SIMPLIFY

$$5 - 4x + 2$$

$$4x + 7$$

A Worked Example for

To reduce algebraic misconceptions in middle school, combine worked examples and self-explanation prompts.

SIMPLIFY

$7 - 4x + 2$

☑ $9 - 4x$

Creating Worked Examples

Kelly M. McGinn, Karin E. Lange, and Julie L. Booth

Researchers have extensively documented, and math teachers know from experience, that algebra is a "gatekeeper" to more advanced mathematical topics. Students must have a strong understanding of fundamental algebraic concepts to be successful in later mathematics courses (Star and Rittle-Johnson 2009). Unfortunately, algebraic misconceptions that students may form or that deepen during middle school tend to follow them throughout their academic careers (Cangelosi et al. 2013). In addition, the longer that a student holds a mathematical misconception, the more difficult it is to correct (Kilpatrick, Swafford, and Findell 2009). Therefore, it is imperative that we, as teachers, attempt to address these algebraic misconceptions while our students are still in middle school. One tool commonly used to do such a task is the combination of worked examples and self-explanation prompts (see **fig. 1**) (Aleven and Koedinger 2002). This article will describe not only the benefits of using this strategy but also how it connects to the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010). It will also provide instruction on creating worked-example and self-explanation problem sets for your own students.

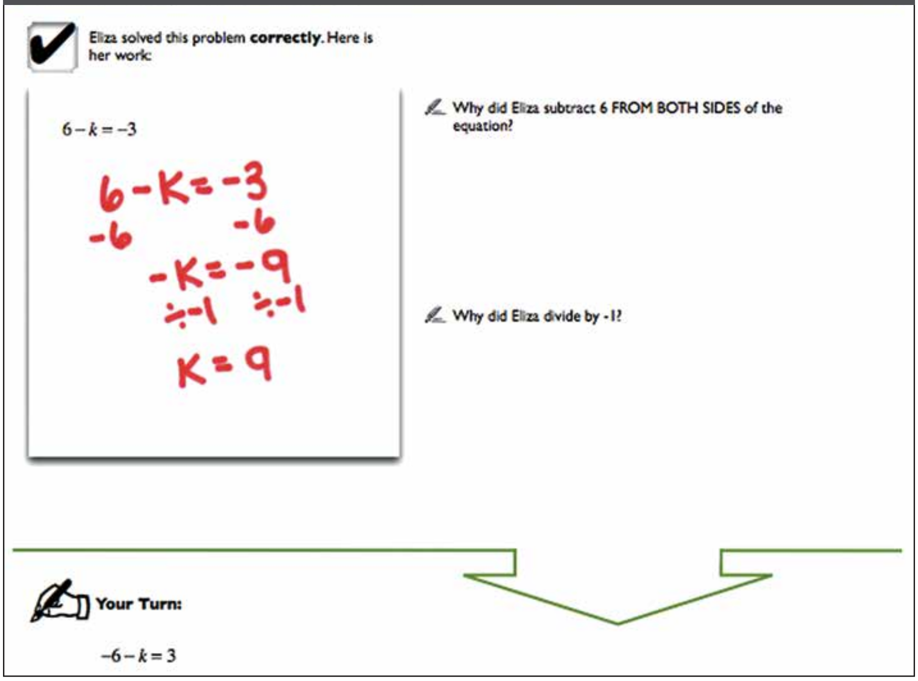
BENEFITS OF WORKED EXAMPLES AND SELF-EXPLANATION PROMPTS

A worked example in mathematics is a problem that has been fully completed to demonstrate a procedure (Clark, Nguyen, and Sweller 2011). Worked examples, in combination with self-explanation prompts (questions that encourage students to explain the problem back to themselves) have been found to increase algebra learning (Booth et al. 2015). Students who receive worked examples make fewer errors, complete follow-up problems faster, and require less teacher assistance (Sweller and Cooper 1985; Carroll 1994). This practice also improves both conceptual and procedural knowledge by promoting the integration of new knowledge with what students already know, helping students make their new knowledge explicit, and focusing students' attention on important mathematical principles (Rittle-Johnson 2006; McEldoon, Durkin, and Rittle-Johnson 2013).

