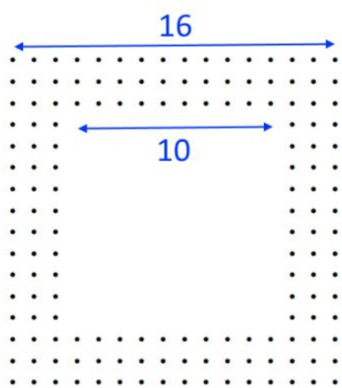


How could you quickly work out the number of dots in each?

Adapted from nrich.maths.org

Now that you've answered the question, how can you make this problem interesting again?





- A general has 960 soldiers. How many different ways can he arrange his battalion in a symmetric hollow square?
- What can you say about battalion sizes that can't be arranged as symmetric hollow squares?
- Can you find a general strategy for arranging any possible battalion into all possible symmetric hollow squares?
- What about hollow squares that are not symmetric?

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As a group, select some mathematical values and create a rubric you could use to give feedback to students on this task.

Example rubric

	Next steps/areas to improve	Expectation	Stands out
Explore and organize		Student creates interesting extension or variation for problem	
		Student explores mathematical situation and organizes ideas in multiple ways	
		Student uses annotations to identify and develop promising leads	
Generalize and test		Student identifies patterns and makes conjectures	
		Student tests conjectures and explores why they might be true	
Prove		Student constructs logical reasoning for why conjectures are true or false	
Communicate clearly		Student's written work clearly communicates problems and process to unfamiliar audience	
		Math work is easy to follow, organized, and labeled	

Rubric Creation Hot Tips

- Single point rubric with space to write “next steps” and “stands out”
- Use language students are familiar with
- Use qualitative descriptors over quantitative ones
- Keep it short and sweet
- Rubrics are general enough they can be used for multiple tasks/projects

Analyzing and sharing feedback with students

Identifying patterns of strength and overall progress

Brian's Check-In Rubric in Action

How do you think you're doing recently?

Metric			
Mathematical Proficiency (How's Your Math)	Advanced	Proficient	Basic
Level of Participation (How You Act)	Engaged	Productive	Elementary
Habits of Mind (How You Think)	Exemplary	Competent	Foundational

Comments:

Here's how Mr. Abend thinks you're doing:

Metric			
Mathematical Proficiency (How's Your Math)	Advanced	Proficient	Basic
Level of Participation (How You Act)	Engaged	Productive	Elementary
Habits of Mind (How You Think)	Exemplary	Competent	Foundational

Comments:

Brian’s Mathematical Proficiency Rubric

Metric	Advanced 4/A Let me show you.	Proficient 3/B I can do this.	Basic 2/C Tell me what to do.
Conceptual Understanding	Solutions are correct and supported by creatively and succinctly linking key course concepts, theorems, axioms, and postulates.	Solutions are often correct and supported by utilizing key course concepts, theorems, axioms, and postulates.	Solutions are sometimes correct and supported by a limited number of key course concepts, theorems, axioms, and postulates.
Procedural Fluency	Solutions demonstrate a clear, linear, and logical path from start to finish, include all important steps, and are direct and to the point.	Solutions demonstrate a legitimate path from start to finish, include key steps, and are appropriate given the prompt.	Solutions are either not started or finished properly, suffer from a lack of direction or circuitous thinking, or lead to answers that do not relate to the problem.
Mechanical Skill	Solutions are algebraically or arithmetically clean and are presented in simplest form.	Solutions sometimes include minor algebraic errors and/or may not be simplified.	Solutions often include major algebraic or arithmetic errors and are not simplified.

Brian's Check-In Rubric

Metric	Advanced	Proficient	Basic
Mathematical Proficiency (Conceptual, Procedural, Mechanical)	Student convincingly demonstrates their conceptual understanding, procedural ability, and mechanical skills by drawing connections, evaluating approaches, and strategically solving unique and non-routine problems.	Student effectively demonstrates their conceptual understanding, procedural ability, and mechanical skills by solving routine and structured problems.	Student demonstrates familiarity with the material and takes an algorithmic (rather than strategic) approach to solve simple, common, and highly structured problems.
Metric	Engaged	Productive	Elementary
Level of Participation	Student proactively communicates and collaborates with others in the classroom and contributes positively to the overall learning environment and classroom culture.	Student actively communicates and collaborates with others in the classroom, putting in effort to engage productively in improving classroom culture.	Student communicates or collaborates with a limited number of others, dos not disrupt or degrade the classroom culture, and participates when called upon.
Metric	Exemplary	Competent	Foundational
Habits of Mind	Student looks to pose insightful questions, imagines new connections between concepts, experiments with and evaluates the usability of multiple approaches, and seeks to extend the conversation beyond the problem.	Student asks thoughtful questions about problems, tries alternative approaches, assesses their own understanding, and engages with problems to find real-world or personal connections.	Student accepts problems at face value, asks mostly clarifying questions, generates obvious solutions, sticks to a single approach, and struggles to persevere when attempting more complex problems.

