Using Letter-Writing to Deepen Mathematical Understandings

William W. DeLeeuw
Arizona State University
William.DeLeeuw@asu.edu

James C. Willingham
James Madison University
willinjc@jmu.edu

Heidi Eisenreich
Georgia Southern University
heisenreich@georgiasouthern.edu

Casey Hawthorne
Furman University
casey.hawthorne@furman.edu

Agenda

1. Introduction to letter-writing activities and our project.
2. The Task
3. Discussion of anticipated affordances of letter-writing
4. Our Results
5. Discussion about how letter-writing activities could be used in mathematics classrooms to deepen mathematical understandings
6. Questions

The Letter-Writing Project

Letter-writing projects have been used for many different purposes:

- Examine student thinking, skills, and attitudes about mathematics (Fennell, 1991; Phillips & Crespo, 1996);
- Assess and develop PSTs knowledge and beliefs about teaching mathematics (Crespo, 1998, 2003);
- Examine PSTs feedback provided to students (Kastberg, Lischka, & Hillman, 2016).

The Letter-Writing Project

The purpose of this project was to examine the affordances and constraints of letter-writing exchanges for the development of preservice K-8 mathematics teachers' understanding of mathematics and mathematical feedback processes.

The Letter-Writing Project

Earlier studies supported our work in key ways:

- Identifying pedagogical benefits for PSTs that justified the approach;
- Identifying potential pitfalls such as funneling questions and providing non-actionable feedback (Kastberg,Lischka, & Hillman, 2016; Norton & Kastberg, 2012);

The Letter-Writing Project

Earlier studies supported our work in key ways:

- Highlighting shifts in mathematical questioning (Crespo, 2003; Norton & Kastberg, 2012);
- Noting the importance of the nature of formative feedback offered in letter-writing exchanges (Kastberg,Lischka, & Hillman, 2016) and participant's reflections on these processes (Norton & Kastberg, 2012).

The Letter-Writing Project

187 PSTs across 4 universities

- Shared task focused on algebraic reasoning
  - Each individual completed the task.
  - Exchanged solution with a partner from another university in the form of written letters and provided feedback.
  - Upon receiving feedback from their partners, participants revised their written solutions, composed new letters, and engaged in a second feedback cycle.
  - Students completed reflections throughout the process.

The Letter-Writing Project

Participants were from 4 different Universities

- 1 Mathematics teacher educator and 1-3 classes of PSTs at each university
- Content and methods courses
- All participants were K-8 preservice teachers.

The Letter-Writing Project

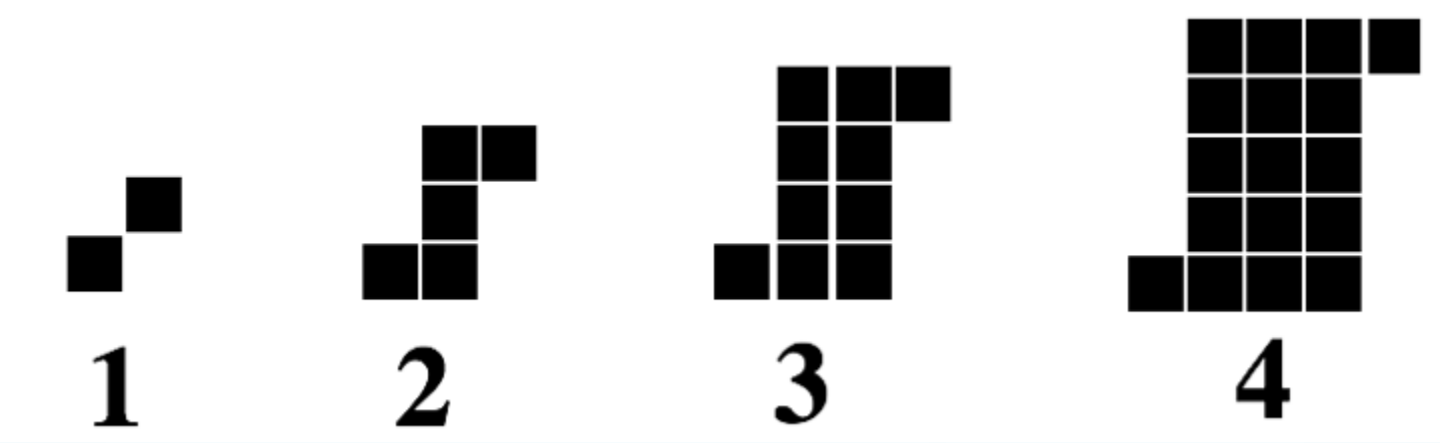
The purpose of this project was to examine the affordances and constraints of letter-writing exchanges for the development of preservice K-8 mathematics teachers' understanding of **mathematics** and mathematical feedback processes.

The Letter-Writing Project

This session will explore the affordances of letter-writing activities around a task to write an algebraic expression describing the general term of a growing pattern of blocks.

