Conceptual vs. Procedural Understanding: Empowering All Students through Concept Development

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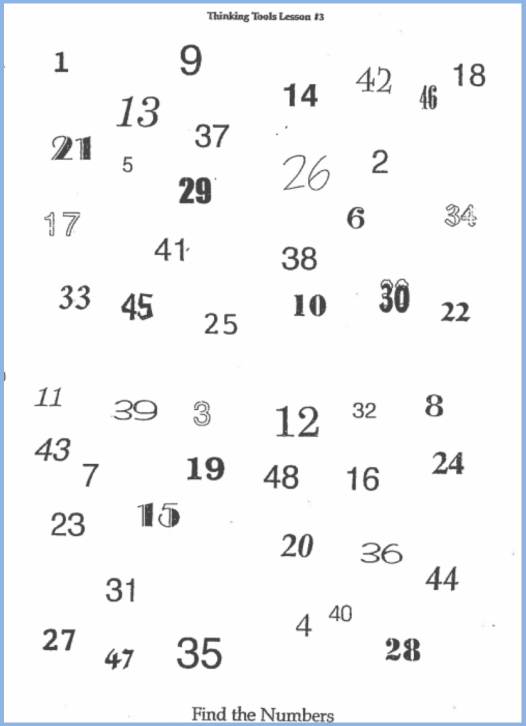
Cross off the numbers 1-48 in ORDER



Today we will:

- Explore conceptual and procedural understanding within the context of equality, multiplication, fraction comparison, and division.
- Use manipulatives to explore conceptual understanding.

Conceptual Knowledge of Mathematics

- ✓ Logical relationships constructed internally - exists in the mind as a network of ideas (Van de Wall, 2004)
- ✓ Knowledge that is <u>understood</u> (Hiebert & Carpenter, 1992)

Examples of Math Concepts

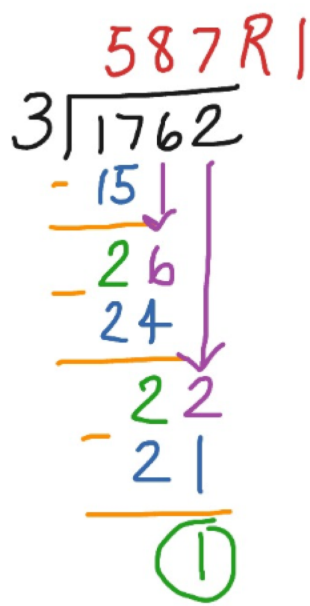
- Rectangle
- Sum
- Product
- Equivalent

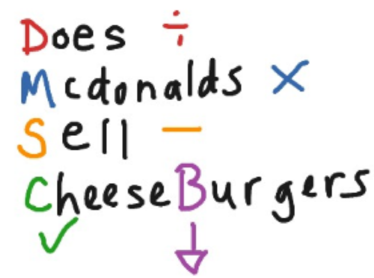
Procedural Knowledge of Mathematics

- Knowledge of the rules and step-by-step procedures to complete a mathematical task (Van de Wall, 2004).

Examples of Procedural Knowledge

- Does
- McDonalds
- Sell
- CheeseBurgers





What do students NEED to be successful in Math?

BOTH!

Procedural rules should never be taught in the absence of a concept.

(Van de Wall, 2004)

"Procedural knowledge without conceptual knowledge is shallow and is unlikely to transfer to new contexts, but conceptual knowledge without procedural knowledge is ineffectual".