Brian’s Test Reflections

Test Reflections Cover Sheet

Name:________________________________

Test #: _______ **Today’s Date:** _________

Evaluating one’s own work is a valuable process both in school and in the outside world. This process involves three steps: <u>revisiting your work, realizing your mistakes, and rebuilding your knowledge</u>. To help encourage this type of independent and thoughtful reflection, you will be given the opportunity to improve your grade through test corrections on your summative, end-of-unit assessments. However, **credit is earned only when this process is clear and deliberate**. This means that your language, explanations, and answers should be correct; your self-evaluation should be concise and honest; and your corrections sheets should demonstrate that you have participated fully in the reflection process.

In order to participate in this self-evaluation process, please complete a **Test Reflections Form** for each incorrect problem and problem part. There are four spaces on the form and each must be filled out properly and completely. Failure to follow this process will result in no credit being earned. Fill out <u>one</u> **Test Reflections Cover Sheet** (this sheet) when you are finished, put the correction forms behind the cover sheet in the correct order, and (if you’d like) staple your original test to the back of the corrections.

<u>In addition to your corrections, please complete the following form regarding your process and reflection.</u>

Did you correct all incorrect problems? Yes No

Did you complete each reflection form? Yes No

Did you try problems again from the beginning? Yes No

Did you consult with anyone? Yes No

If “Yes”, then with whom did you consult? Mr. Abend Classmate(s) Teacher(s) Tutor Parent

In the space below, analyze your original test performance. What was the cause of most of your mistakes? What will you do differently in the future before the test to be more prepared? Do you feel your grade is reflective of your knowledge and ability?

Teacher Comments:

Grade:

Test Reflection Sheet

Problem #/part(s): _________

Question:

Original Work & Thought:

Misconceptions & Mistakes:

Correct Solution:

Teacher Comments:

A+ A B C Not Yet

Anna’s Collaboration Quiz (stolen with love from Sam Shah)

Group 1	Group 2	Group 3	Group 4
Waiting for marker	One person shares, everyone else listening actively	"That doesn't make sense."	Goes to the whiteboard, active listening
Looking at a cube, all involved	"Let's go to the board"	One person explaining, everyone else listening actively.	"Why don't you explain what you're doing?"
Two people are discussing, one person writing up previous work	Asks another group a question: "What if it's not a square?"	Another person adds on to earlier explanation	"Do you get it?" "Yeah. I was confused because the x was 1, but I get it now."
All involved in conversation	Everyone involved, discussing	One person working independently, two are discussing earlier thinking	"I can send the picture to you guys."
	"Can I please write that? Just dictate out loud."		Working quietly on their own
			One member of the group is helping group 3
			Still working on their own, no discussion

Anna's Unit Self-Evaluation

Algebra and Functions Self-Evaluation

This self-eval will be included in your mid-semester portfolio and narratives.
Look over your work in this unit and find examples to help you decide if you're "Practicing" or "Demonstrates Consistently" for each habit of learning. Mark the column you think describes you best (you can mark both if you think you're in between).

Mathematical Habit	Practicing	Demonstrates Consistently
Explore and organize: explore a mathematical situation, generate data and organize it in different ways, identify and develop promising leads, tweak and generate your own problems		
Generalize and test: identify patterns and relationships, describe and test them to understand the conditions under which they hold		
Abstract and symbolize: understand and use different symbols and notation, define terms clearly, invent terms as needed		
Represent, retrieve, and connect: represent problems and ideas in multiple ways, analyze and compare different approaches, look for connections between approaches and topics		
Prove: make convincing arguments to support ideas, analyze and build on others' arguments		
Communicate clearly: explain ideas clearly and fully		
Think precisely: think critically about assumptions, estimate and check for reasonability, attend to precision and accuracy		
Show a growth mindset: take responsibility for learning; value productive struggle, challenge, and growth; take risks; ask for help and use resources; use feedback to improve; appreciate mathematics as a subject		
Respect community: encourage, build-on, and connect others' ideas, ask for and give help, support others' thinking and growth		
Reflect: give yourself feedback, plan and work on reaching goals, think about yourself as a learner and mathematician and how to make progress		
Develop academic habits: take and organize notes, review past work, come to class prepared and on time, complete work on time, be productive and focused during class		

Open your Math Progress spreadsheet and read over your recent reflections and fall/winter feedback in the Mathematical Habits tab. Please respond to the following prompts using full sentences.

