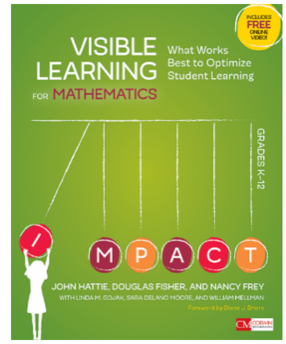

Making Mathematics Learning Visible:

An Introduction to Visible Learning for Mathematics

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Learning Intention and Success Criteria

- I am learning to define the three phases of learning and the unique importance of each.
  - *I can describe surface, deep and transfer learning.*
  - *I can explain the connectivity between surface, deep, and transfer to my peers.*

SURFACE, DEEP, TRANSFER

- *I can define surface, deep and transfer*
- *I can identify examples of surface, deep, and transfer of learning within a mathematical task*

High Impact Approaches at Each Phase of Learning

Surface Learning		**Deep Learning**		**Transfer Learning**	
Strategy	**E.S.**	**Strategy**	**E.S.**	**Strategy**	**E.S.**
Manipulatives	0.50	Questioning	0.48	Peer Tutoring	0.55
Direct Instruction	0.59	Multiple Representations	0.50	Cooperative Learning	0.59
Note-taking	0.59	Concept Mapping	0.60	Problem solving teaching	0.61
Summarizing	0.63	Study Skills	0.63	Metacognitive strategies	0.69
Number Talks	0.64	Self-Questioning	0.64	Formal discussions (debate)	0.82
Leverage prior knowledge	0.65	Reciprocal Teaching	0.74	Transforming conceptual knowledge	0.85
Vocabulary Instruction	0.67	Class Discussion	0.82	Organizing conceptual knowledge	0.85
Spaced Practice	0.71	Organizing and transforming notes	0.85	Identifying similarities and differences	1.32
Student-teacher relationships 0.72					
Teacher clarity 0.75					
Feedback 0.75					
Teacher credibility 0.90					
Assessment-capable learners 1.44					
Collective teacher efficacy 1.57					
Teacher estimates (expectations) of student learning 1.62					

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Surface Learning

In mathematics, we can think of **surface learning** as having two parts. First, it is initial learning of concepts and skills. When content is new, all of us have a limited understanding. That doesn't mean we're not working on complex problems; it's just that the depth of thinking isn't there yet. Whether a student is exposed to a new idea or information through an initial exploration or some form of structured teacher-led instruction (or perhaps a combination of the two), it is the introductory level of learning—the initiation to, and early understanding of, new ideas that begins with developing conceptual understanding—and at the right time, the explicit introduction of the labels and procedures that help give the concepts some structure. Let us be clear: surface learning is not shallow learning. It is not about rote skills and meaningless algorithms. It is not prioritizing "superficial" learning or low-level skills over higher order skills. It should not be mistaken for engaging in procedures that have no grounding in conceptual understanding. Second, surface learning of concepts and skills goes beyond just an introductory point; students need the time and space to begin to consolidate their new learning. It is through this early consolidation that they can begin to retrieve information efficiently, so that they make room for more complex problem solving. For example, counting is an early skill, and one that necessarily relies initially on memorization and rehearsal. Very young children learn how to recite numbers in the correct order, and in the same developmental space are also learning the one-to-one correspondence needed to count objects. In formal algebra, surface learning may focus on notation and conventions. While the operations students are using are familiar, the notation is different. Multiplication between a coefficient and a variable is noted as $3x$, which means 3 times x. Throughout schooling, there are introductions to new skills, concepts, and procedures that, over time, should become increasingly easier for the learner to retrieve.

Surface learning is the initiation to new ideas. It begins with development of conceptual understanding, and then, at the right time, labels and procedures are explicitly introduced to give structure to concepts.

Surface learning is not shallow learning. It is not about rote skills and meaningless algorithms.

Importantly, through developing surface learning, students can take action to develop initial conceptual understanding, build mathematical habits of mind, hone their strategic thinking, and begin to develop fluency in skills. For example, surface learning strategies can be used to help students begin developing their metacognitive skills (thinking about their thinking). Alternatively, surface learning strategies can be used to provide students with labels (vocabulary) for the concepts they have discovered or explored. In addition, surface learning strategies can be used to address students' misconceptions and errors.

One challenge with surface learning is that there is often an overreliance on it, and we must think of the goal of mathematics instruction as being much more than surface learning. When learning stalls at the surface level, students do not have opportunities to connect conceptual understandings about one topic to other topics, and then to apply their understandings to more complex or real-world situations. That is, after all, one of the goals of learning and doing mathematics. Surface learning gives students the toolbox they need to build something. In mathematics, this toolbox includes a variety of representations (e.g., knowing about various manipulatives and visuals like number lines or bar diagrams) and problem-solving strategies (e.g., how to create an organized list or work with a simpler case), as well as mastering the notation and conventions of mathematics. But a true craftsman has not only a repertoire of tools, but also the knowledge of which tools are best suited for the task at hand. Making those decisions is where **deep learning** comes to the forefront, and, as teachers, we should always focus on moving students forward from surface to deep learning.