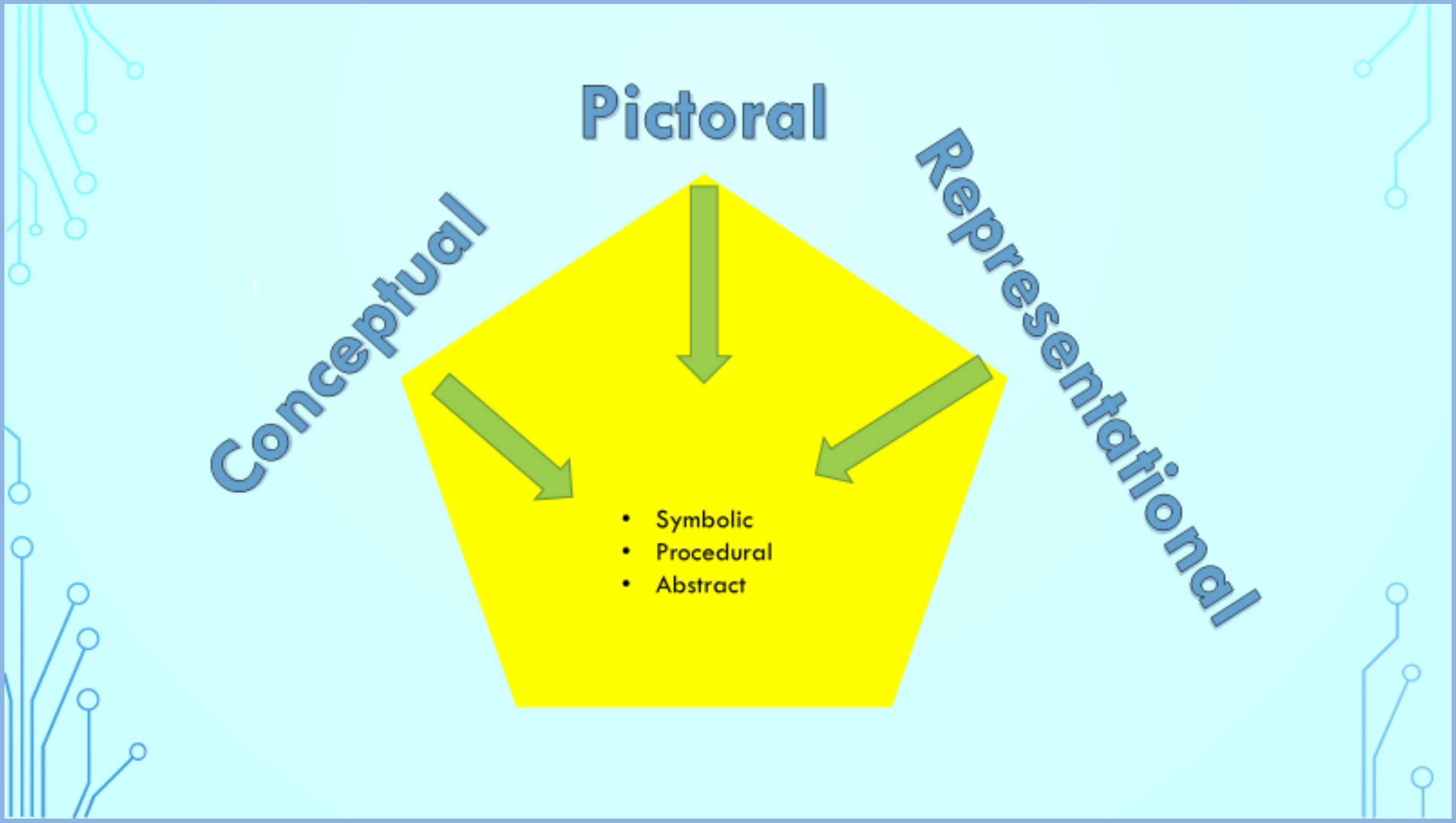
(Willingham, 2010)

When the
WHY is clear,
the HOW is easy!

True or False

Manipulatives = Conceptual Teaching

Where do manipulatives fit?



Making the Connections

- Use CONTEXT (word problems)
- Manipulatives
- Visuals

(Dixon, et al., 2016)

Context

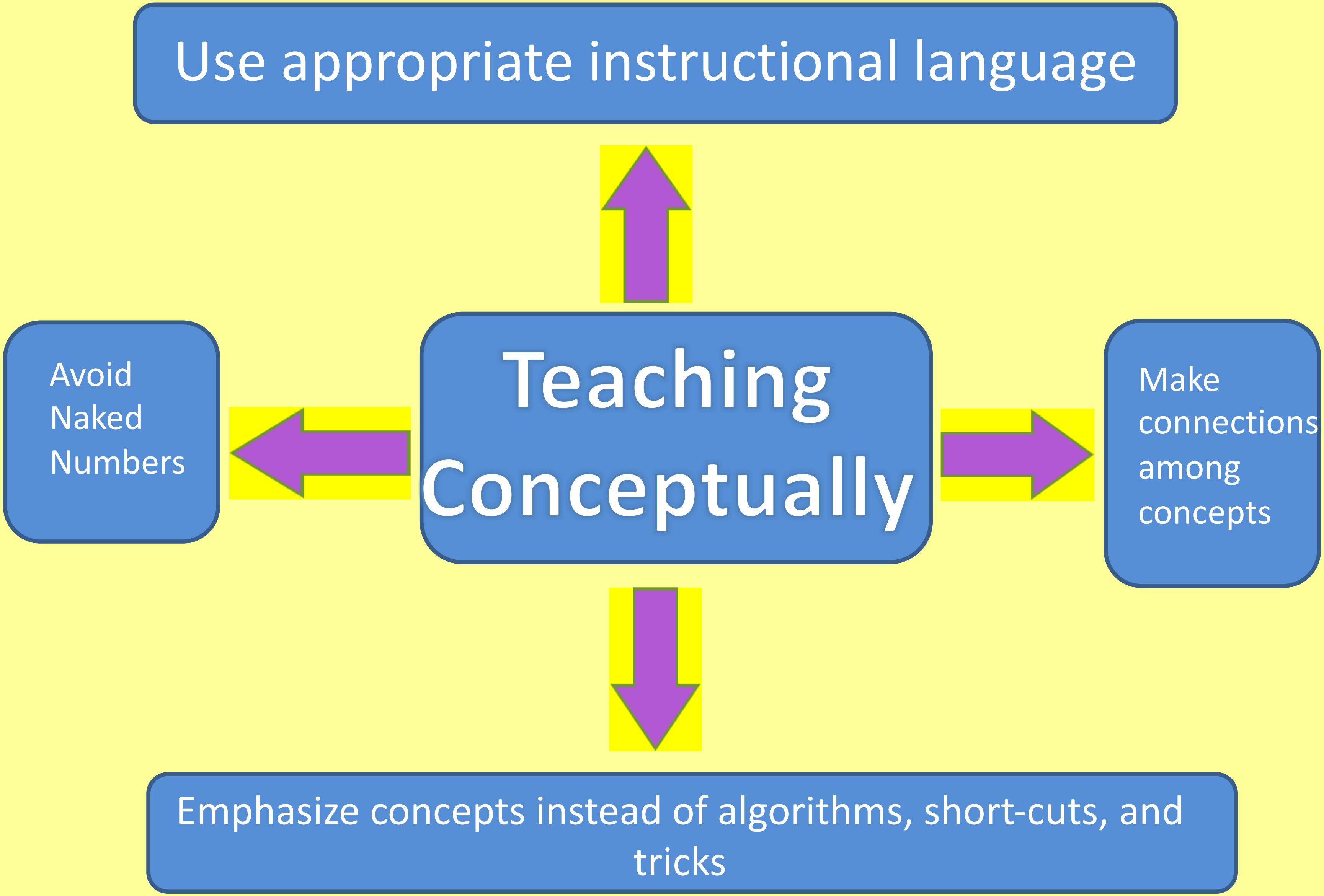
Context is essential to the development of understanding. It should be used from the start as a means of "construction" rather than as a culminating or extending activity for "application" at the end of a unit of study.

