NCTM 2019
Annual Meeting & Exposition

Empowering the Mathematics Community

San Diego, California
April 3-6, 2019

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Increasing Access to Algebra by Examining Worked Examples

Session 343

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Session Agenda

- Welcome & Introductions
- Access, Barriers, and Algebra
- NCTM's *Principles to Actions*
- Effective Mathematics Teaching Practices
- Worked Examples
- Math Activity
- Reflecting on Math Activity and Effective Mathematics Teaching Practices
- Wrap Up

Access, Barriers, & Algebra

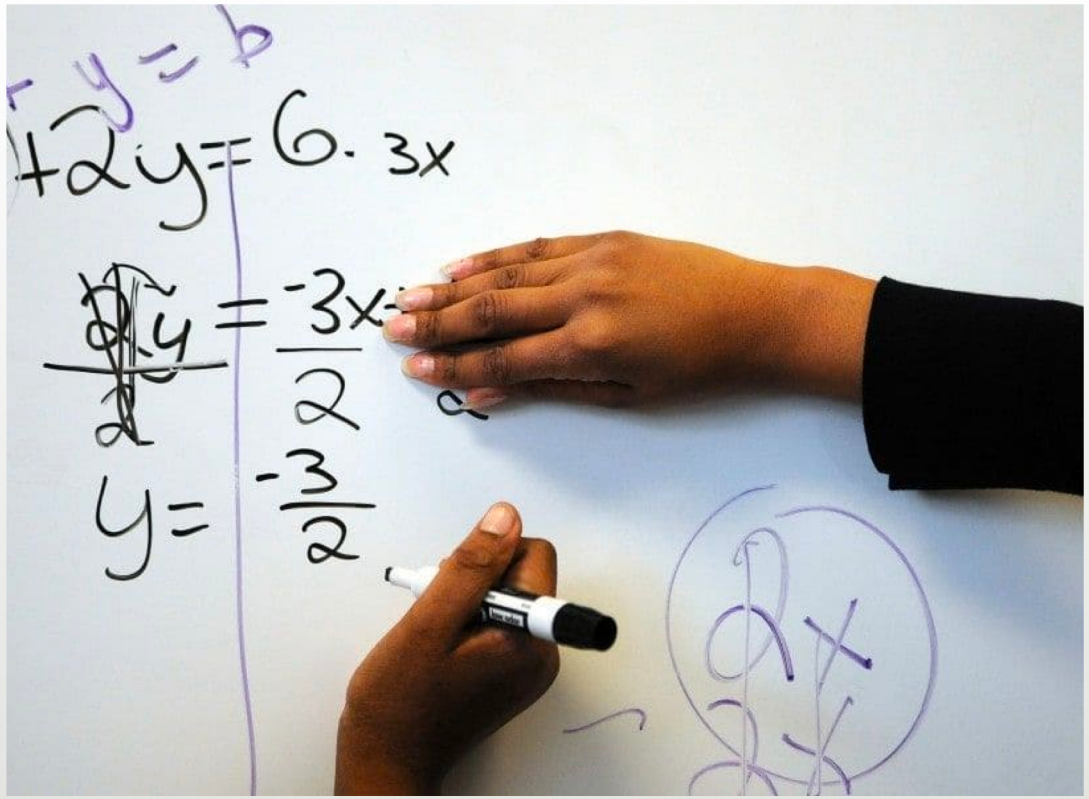
- **Algebra requirements have increased, yet the majority of US students fail to reach proficiency.**

Lucille Packard Foundation, 2014

- **One of the primary reasons for this is that they arrive mathematically unprepared to succeed in formal Algebra courses.**