Deep learning is about consolidating understanding of mathematical concepts and procedures and making connections among ideas.

Deep Learning

The deep phase of learning provides students with opportunities to consolidate their understanding of mathematical concepts and procedures and make deeper connections among ideas. Often, this is accomplished when students work collaboratively with their peers, use academic language, and interact in richer ways with ideas and information.

Mrs. Graham started the school year for her fourth graders working with factors and multiples, connecting this work to previous third-grade experiences with arrays as models for multiplication, and extending these ideas to understanding prime and composite numbers. Students started by building and describing rectangular arrays for numbers from 1 to 50 (some students continued on to 100) and then discussed their answers to a variety of questions that developed the idea of prime and composite numbers. Class discussion incorporated mathematical vocabulary so it became a natural part of the student conversations (surface learning). The next day, students played a game called Factor Game (http://www.tc.pbs.org/teachers/mathline/lessonplans/pdf/msmp/factor.pdf) in which an understanding of primes and composites was crucial to developing strategies to win (deep learning is now occurring). However, the story doesn't end there. In March, students were beginning to study area and perimeter of rectangles. Following an initial exploration, several students approached Mrs. Graham to comment, “This is just like what we did last September when we were building arrays and finding primes and composites!” Talk about making connections!

As you can see, students move to deep learning when they plan, investigate, and elaborate on their conceptual understandings, and then begin to make generalizations. This is not about rote learning of rules or procedures. It is about students taking the surface knowledge (which includes conceptual understanding) and, through the intentional instruction designed by the teacher, seeing how their conceptual understanding links to more efficient and flexible ways of thinking about the concept. In Mrs. Graham’s class, students began by developing surface knowledge of factors and multiples using concrete models and connected that to primes and composites. Mrs. Graham’s use of the Factor Game provided students a way to apply their surface knowledge to developing strategies to win a game . . . deep knowledge. A teacher who nurtures strategic thinking and action throughout the year will nurture students who know when to use surface knowledge and when deep knowledge is needed.

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We need to balance our expectations with our reality. This means more explicit alignment between what teachers claim success looks like, how the tasks students are assigned align with these claims about success, and how success is measured by end-of-course assessments or assignments. It is not a matter of all surface or all deep. It is a matter of being clear about when surface and when deep is truly required.

Consider this example from algebra. A deep learning aspect of algebra comes when students explore functions—in particular, the meaning of the slope of a line. Surface knowledge focuses on understanding the term mx in the slope-intercept ($y = mx + b$) form to mean m copies of the variable x. Deep learning requires students to understand and show that this term represented visually is the steepness or flatness of the slope of a line and the rate of change of the variables. Such learning might come from working collaboratively to explore a group of functions represented in multiple ways (equations, tables of values, and graphs) and make inferences about the slope in each representation. At this point, students are connecting their conceptual knowledge of ratio to their surface knowledge of algebraic notation and the process of graphing. This is deep learning in action.

Transfer Learning

The ultimate goal, and one that is hard to realize, is transfer. Learning demands that students be able to apply—or transfer—their knowledge, skills, and strategies to new tasks and new situations. That transfer is so difficult to attain is one of our closely kept secrets—so often we pronounce that students can transfer, but the processes of teaching them this skill are too often not discussed, and we'll visit that in Chapter 6.

Transfer is the phase of learning in which students take the reins of their own learning and are able to apply their thinking to new contexts and situations.

Transfer is both a goal of learning and also a mechanism for propelling learning. Transfer as a goal means that teachers want students to begin to take the reins of their own learning, think metacognitively, and apply what they know to a variety of real-world contexts. When students reach this level, learning has been accomplished.

Nancy once heard a mathematics teacher say that transfer is what happens when students do math without someone telling them to do math. It's when they reach into their toolbox and decide what tools to employ to solve new and complex problems on their own.

For example, transfer learning happens when students look at data from a science or engineering task that requires them to make sense of a linear function and its slope. They will use their surface knowledge of notation and convention, along with their deep understanding of slope as a ratio, to solve a challenge around designing an electrical circuit using materials with a variety of properties. Ohm's law ($V = iR$, where V represents voltage, i represents the current, and R represents resistance) is the linear function that relates the relevant aspects of the circuit, and students will use their mathematics knowledge in finding their solution.

One of the concerns is that students (often those who struggle) attempt to transfer *without* detecting similarities and differences between concepts and situations, and the transfer does not work (and they see this as evidence that they are dumb). Memorizing facts, passing tests, and moving on to the next grade level or course is not the true purpose of school, although sadly, many students think it is. School is a time to apprentice students into the act of becoming their own teachers. We want them to be self-directed, have the dispositions needed to formulate their own questions, and possess the tools to pursue them. In other words, as students' learning becomes visible to them, we want it to become the catalyst for continued learning, whether the teacher is present or not. However, we don't leave these things to chance. Close association between a previously learned task and a novel situation is necessary for promoting transfer of learning. Therefore, we teach with intention, making sure that students acquire and consolidate the needed skills, processes, and metacognitive awareness that make self-directed learning possible.