(Fosnot & Dolk, 2001, as cited in Caldwell et al. 2014)

By presenting a situation and leaving students to chart their own path to a solution, you give them the opportunity to construct the mathematical ideas rather than simply apply an algorithm…

...If the context is "real" they will discuss and make sense of the situation, not just describe the steps in the procedure. This moves the focus from rules and procedures to CONCEPTUAL problem solving.

(Caldwell et al. 2014)


Use appropriate instructional language
Avoid Naked Numbers
Teaching Conceptually
Make connections among concepts
Emphasize concepts instead of algorithms, short-cuts, and tricks

(Molina, 2014)

Emphasize concepts instead of procedures, short-cuts, and tricks

Equality

What is it?

Do your students understand it?

Equality

"By some estimates, as few as 25 percent of American sixth-graders have a deep understanding of the concept of equality."

(Willingham, 2010)

Fill in the missing numbers.

$7 + 3 = \boxed{10} + 4$

I added 7+3 with fingers

to figur out sum.

Fill in the missing numbers.

$$7 + 3 = \boxed{10} + 4$$

Because I added 7 Plus 3.

Then Up poped the anser.

$8 - \boxed{5} = 3 + 3$

The same thing but I

counted down.

Fill in the missing numbers.

7 + 3 = 10 + 4

by doing this Pomelom

7+3=10

8 − 5 = 3 + 3

by doing this pramme

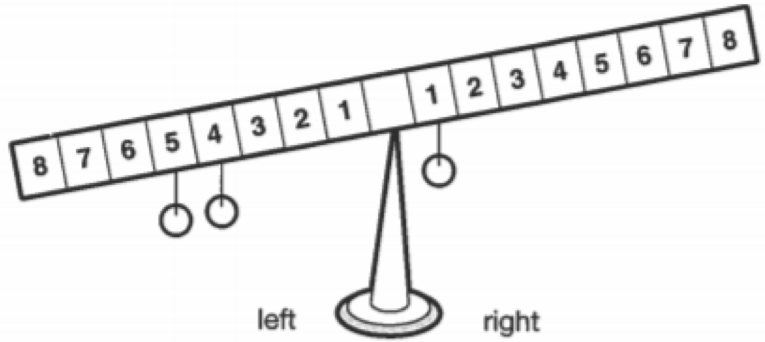
8 − 5 = 3

Fill in the missing numbers.

$7 + 3 = \boxed{14} + 4$

because 7+3+4 is 14.

Balance Beam 3



1. Make the beam balance.

   Draw one  on the right side.