National Mathematics Advisory Panel, 2008

Access, Barriers, & Algebra



- **Not just a math education issue…**
- **…but a social justice issue**

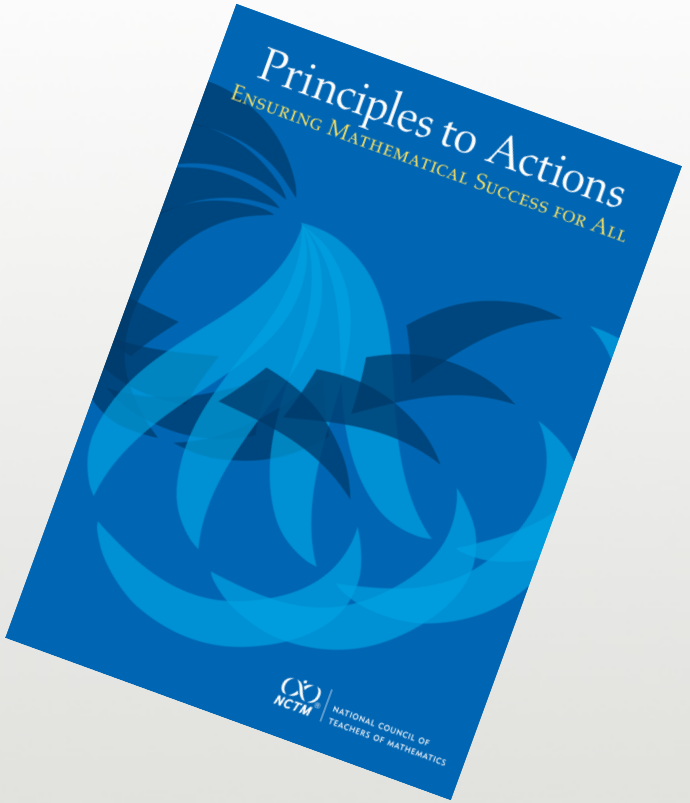
Access, Barriers, & Algebra

- College & career gatekeeper
- Required for virtually all 2- and 4-year degrees
- 50-70% of 2-year college students arrive unqualified for college math
- 80% of these do not complete any college-level math courses within 3 years
- Many spend years repeating class & leave without a degree

Baily, Jeong, & Cho, 2010;
Complete College America, 2012

NCTM's *Principles to Actions*

The primary purpose of *Principles to Actions* is to fill the gap between the adaption of rigorous standards and the enactment of practices, policies, programs, and actions required for successful implementation of those standards.



NCTM's Guiding Principles for School Mathematics

- Teaching and Learning
- Access and Equity
- Curriculum
- Tools and Technology
- Assessment
- Professionalism

Effective Mathematics Teaching

Effective teaching is the non-negotiable core that ensures that all students learn mathematics at high levels.

- Student learning of mathematics "depends fundamentally on what happens inside the classroom as teachers and learner interact over the curriculum." (Ball and Forzani 2011, p. 17)

Worked Examples

Discuss With Your Neighbor

- **First, introduce yourself to your neighbor**
- **Then, share with your friend:**
  - What do you know about worked examples or error analysis?
  - What types of worked examples do you use (correct, incorrect, etc.) in your classroom or setting?

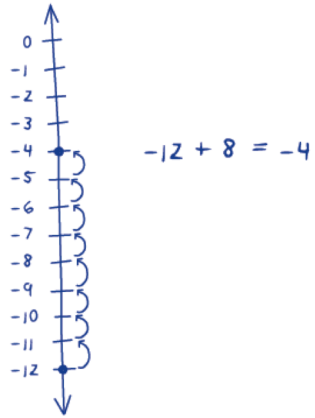
Worked Example Types

Correct Solution


Aurora

I drew a vertical number line that looks like a thermometer. 12 degrees below zero is like –12. Since the temperature increased 8 degrees, I moved 8 units up. I ended up at –4, so it's 4 degrees below zero. I could also just add 8 to –12.

OK



$-12 + 8 = -4$

Incorrect Solution


Ken

12 and 8 more equals 20. It's 20 degrees below zero.

Pitfall

$12 + 8 = 20$

Partially-worked Solution

3. At the start of this week, Mei's checking account balance was $13. During the week, she wrote a check for $35 and made a deposit of $20. What is her balance at the end of the week?

Fill in the blanks in Jacob's work below.

Jacob's work

$13 - 35 + 20$

$= (13 + 20) - 35$

$= \_\_\_\_\_ - 35$

$= \_\_\_\_\_$ Mei's balance is _____.