Furthermore, research has found that the use of both correct and incorrect worked examples can improve student learning (Booth et al. 2013). Often, teachers are hesitant to use incorrect examples because they feel that exposing students to incorrect

I find that either correct examples or incorrect examples help kids identify themselves with somebody else easily. [With misconceptions], kids can be really stubborn, and they really don't believe you that it's wrong. To see a kid look at an incorrect example and say, "No, but this is correct," and kind of have that moment of "Oh, I really was off, and now I understand it more" . . . I think they are allowed to engage with that problem more so than if it was just a standard practice.

Fig. 1 This task combines a worked example with a self-explanation prompt.



procedures may increase misconceptions. However, this is unfounded; incorrect examples help students recognize incorrect procedures and think about the differences between them and the correct procedures, which can increase students' conceptual and procedural knowledge (Booth et al. 2013).

To maximize the impact of worked examples, a similar practice problem can be included, thus allowing the students to practice their newly learned concept or skill (Atkinson et al. 2000). Students who receive alternating worked examples and practice problems outperform those who receive all worked examples followed by all practice problems (Trafton and Reiser 1993). When students take time to study a worked-out example, answer questions that require them to explain the problem to themselves, and then practice a similar problem immediately afterward, they begin to break down their previously stubborn misconceptions in ways that help strengthen their understanding of algebraic concepts.

TEACHER PERSPECTIVES

Although empirical research has shown the benefits of providing middle school students with worked examples and self-explanation prompts, reviewing the perspective of the teacher can be just as valuable (all names are pseudonyms).

Jane, an eighth-grade algebra teacher from a public school in the Midwest, points out the benefit of using incorrect examples by stating,

> The incorrect examples are actually sometimes the ones that really are better for showing students. The incorrect examples are often the best learning tool. . . . Forcing them to say, "Well, that's what I do, what should I do then? If I'm doing the same thing as this boy in this problem, what's wrong with that?"

Alyssa, another eighth-grade algebra teacher from a second school district, voices another view, finding that there are benefits to using both types of examples. She says,

Teachers who have used worked-example–self-explanation problems have reported overwhelmingly positive results. Most important, they see their students finally mastering concepts that had previously eluded them because of persistent misconceptions. By using standards-aligned instructional strategies that help middle school students gain access to algebraic concepts, teachers are helping to ensure that all students can achieve success in higher-level mathematics courses. Peter, a mathematics teacher leader, summed it up by saying,

> Too often in math class, it is about just getting the answer, it's not about the process. And when you're analyzing someone else's work . . . you're dealing with the process, not just the answer. I think that's extremely valuable for students!

Although it is evident that the use of worked examples and self-explanation prompts can improve student learning and help students confront their misconceptions, it is also important to ensure that this strategy aligns with the principles of the Common Core State Standards for Mathematics (CCSSM).

ALIGNMENT WITH THE COMMON CORE

Worked-example problems with self-explanation prompts can be paired with any CCSSM content standard; however, this strategy also helps teachers integrate the Standards for Mathematical Practice (SMP) into his or her classroom.

The first standard, make sense of problems and persevere in solving them, is a great example (CCSSI 2010). The first line of the standard's description is, "Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution" (CCSSI 2010, p. 6). The worked-example instructional strategy helps scaffold understanding so that students can work through this process. The use of worked examples allows students to study a sample entry point, whereas the self-explanation questions prompt students to explain to themselves the meaning of the problem. This process not only necessitates that students think through the problem in a new way but also helps situate students to be able to understand the different approaches that a classmate might use.

Worked examples and self-explanation prompts also align with SMP 2: Reason abstractly and quantitatively (CCSSI 2010). Specifically, the standard emphasizes the importance of "making sense of quantities and their relationship in problem situations" and "attending to the meaning of quantities, not just how to compute them" (CCSSI 2010, p. 6). Carefully worded self-explanation prompts help students accomplish this goal. For instance, a student can be explicitly asked to explain what the y-intercept represents in a given word problem, a task that is often left to discussion or implication with traditional solution-based assignments.

Finally, SMP 3, Construct viable arguments and critique the reasoning of others (CCSSI 2010), is also addressed through the use of the worked-example–self-explanation strategy. As explained in more detail below, the worked example demonstrates the effort of a fictitious student (see **fig. 1**). Therefore, the actual student may practice "critiquing" another student's reasoning in a safer environment when answering the explanation prompts. For example, a potential prompt may read, "Does Natalie's price for a pen seem reasonable? Why or why not?" In addition, the use of worked examples that are incorrect gives students the opportunity to "distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is" (CCSSI 2010, p. 7).