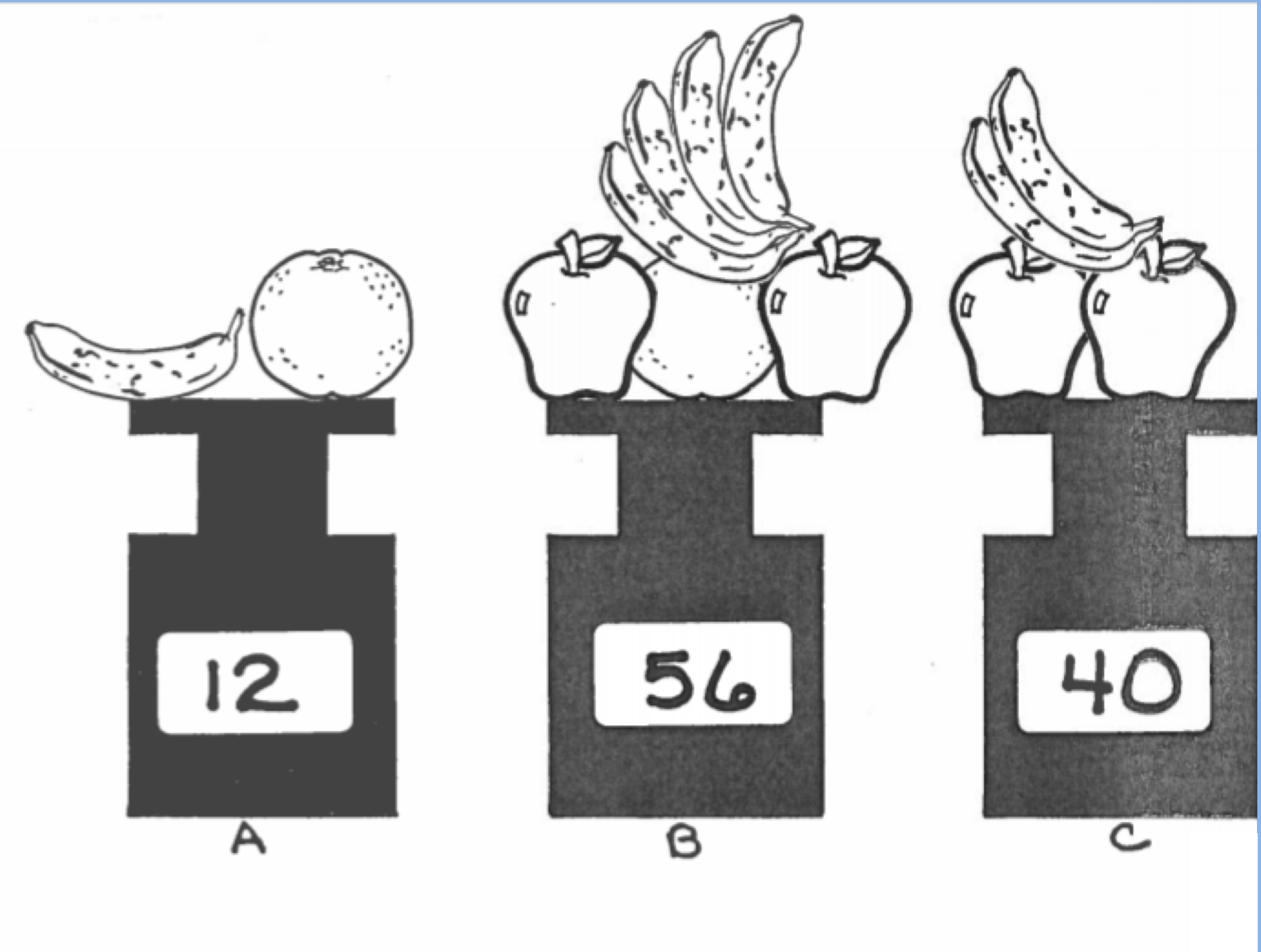
2. Explain why the beam balances. ____________________

