Algebraic Reasoning and Proof

Algebraic calculations and reasoning can be used to solve many kinds of practical problems. But they can also be used to perform some amazing magic tricks with numbers. For example, do these numerical operations.

> **Pick an integer between 0 and 20.**
>
> **Add 5 to your number.**
>
> **Multiply the result by 6.**
>
> **Divide that result by 3.**
>
> **Subtract 9 from that result.**

If each class member reports the final number that results from those operations, your teacher will be able to tell the starting number of each student.

If you believe that your teacher performs this number magic by memorizing all the possible starting and ending number combinations, you can increase the range of starting numbers to 0–50 and then 0–100. Your teacher will still be able to find each start number.

Data in the table at the right show some combinations of starting and ending numbers.

Start Number	End Number
4	9
11	23
17	35
33	67
45	91

THINK ABOUT THIS SITUATION

The number magic is not so amazing if you think about it with algebraic reasoning.

a What starting number do you think would lead to an end result of 39? Of 123? Of 513?

b Can you explain how and why your teacher is able to find every starting number when told only the ending number?

In this lesson, you will explore ways to use algebraic reasoning to discover and prove number patterns and relationships like the one used in the number trick. Then you will see how the same reasoning methods can be used to prove other important mathematical principles.

INVESTIGATION 1

Reasoning with Algebraic Expressions

In earlier algebra units of *Core-Plus Mathematics*, you developed a toolkit of techniques for writing algebraic expressions in useful equivalent forms. As you work on the problems in this investigation, look for answers to this question:

How can strategies for manipulating algebraic expressions into equivalent forms be used to explain interesting number patterns?

Algebra and Number Magic Algebraic reasoning skills can be used to create and explain many different number tricks.

1 There is a simple way to discover start numbers in the Think About This Situation number trick: *Subtract one from the end number and divide that result by two.* Explaining why that decoding strategy works and proving that it will always work requires some algebraic reasoning.

a. Use the letter n to represent the start number and build an algebraic expression that shows the steps for calculating the result that will be reported to the teacher.

i. Adding 5 to your number is expressed by … .

ii. Multiplying that result by 6 is expressed by … .

iii. Dividing that result by 3 is expressed by … .

iv. Subtracting 9 from that result is expressed by … .

b. Use what you know about algebra to write the final expression from Part a in simplest form. What relationship between starting and ending numbers is shown by the result?

c. How can the relationship between starting and ending numbers be used to find starting numbers when only ending numbers are known?

d. Suppose that the third step of the number trick, "Divide that result by 3," is replaced by "Divide that result by 2." How would that change in the procedure affect the decoding strategy?