Incorrect examples help students recognize incorrect procedures and think about the difference between them and the correct ones.

To ensure that students receive the full benefits of this strategy as demonstrated in prior research, it is important that care is taken to write problems that specifically address your own students' needs. A step-by-step guide will help to maximize the benefit of each problem on student learning.

STEPS FOR CREATING WORKED EXAMPLES AND SELF-EXPLANATION PROMPTS

Five steps can be used to create a worked-example–self-explanation item.

Step 1

Identify the objective and list a few common misconceptions associated with this objective. Similar to planning a lesson, start by writing the lesson goal or focus objective. For example, the objective for our sample item will be the following: Students will be able to simplify an expression by combining like terms.

This is where you must brainstorm. Think about the mistakes you have seen students make while solving problems associated with the objective in the past. For our sample, the misconceptions and errors associated with combining like terms include a tendency to fail to include the negative sign as a part of the term and to combine non-like terms.

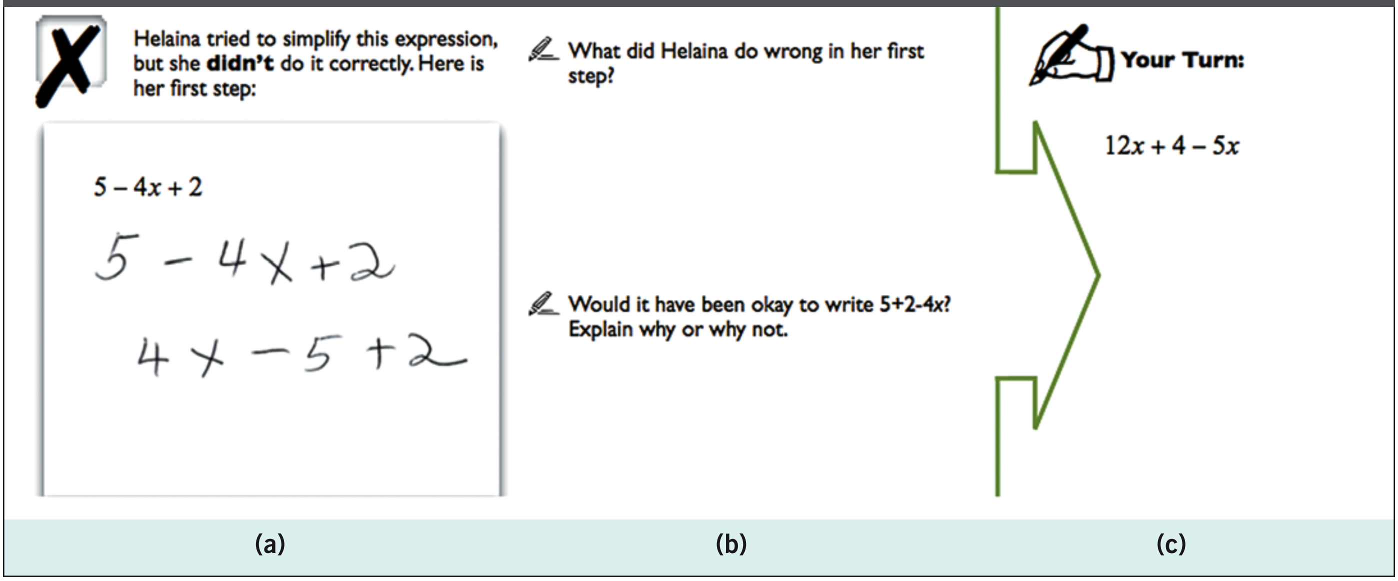
Step 2

Choose one misconception or error for each example. The goal is to focus students' attention on one aspect of the problem at a time; do not overwhelm them with too many errors or ideas. Either make an entire activity sheet focused on one misconception or error or create a sheet that focuses on just a few items. For our sample, we created a worked example focused on the idea that you must include the negative sign with the term when rearranging terms within the expression.

Step 3

Create the worked example using the misconception. Write a worked-out solution to a problem that meets your objective. Although the worked example can be done either correctly or incorrectly, clearly mark the problem as correct or incorrect. It is also helpful to act as if a fictitious student completed this example because the actual student completing the work will connect with that other student and realize that a similar misconception or a similar error is indeed common. Choose students' names that reflect the diversity of the classroom and vary who completes the incorrect and

Fig. 2 A practice problem can be given to students to complete on their own.