   ____

   ____

   ____

Groundworks Algebraic Reasoning, Grade 1
McGraw Hill Publications

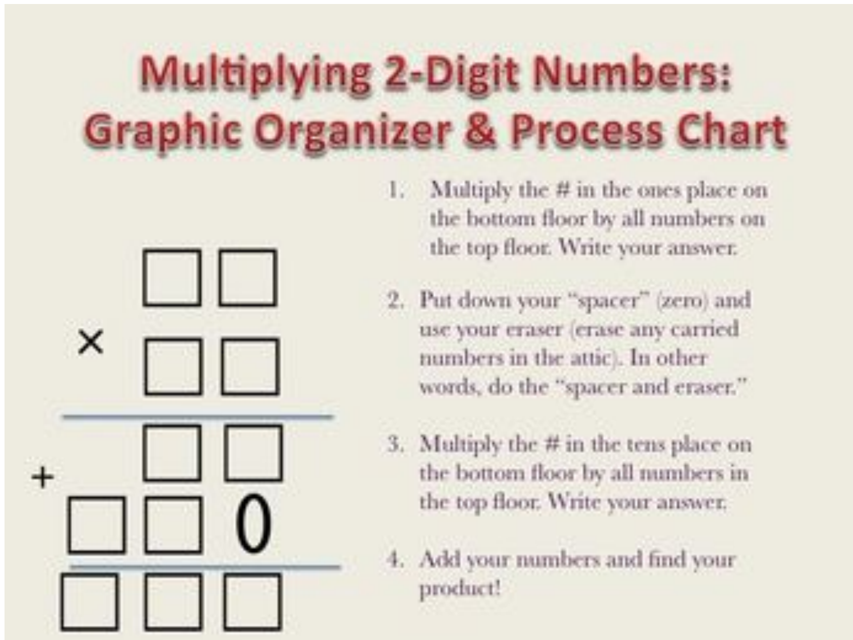


Weight Logic by Marcy Cook
www.marcycookmath.com

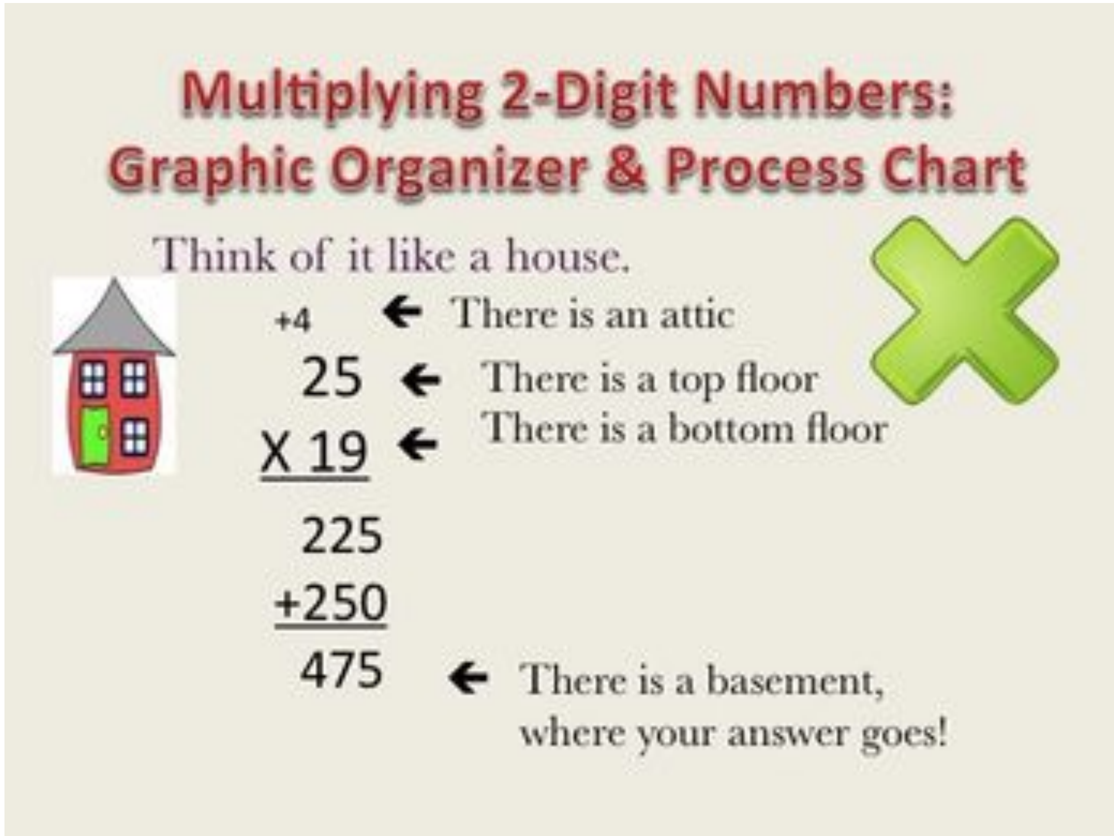
Multiplication

- Students are often taught tricks to help them remember the steps.

"Drop the Egg"


Multiplying 2-Digit Numbers:
Graphic Organizer & Process Chart
×
+
0
1. Multiply the # in the ones place on the bottom floor by all numbers on the top floor. Write your answer.
2. Put down your "spacer" (zero) and use your eraser (erase any carried numbers in the attic). In other words, do the "spacer and eraser."
3. Multiply the # in the tens place on the bottom floor by all numbers in the top floor. Write your answer.
4. Add your numbers and find your product!

House Multiplication



Multiplication

- Area method
- Partial products
- Standard algorithm

Solve 47x34=?

Using Area Model/ Box Method

	30	4
40		
7		

Using Partial Products

$$\begin{array}{r} 47 \\ \underline{\times\ 34} \end{array}$$

(x)
(x)
(x)
________ (x)

Using the Standard Algorithm

$$\begin{array}{r} 47 \\ \underline{\times\ 34} \end{array}$$

Comparing Fractions Number Talk

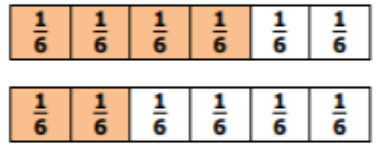
Compare $\frac{2}{3}$ and $\frac{3}{4}$

Turn and talk at your table.

Share out common strategy.