Worked Examples & Effective Teaching Practices

Let's put them together

NCTM's Effective Mathematics Teaching Practices

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Exploring Worked Examples

As we engage in a math problem and related worked examples, keep in mind the following Math Teaching Practices (MTPs):

- **Use and connect mathematical representations.**
- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Build procedural fluency from conceptual understanding.**
- **Elicit and use evidence of student thinking.**

Math Time!

Independently, spend a couple of minutes working on this problem:

There are 8 times as many giraffes as there are cheetahs.

Write an equation to represent the relationship between the number of giraffes and the number of cheetahs. Explain what the variables in your equation stand for.

Consider how students might approach this problem.

OK Solution

Nick

If there are 8 times as many giraffes as cheetahs, this means there are more giraffes. I made a table of possible values. If there is 1 cheetah, there are 8 giraffes. If there are 2 cheetahs, there are 16 giraffes. I saw a pattern: if the number of cheetahs is *c*, then the number of giraffes is 8c. I used *g* to represent the number of giraffes, so my equation is $g = 8c$.

OK

Number of cheetahs	Number of giraffes
1	8
2	16
3	24
c	8c

Number of giraffes = 8 × Number of cheetahs

$g = 8c$

Pitfall Solution

Erica

I used g for giraffes and c for cheetahs and got the equation $8g = c$. There are more giraffes, so it makes sense that the number 8 goes with the g.

Pitfall

8 times as many giraffes as cheetahs

$8 \times g = c$

Take a couple of minutes to share…

- **What stood out to you during this experience with worked examples and group discussion?**

- **What thoughts do you have about worked examples providing access to algebra?**

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How Can Worked Examples Support Algebra Readiness?

Mathematics Teaching Practices Discussion

- On your own, read the MTPs Teacher and Student Actions.
- With a partner(s) discuss 1 or 2 Mathematics Teaching Practices.
- Whole group discussion

Use and Connect Mathematical Representations

Use and connect mathematical representations	
What are teachers doing?	**What are students doing?**
Selecting tasks that allow students to decide which representations to use in making sense of the problems. Introducing forms of representations that can be useful to students. Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.	Using multiple forms of representations to make sense of and understand mathematics. Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations. Sketching diagrams to make sense of problem situations.

National Council of Teachers of Mathematics (2014). *Principles to Actions: Ensuring mathematical success for all.* NCTM: Reston, VA.

Pose Purposeful Questions

Pose purposeful questions	
What are teachers doing?	**What are students doing?**
Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. Allowing sufficient wait time so that more students can formulate and offer responses.	Expecting to be asked to explain, clarify, and elaborate on their thinking. Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly. Reflecting on and justifying their reasoning, not simply providing answers. Listening to, commenting on, and questioning the contributions of their classmates.

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Facilitate Meaningful Mathematical Discussions

Facilitate meaningful mathematical discussions	
What are teachers doing?	**What are students doing?**
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.	Presenting and explaining ideas, reasoning, and presentations to one another in pair, small-group, and whole-class discourse.
Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion	Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.
Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.	Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others.
Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Identifying how different approaches to solving a task are the same and how they are different.

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Build Procedural Fluency from Conceptual Understanding

Build procedural fluency from conceptual understanding	
What are teachers doing?	**What are students doing?**
Providing students with opportunities to use their methods for solving problems. Asking students to discuss and explain why the procedures that they are using work to solve particular problems. Connecting student-generated strategies and methods to more efficient procedures as appropriate. Using visual models to support students' understanding of general methods. Providing students with opportunities for distributed practice of procedures.	Making sure that they understand and can explain the mathematical basis for the procedures that they are using. Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems. Determining whether specific approaches generalize to a broad class of problems. Striving to use procedures appropriately and efficiently.

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Elicit and Use Evidence of Student Thinking

Elicit and use evidence of student thinking	
What are teachers doing?	**What are students doing?**
Identifying what counts as evidence of student progress toward mathematics learning goals. Interpreting student thinking to assess mathematical understanding, reasoning, and methods. Reflecting on evidence of student learning to inform the planning of next instructional steps.	Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse. Asking questions, responding to, and giving suggestions to support the learning of their classmates. Assessing and monitoring their own progress toward mathematics learning goals and identifying areas in which they need to improve.