One of the struggles in teaching mathematics is to determine how much to tell students versus how to support students as they engage in productive struggle on their own, and when to know which is the right step to take. Let's take a look at helping elementary-age children build a toolbox of problem-solving strategies. Linda once attended a workshop for teachers that opened a whole new world of problem-solving strategies to use when solving nonroutine or open-ended problems. She was excited to take these problems back to her students and give them the opportunities to solve rich problems that involved some higher order thinking—that is, solving problems that involve much more than simple calculations. After some careful planning, she started a Monday class with her fifth graders by presenting the following problem.

> Mrs. Thompson, the school cook, is making pancakes for the special fifth-grade breakfast. She needs 49 pounds of flour. She can buy flour in 3-pound bags and 5-pound bags. She only uses full bags of flour. How can she get the exact amount of flour she needs?

Having never solved this type of problem before, the students rebelled. Choruses of "I don't know what they want me to do!" rang out across the classroom. "But they said in this workshop that kids could do this!" Linda thought.

Refusing to give in to the students' lament that the work was too hard, Linda decided that she needed to go about this differently. She resolved to spend each Monday introducing a specific strategy, presenting a problem to employ that strategy for students to solve together, and discuss their thinking. This was followed by an independent "problem of the week" for students to solve. After introducing all of the strategies (surface learning) and following up with independent applications of those strategies for students (deep learning), students continued to work independently or in small groups to solve a variety of open-ended problems on their own using strategies of their choice (transfer learning). Later that year, a group of girls approached Linda asking why she had saved all of the easy problems for the end of the year. That's transfer!

It's important to note that within the context of a year, a unit, or even a single lesson, there can be evidence of all three types of learning, and

THE RELATIONSHIP BETWEEN SURFACE, DEEP, AND TRANSFER LEARNING IN MATHEMATICS

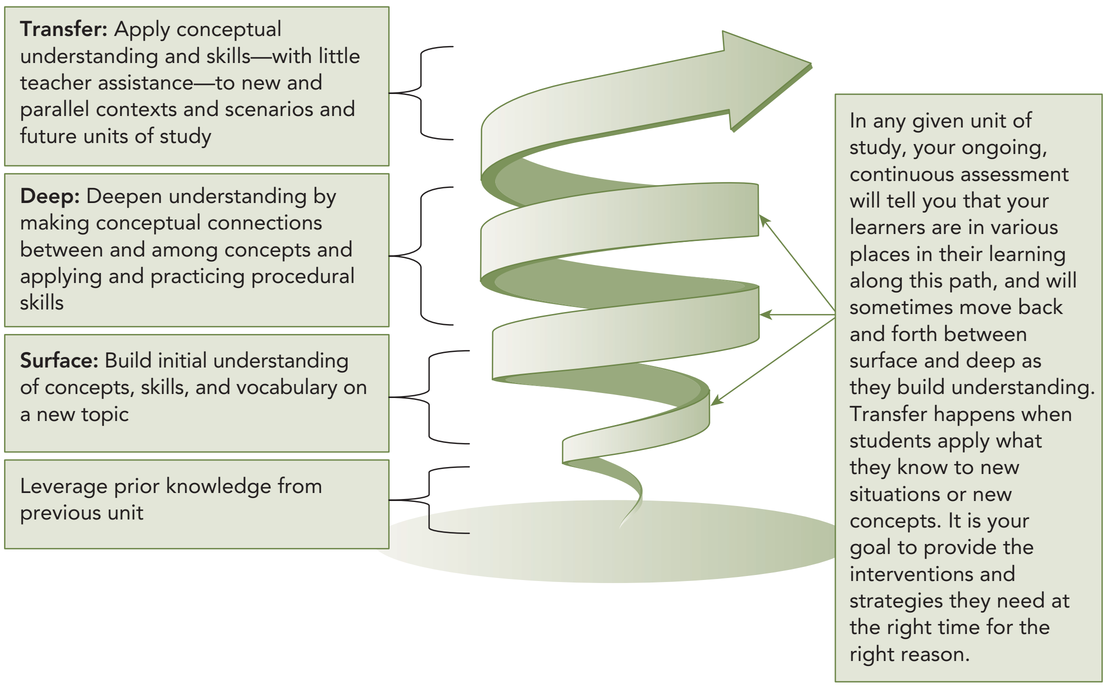

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Figure 1.4

that students can sometimes move among various kinds of learning depending on where they are as individual learners. Figure 1.4 describes the relationship between surface, deep, and transfer learning.

Surface, Deep, and Transfer Learning Working in Concert

As mentioned before, when it comes to the surface, deep, and transfer phases of learning, knowing *what* strategies to implement *when* for maximum