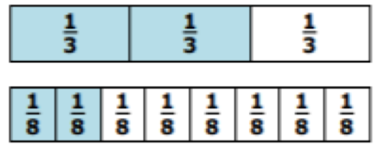
Strategies for
Comparing Fractions

1. Same <u>denominator</u>?
   → same <u>size</u> parts, so you want more parts



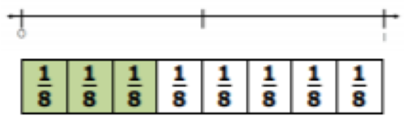
$\frac{4}{6} > \frac{2}{6}$

2. Same <u>numerator</u>?
   → same <u>number of</u> parts, so you want the bigger parts



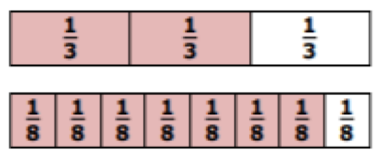
$\frac{2}{3} > \frac{2}{8}$

3. Compare to one-half
   → is the numerator more than, less than, or equal to half the denominator?



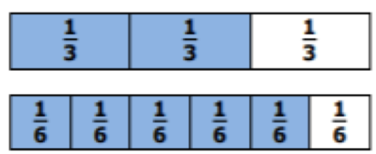
$\frac{3}{8} < \frac{4}{6}$

4. Are they one unit fraction from a whole?
   → the smaller unit fraction is closer to a whole



$\frac{2}{3} < \frac{7}{8}$

5. Create an equivalent fraction so they have a common numerator or denominator



$\frac{2}{3}$ is equivalent to $\frac{4}{6}$ which is $< \frac{5}{6}$

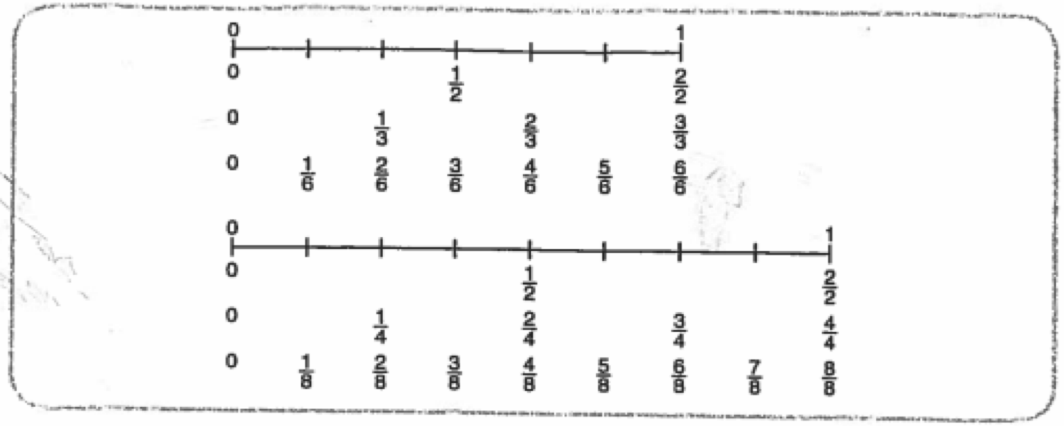
www.mathcoachescorner.com

Teachers Pay Teachers store:
Math-Coaches-Corner

Comparing Fractions Using the Butterfly method

Butterfly Method

Practice 6



Directions: Use the number lines to help you compare the fraction pairs shown here. Use > (greater than), < (less than), or = (equal to) to compare each pair.

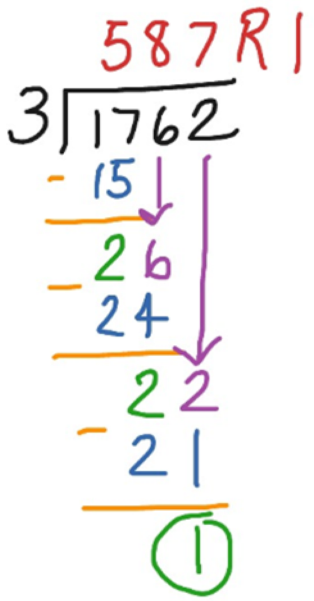
1. $\frac{1}{2}$ < $\frac{3}{4}$	2. $\frac{5}{8}$ > $\frac{1}{2}$	3. $\frac{3}{6}$ > $\frac{1}{3}$	4. $\frac{4}{6}$ = $\frac{2}{3}$	5. $\frac{7}{8}$ > $\frac{3}{4}$
6. $\frac{1}{8}$ < $\frac{1}{4}$	7. $\frac{1}{4}$ < $\frac{3}{8}$	8. $\frac{5}{6}$ > $\frac{1}{2}$	9. $\frac{4}{4}$ > $\frac{5}{8}$	10. $\frac{1}{3}$ < $\frac{5}{6}$
11. $\frac{1}{2}$ < $\frac{1}{3}$	12. $\frac{4}{4}$ > $\frac{7}{8}$	13. $\frac{1}{3}$ = $\frac{2}{6}$	14. $\frac{3}{8}$ < $\frac{2}{4}$	15. $\frac{8}{8}$ > $\frac{3}{4}$
16. $\frac{3}{6}$ < $\frac{5}{8}$	17. $\frac{4}{8}$ = $\frac{3}{6}$	18. $\frac{5}{8}$ < $\frac{3}{4}$	19. $\frac{4}{8}$ < $\frac{3}{4}$	20. $\frac{3}{6}$ < $\frac{2}{3}$
21. $\frac{5}{6}$ < $\frac{3}{3}$	22. $\frac{4}{4}$ > $\frac{5}{6}$	23. $\frac{1}{8}$ > $\frac{1}{3}$	24. $\frac{1}{4}$ = $\frac{2}{8}$	25. $\frac{1}{6}$ < $\frac{1}{2}$
26. $\frac{4}{6}$ > $\frac{4}{8}$	27. $\frac{1}{4}$ < $\frac{1}{3}$	28. $\frac{1}{3}$ < $\frac{3}{6}$	29. $\frac{1}{3}$ > $\frac{2}{8}$	30. $\frac{1}{2}$ < $\frac{5}{8}$

Comparing Fractions

- Working with a partner, pull two fraction bars or cards from the pile or bag.
- Write the fractions, compare them, and write the strategy that you used next to the fractions you are comparing.
- Turn and Talk to your table group.