1. Which habits do you think have been strengths for you this semester? For each habit you think is a strength, give an example that shows how you are strong in it.
2. Which habits would you like to work on for the remainder of the year? For each habit you'd like to work on, write down one "next step" that can help you make progress.
3. What have you been doing to meet your previous goals? What should you keep doing/start doing/stop doing?
4. What content topics are you still working to master? What are your next steps here?
5. What can Anna and/or your parents do to help support you?
6. Look at the Geometry tab of your spreadsheet. Which topics are you most looking forward to? Which topics might present a challenge?

Remember to submit this on Google classroom when you are finished.

Anna's Feedback Meetings

- Students self-assess content and practices work using a rubric
- Students reflect on goals from previous meeting and set actionable next steps
- Anna takes notes in a spreadsheet that is shared with students and parents
- For classes that get a grade, this is how we get a grade

Example	Your content understanding has been very strong this semester. You just reviewed the one topic that you had missed from last unit: Understand the categorization of real numbers and their subsets. You are now all caught up with content – yay! While you're doing a great job of mastering the main content objectives, you can do more to push yourself and improve your depth of understanding and communication and proof. You plan on spending more time on the Pushing/Pondering problems on future homework assignments. You have been doing a fantastic job during class, George! Very high engagement and working well with peers, pushing back on their thinking. You are going to revise Marcella's Bagels by next Friday, March 15. Use the comments in Google Classroom as a reminder of what to revise.

Anna’s Portfolios

Students can include:

- Picture of class work or homework problem
- Desmos link
- Video of you explaining something
- Link to something you coded
- Written explanation of a problem or concept
- Summary of discussion you had with peers/teacher
- Correction of previous work
- Problem from the internet or other resource you worked on
- Question you are wondering about
- Reflection about your learning or something that was confusing to you

Small changes to your practice

Ask students to:

- Reflect on a math practice used and/or a practice they'd like to develop
- Write up a solution, emphasizing multiple approaches, representations, or connections to other problems
- Revise and reflect on corrections
- Self-assess understanding

You:

- Use rich problems to elicit rich responses
- Identify mathematical values and feedback you can give that's aligned to them
- Create rubrics with students to build understanding of exemplary work
- Start small to build success
- Be explicit with students re purpose

Bigger changes to your practice

- Ongoing student journal to document process and thinking about problems and progress
- Portfolio of student work that includes self-reflection
- Larger unit projects with revision cycles
- Incorporating qualitative feedback into students' grades

Read your earlier reflections - have you made strides in the mathematical practices you identified as ones needing improvement? What should you keep doing? What changes should you make to your plan?

I feel that I definitely made some improvement. Most of the trig identities are all connected with each other and I was able to use their connection to solve problems. For example, I used the unit circle to prove sin(-x)=-sin(x), and I realized that I can use the same way to prove cos(-x)=cos(x). I think what I should keep doing is to look the problems more carefully to see whether they are similar so that I can use the same pattern to solve them. I guess I should not just do random practice problems and instead I should concentrate on a very specific topic so that I can be more familiar with the connections between similar problems.

I have definitely improved at the community skill by asking for help from my peers when I'm confused, and communicating my ideas and level of understanding with others. For a next step, I want to make sure that I am creating an environment where others can do the same, and be more conscious of what they are thinking as well. In my initial reading reflection, I also made the goal to ask more questions when starting problems for the math practice of exploring and organizing. I don't think that I have made a lot of progress on this yet, so I'm hoping to reframe this goal by generating other questions every time an open ended question is asked in class or for homework.

References

- “The Power of Feedback,” Hattie & Timperley, 2007
- “Mathematical Habits of Mind,” Levasseur, Cuoco (first two chapters are online for free)
- http://www.withoutgeometry.com/2010/09/habits-of-mind.html (Avery’s blog, @woutgeo)
- https://parkmath.wordpress.com/mathematical-habits-of-mind/
- http://mathforum.org/library/problems/
- “The Art of Problem Posing,” Brown, Walter
- http://www.peterliljedahl.com/teachers/good-problem
- https://nrich.maths.org/

Electronic versions of all the things
(if you’re looking at the pdf handout, go to https://bit.ly/2UiLecx for working links)

- Brian’s Rubrics (Math Proficiency and Check-In)
- Brian’s Test Reflections (Cover and Form)
- Anna’s Unit Self-Evaluation
- Sam Shah’s blog post on collaboration quizzes
- Anna’s Portfolio directions
- Anna’s blog post about feedback meetings
- Brian’s blog post about formative grading