Sample Prompts

Although it is acceptable to ask procedural questions, be sure to ask students to explain and/or justify their reasoning.

1. Why is __________ not included in the answer?
2. What did [student name] __________ do as his first step?
3. What should [student name] have done to __________?
4. Would it have been OK to write __________? Why or why not?
5. Why did [student name] combine __________ and __________?
6. Why did [student name] first _______ then _______?
7. Is _______ the same expression as _______? Explain.
8. Would [student name] have gotten the same answer if he [or she] _______ first?
9. Why did [student name] change __________ to __________?
10. Explain why __________ would have been an unreasonable answer.
11. How could [student name] have figured out that his [or her] answer did not make sense?
12. How did [student name] know that __________ was not equal to __________?
13. What did the __________ represent in this word problem?
14. How did the __________ in the equation affect the graph?
15. Why did [student name] __________ from both sides of the equation?

correct examples. **Figure 2a** shows our sample worked example. Note that the item is clearly marked incorrect, that a fictitious student completed the item, and that the fictitious student only made one error.

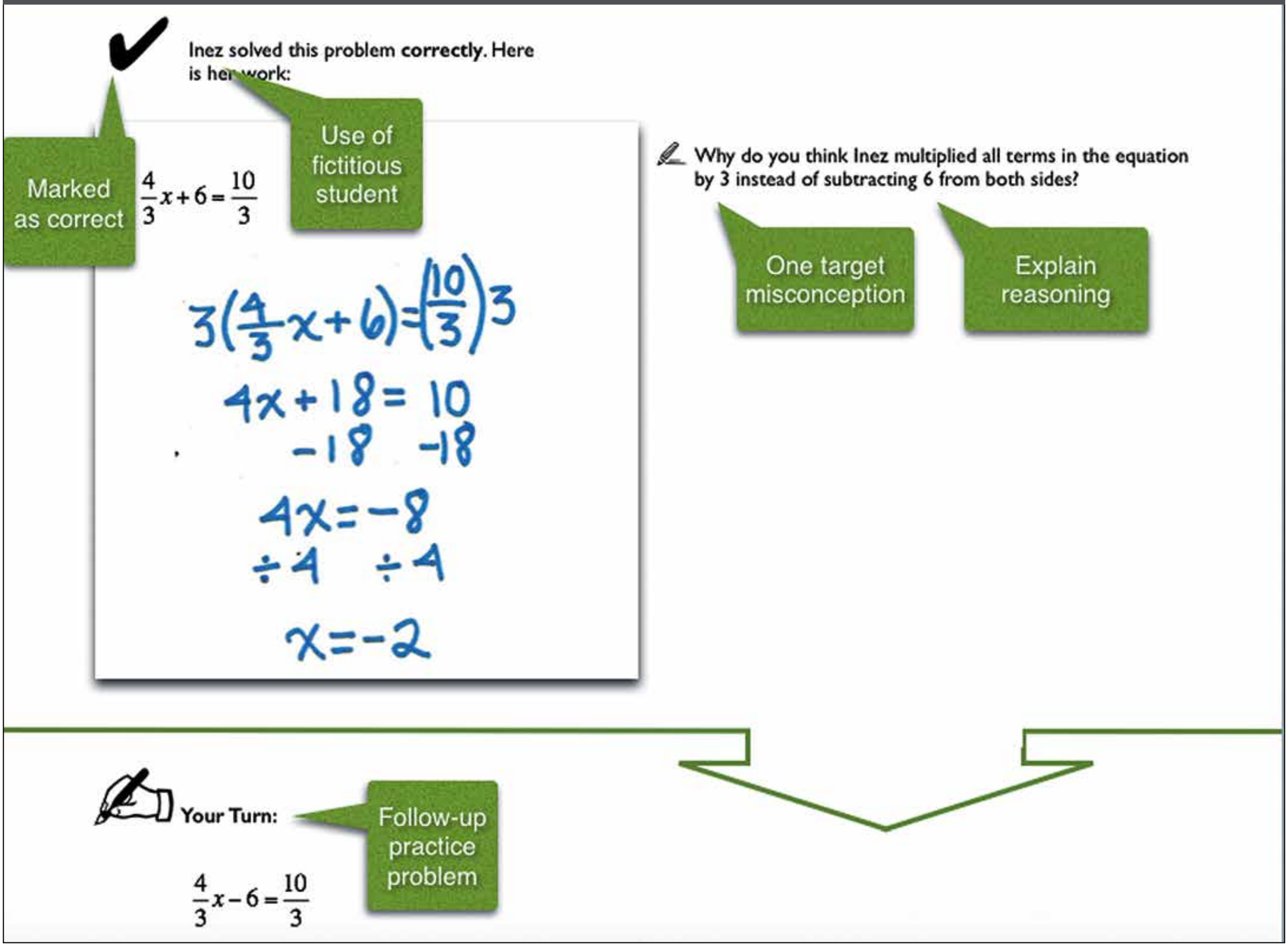
Step 4

Write the self-explanation prompt, focusing on the target misconception or error. This is the trickiest part of the process. You will want to write one or two questions that specifically ask the student to examine his or her own misconception through the work done by the fictitious student. At first, teachers often have trouble creating self-explanation prompts; however, it gets easier with practice. Avoid only asking such "what" questions as these:

1. What is wrong with the example?
2. What mistake was made?
3. What is the correct answer?

Instead, focus on writing "why" questions. You want to have students explain their reasoning, not just state the procedure. It is important to call students' attention to the features of the problem that you think are important; in other words, draw their

Fig. 3 This version shows a second worked-example–self-explanation combination.



attention to the target misconception or error you chose in step 2. See **figure 2b**. Although the student was asked a "what" question, it was followed up with a question that prompted the student to explain his or her reasoning. This is important. Although it is fine to ask procedural questions, follow it up with a conceptual question. In fact, research has found greater learning gains when students are asked to explain the concept, rather than the procedure (Matthews and Rittle-Johnson 2009).

Step 5

Create a practice problem similar to the worked example. Give this problem to students to complete on their own after they have studied the worked example and answered the self-explanation prompts, thus allowing them time to practice. It will also reinforce the new information related to their misconception. Note in **figure 2c** that the structure of the "Your Turn" problem is identical to the worked example. The only aspects that changed were the numbers and order of the terms.