Division

We were taught this way. How many of you were taught this way?



Does ÷
Mcdonalds ×
Sell −
CheeseBurgers ✓ ↓

Partial Quotient Division

Step 1: Determine how many groups of 3 you can make, using facts. For example, you know that 3 x 9 = 27, and 3 x 90 = 270.

Step 2: Write 3 x 90 next to where you wrote 270.

Step 3: Subtract the total number of groups you just created (270) from the dividend (287).

Step 4: Determine how many groups of 3 you can Make from 17 (5 groups because 3x5=15)

Step 5: Write 3 x 5 next to where you wrote 15.

Step 6: Subtract again.

Step 7: You can't make any more groups of 3, so add 90 + 5 to get your quotient of 95 and the remainder is 2.

		95 R2
	3	287
3 x 90		-270
+		17
3 x 5		- 15
95		2

Special Populations

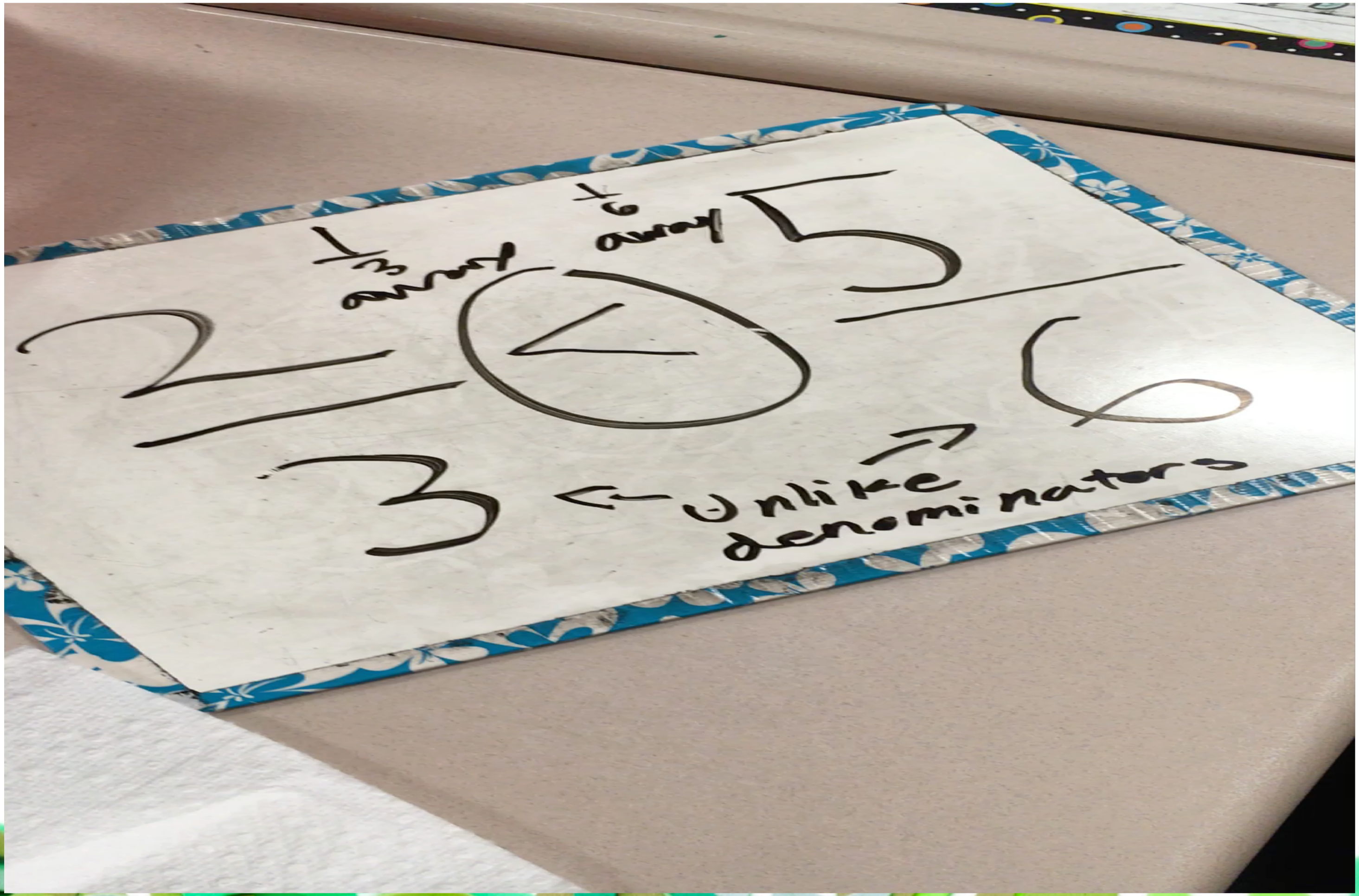
- EL
- SPED
- GIFTED

