Constructive Proofs: Use Web Sketchpad to Prove SSS and More

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The CCSSM expects students to understand congruence and similarity in terms of transformations and superposition, an approach that's both clearer and more generally applicable than the traditional approach.

Another benefit of the transformational approach is that SAS, SSS, and ASA are no longer postulates: now students can actually prove them!

To take advantage of this opportunity, we need new approaches to transformations and proofs. Proof is been a difficult area for geometry students to master, and almost all of us who teach secondary math today had minimal exposure to transformations during our own high school days.

Constructionism

Richard Feynman said *What I cannot create, I do not understand.* By using Web Sketchpad (WSP) to create transformations and develop congruence proofs , students can develop both a sense of ownership and a deeper understanding of important mathematical concepts such as transformation, congruence, and proof.

Dancing to Understand Transformations

The transformation lessons make use of dance by having students create dances based on geometric transformations and by encouraging them to think of the independent and dependent variables of a WSP "geometric function" as being engaged in a dance as they move about on the computer screen. The concrete aspect of students' physical dance experiences are intentionally "faded" by creating virtual dances using WSP.

SSS Proof as a Construction Problem

The SSS lesson provides students with WSP transformation tools that they use to superpose the image of one triangle on another triangle with all three sides equal in length to the first. The steps of a successful construction correspond to the steps required to prove the SSS Theorem.

Concreteness Fading*

Math-education researchers have shown that students' understanding of abstract concepts is enhanced by connecting those concepts to students' concrete experiences through a process that gradually fades the concrete aspects of the original experience while emphasizing the abstract (usually symbolic) form. By using "concreteness fading" we can help students make important mathematical connections and strengthen their understanding of the target concept.

Presentation web page: https://bit.ly/2uCpwkK

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Thanks to Zal Usiskin for his pioneering work promoting the transformation approach. Thanks also to Jim King and PCMI for creating an excellent online course for teachers.

We are particularly grateful to McGraw-Hill Education for making WSP available for non-commercial use.

* "The benefits of concreteness fading may be explained by the fact that it starts with a well-understood concrete format, and explicitly links and fades it to the abstract symbols. Concrete materials are advantageous initially because they allow the math concept to be grounded in easily understood, real-world scenarios."
—Fyfe, E. R., McNeil, N. M., & Borjas, S. (2015). Benefits of "concreteness fading" for children's mathematics understanding. *Learning and Instruction,* 35, 104-120.

Constructive Proofs: Workshop Agenda

Workshop Description

Experience proving triangle congruence using a transformation approach. Using Web Sketchpad (WSP) technology, you'll construct transformations that superpose one triangle on another. With WSP's ability to capture your construction steps, the proof of the SSS Theorem is "transformed"! Bring a laptop or tablet to try free, classroom-ready lessons.

Technology

The website for this workshop is here: geometricfunctions.org/links/nctm2019/sss . You can use any wifi-enabled laptop or tablet.

We'll use Web Sketchpad (WSP) as our dynamic geometry platform. Each WSP activity provides only a small number of tools, simplifying the user interface and encouraging productive exploration. When you tap to activate a tool, the entire construction appears, ready for you to place its glowing objects where you want them, or to match them to existing objects in the sketch.

Intro: the Problem with Congruence

The traditional approach to congruence is fraught with difficulties, leaving triangle congruence properties based on a postulate. A transformation approach overcomes those difficulties, opening a path for students to confront and solve the challenge of proving SSS as a theorem.

Transformation/Function Dances

Students need to "understand congruence … using physical models." [CCSS 8.G] We'll do a reflection dance, while using the language of functions. [CCSS G-CO.2]

Reflection Fixed Points: Patty Paper & WSP

The fixed points of the reflect function are critical to understanding this function and using it in proofs. We'll use patty paper to develop the connection between reflection fixed points, perpendicular bisectors, and segment congruence. [CCSS G-CO.12]

Then we'll move from the concrete physical model (patty paper) to dynamic geometry to informally develop connections between reflection fixed points, the perpendicular bisector theorem, and Euclid's Proposition 1. [CCSS G-CO.9]

Investigate Triangle Congruence

We'll use the Mirror, Mirror WSP activity to build fluency in transforming one congruent triangle to another.

Do the Side-Side-Side Dance (15 min.)

This experimental activity takes us back to the physical world of dance; we will try it out if time permits.

Prove the Segment Congruence Theorem

To take the process of concreteness fading to a new level of abstraction, we undertake a rigorous proof of the congruence of two segments of equal length.

Prove the Side-Side-Side Theorem

In this concluding activity, we employ the same techniques we just applied to proving segment congruence. The result is a reasonably rigorous proof of the Side-Side-Side Theorem. [CCSS G-CO.8]

Additional sessions using WSP:

Connecting Geometric Transformations and Functions Using Technology
Workshop 368, Fri. 9:45, Hilton Sapphire CD
Come construct, dance, and play Web Sketchpad games that support students in making connections between geometric transformations and functions. Experience embodied learning activities as you choreograph and perform transformation dances that feature independent and dependent variables. Bring a laptop or tablet.

Not So Complex: A Geometric Approach to Complex Numbers from Addition through Euler's Formula
Session 453, Fri. 1:30, Hilton Aqua 311
Take your understanding of complex numbers to a new level using the dynamic visualization capabilities of Web Sketchpad. We'll gain mathematical insights into complex number arithmetic, de Moivre's theorem, and Euler's formula through applying a geometric transformations approach to these topics. Free webbased materials will be provided.