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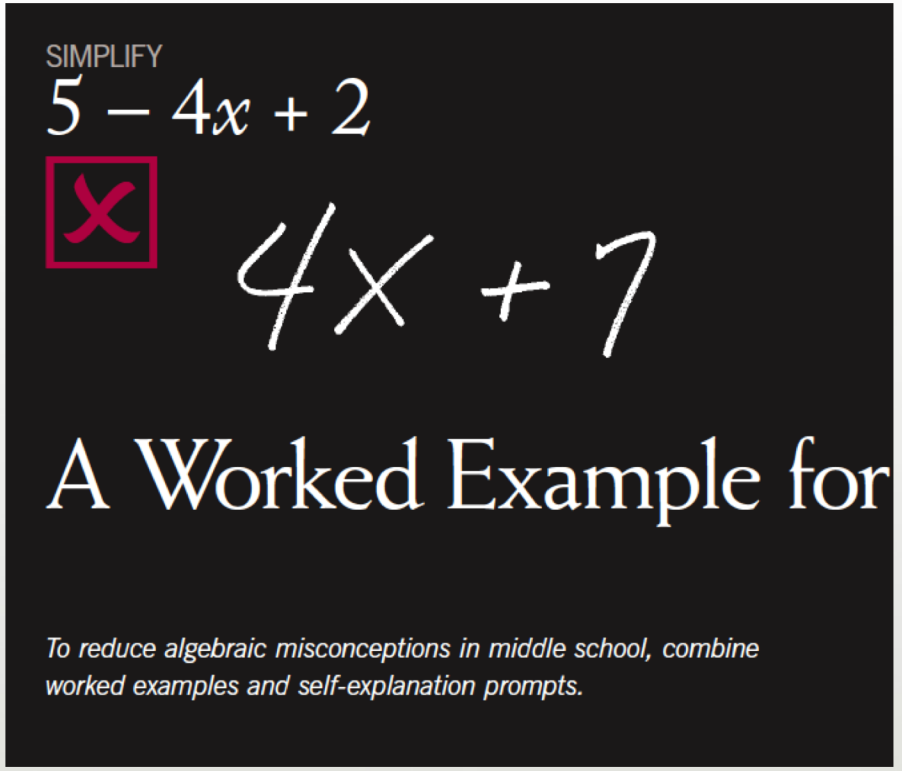
A Resource for Creating Your own Worked Example

On pages 29-31, article outlines 5 steps to create your own worked example with prompts.

Consider using this type of worked example in your classroom.



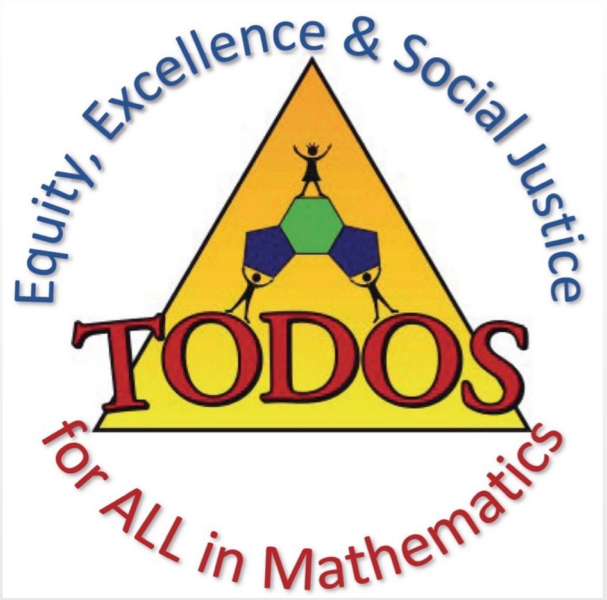
Wrapping It Up

- What are some thoughts you have from our experience today?
- Any new insights about worked examples? If so, what are they?
- What implications does this have for your classroom instruction to create more access and success with algebra for your students?

Acknowledgements

This material is based upon work supported by the National Science Foundation under Grant No. (DRL-1314416, PI Davenport). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

The mission of TODOS: Mathematics for All is to advocate for equity and high quality mathematics education for all students, in particular, Latinx students.



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Thank you!

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