ONE MORE EXAMPLE OF A WORKED EXAMPLE

It is always helpful to see more than one example when learning a new skill. **Figure 3** illustrates a second worked-example–self-explanation combination. Note a few important features:

1. Marked example: Be sure to mark the example as correct or incorrect. This example is clearly marked as correct.
2. Use of fictitious student: Inez completed this problem.
3. One target misconception or error: Although many different misconceptions and errors can be associated with a particular problem, be sure to only focus on one at a time. In this instance, the item was designed to promote alternate problem-solving procedures and counter the misconception that there is only one way to solve a problem. With the same example, one could also target the common error of not selecting the

appropriate number to multiply both sides by or not multiplying all terms in the equation.

4. Explain reasoning: Although it is sometimes beneficial to ask for procedural explanations, be sure to also prompt students to explain their reasoning. For example, when students were asked, “Why do you think Inez multiplied all terms in the equation by 3 instead of subtracting 6 from both sides?” they answered in these ways:

    - To remove the denominators, because then it would be more complicated than it needs to be
    - So we could get rid of the fractions
    - To get rid of the denominator
    - To remove the bottom #!

This prompt was meant to highlight the possibility of additional problem-solving strategies.

5. Follow-up practice problem: Finally, be sure to allow students to complete a problem similar to the worked example to practice the skill.

EXTENDED UNDERSTANDING OF SELF-EXPLANATION PROMPTS

As mentioned above, writing self-explanation prompts is the most difficult part of the process, yet it is also the most crucial component in creating a problem that successfully addresses students’ misconceptions. See the **sidebar** (on p. 30) for a few sample prompts to help you get started. Recall that although it is acceptable to ask procedural questions, do not solely rely on those types of questions. Be sure to ask students to explain and/or justify their reasoning. The simplest way to do this is to ask “why” questions.

FINAL THOUGHTS

Worked examples paired with self-explanation prompts show promise as being a new strategy to accelerate student understanding and success in algebra, especially for students who have held persistent misconceptions over time. Teachers will find the most success when they target their students’ individual needs and misconceptions. By giving teachers the tools to respond to students and create their own examples and prompts, it is hoped that all students can achieve success in understanding foundational components of algebra.

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