- The task will provide the opportunity to explore the conceptual connections between and among multiple solutions (equivalent explicit expressions), solution types (i.e. recursive, explicit, etc.), solution types, and representations.

The Growing S-Pattern Task

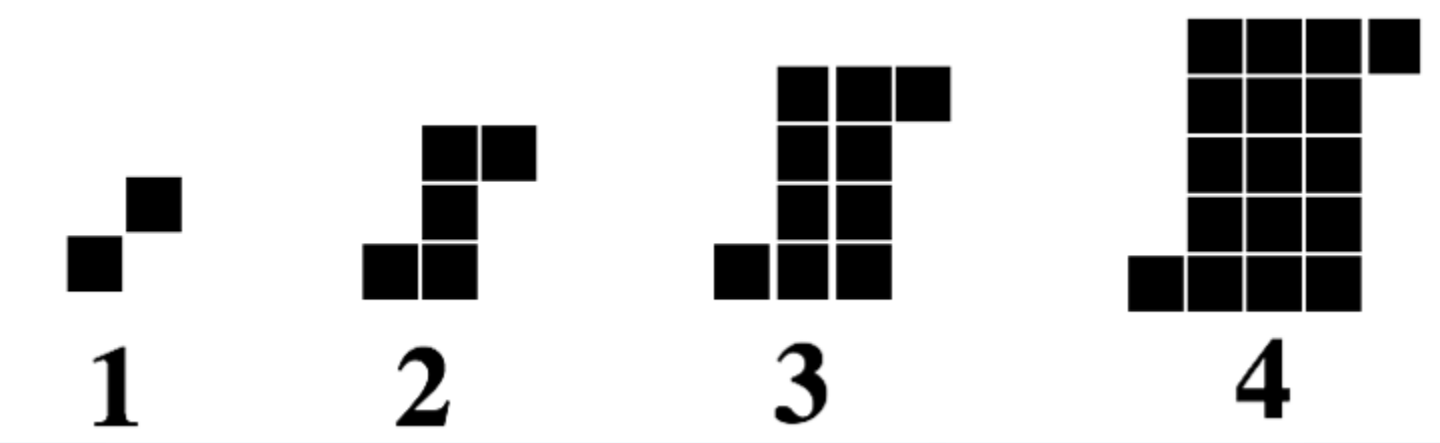


1. What would the next few figures in the sequence look like?
2. How would you describe a figure in the sequence that is larger than the 20th figure?
3. Do these patterns suggest a specific equation for the total number of tiles in any figure in the sequence?

The Growing S-Pattern Task

Take about 5 minutes to ***individually*** begin to describe the pattern in this growing sequence of blocks. Work towards an algebraic expression to describe the total number of blocks in any figure.

The Growing S-Pattern Task



1. What would the next few figures in the sequence look like?
2. How would you describe a figure in the sequence that is larger than the 20th figure?
3. Do these patterns suggest a specific equation for the total number of tiles in any figure in the sequence?

The Growing S-Pattern Task

Compare with your neighbor your solution paths, solution types, and representations.

The Growing S-Pattern Task

Thinking about just the task:

What are some of the affordances that this task provides?

The Growing S-Pattern Task

What are some of the affordances that doing this task as an iterative letter-writing project and receiving feedback?

- Writing-up this solution as a letter could …
- Seeing another student's work could ...
- Having this rubric could …
- Receiving feedback from another student could ...

Results

Participant A:

- Initial work was strong; feedback was average
- Correspondence introduced different way of thinking.
- Letter exchanged inspired different mathematical approach (connection to the derivative), even though this way of thinking was not provided.
- Case demonstrates that exposure to different ways of thinking can promote a deepened reflection on the mathematics of a task, even when feedback is not specific.

Results

Participant C:

- Initial work consisted of a systematic and quantitative approach, but relationships with stage number were not observed. Student was unable to write an explicit formula. Feedback was detailed.
- Exchange prompted Participant C to understand the pattern differently, but was ultimately unable to write an explicit algebraic expression.
- Case demonstrates that although feedback process may be helpful, growth in understanding may be be masked by examining final results in isolation.

Results

Comments from participants through reflections:

- Received honest feedback from an anonymous peer - Not pressured to hold back critical feedback, as they will not see these students.
- Authenticity – interacting with student peers promoted understanding of content and feedback processes.
- Participants from another university often provided a different perspective on the task.
- Deepening their mathematical knowledge by looking at strategies they may not have initially thought about.
- Self-reported improvement in self-efficacy is one factor in mathematical motivation and confidence.

Iterative Letter-Writing Activities

Affordances for learning mathematics:

- Provides students with the opportunities to engage with another person's perspective.
- This often leads to not only other ways of thining, but new connections with original ways of thinking.

Iterative Letter-Writing Activities

Challenges:

- Logistics can get complex.
- Having students provide feedback requires some instruction to be effective.

Questions?

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