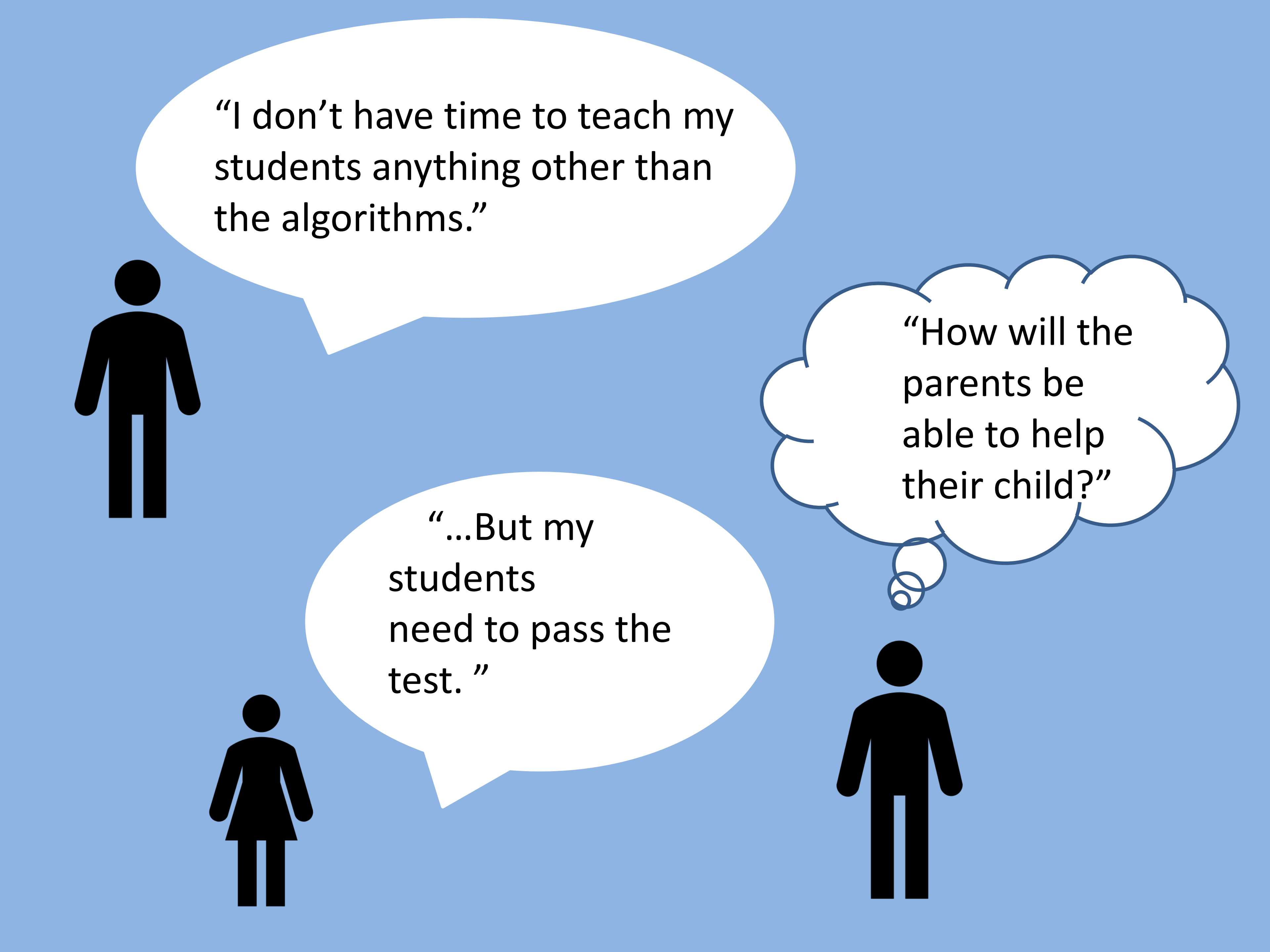
The Conceptual Classroom

- What does it look like?

- What does it sound like?

Student Example




"I don't have time to teach my students anything other than the algorithms."
"How will the parents be able to help their child?"
"...But my students need to pass the test. "

Resource List

Resources to Help with Conceptual Learning

www.youcubed.org/tasks/

https://nrich.maths.org/10334

www.gregtangmath.com

http://www.mathcoachscorner.com/

http://www.stevewyborney.com (SPLAT!)

www.playwithyourmath.com

www.mathpickle.com

Facebook Communities

1. Building Math Minds
2. Teaching and Coaching Conceptually
3. Jo Boaler's How to Learn Math
4. You Cubed
5. The Recovering Traditionalist
6. Graham Fletcher Mathematics

Thank you for coming!

We hope that you found this session useful!