

Stop Teaching *Strategies*,

Start Teaching *Sense*

WITH

**CHRISTINA TONDEVOLD**





# Christina Tondevold



Build Math Minds  
[@buildmathminds](https://twitter.com/buildmathminds)

#NCTMSD19

#mathisNOTaWorksheet





## *During our time*

- Strategies for Addition & Subtraction
- Why we shouldn't be teaching strategies
- How to help kids develop them



Grade	Standards
K	<ul style="list-style-type: none"> <li>•Solve +/ - word problems within 10 using objects or drawings.</li> <li>•Fluently +/ - within 5.</li> </ul>
1	<ul style="list-style-type: none"> <li>•Solve +/ - word problems within 20 using objects, drawings, or equations.</li> <li>•Fluently +/ - within 10, have strategies for problems within 20.</li> <li>•Add/Subtract within 100 (2-digit/1-digit and 2-digit/multiple of 10) using objects, drawings, and strategies.</li> </ul>
2	<ul style="list-style-type: none"> <li>•Solve +/ - word problems within 100 using drawings or equations.</li> <li>•Fluently +/ - within 20, know from memory within 18.</li> <li>•Fluently +/ - within 100 using strategies.</li> <li>•Add/Subtract within 1000 using objects, drawings, and strategies.</li> </ul>
3	<ul style="list-style-type: none"> <li>•Fluently add/subtract within 1000 using strategies.</li> </ul>
4	<ul style="list-style-type: none"> <li>•Fluently add/subtract multi-digit whole numbers using the standard algorithm.</li> </ul>

CCSS



# Mental Math

57-19



$$57 - 19$$

$$57 - 20 = 37$$

$$+ 1$$


---


$$38$$

$$\begin{array}{r} 4 \\ 57 \\ - 19 \\ \hline 38 \end{array}$$

$$19 \xrightarrow{+1} 20 \xrightarrow{+37} 57$$

$$57 - 19$$

$$58 - 20 = 38$$

$$57 \xrightarrow{-7} 50 \xrightarrow{-10} 40 \xrightarrow{-2} 38$$

$$\begin{array}{r} 57 \\ - 19 \\ \hline 40 \\ - 2 \\ \hline 38 \end{array}$$



# Mental Math

66-48

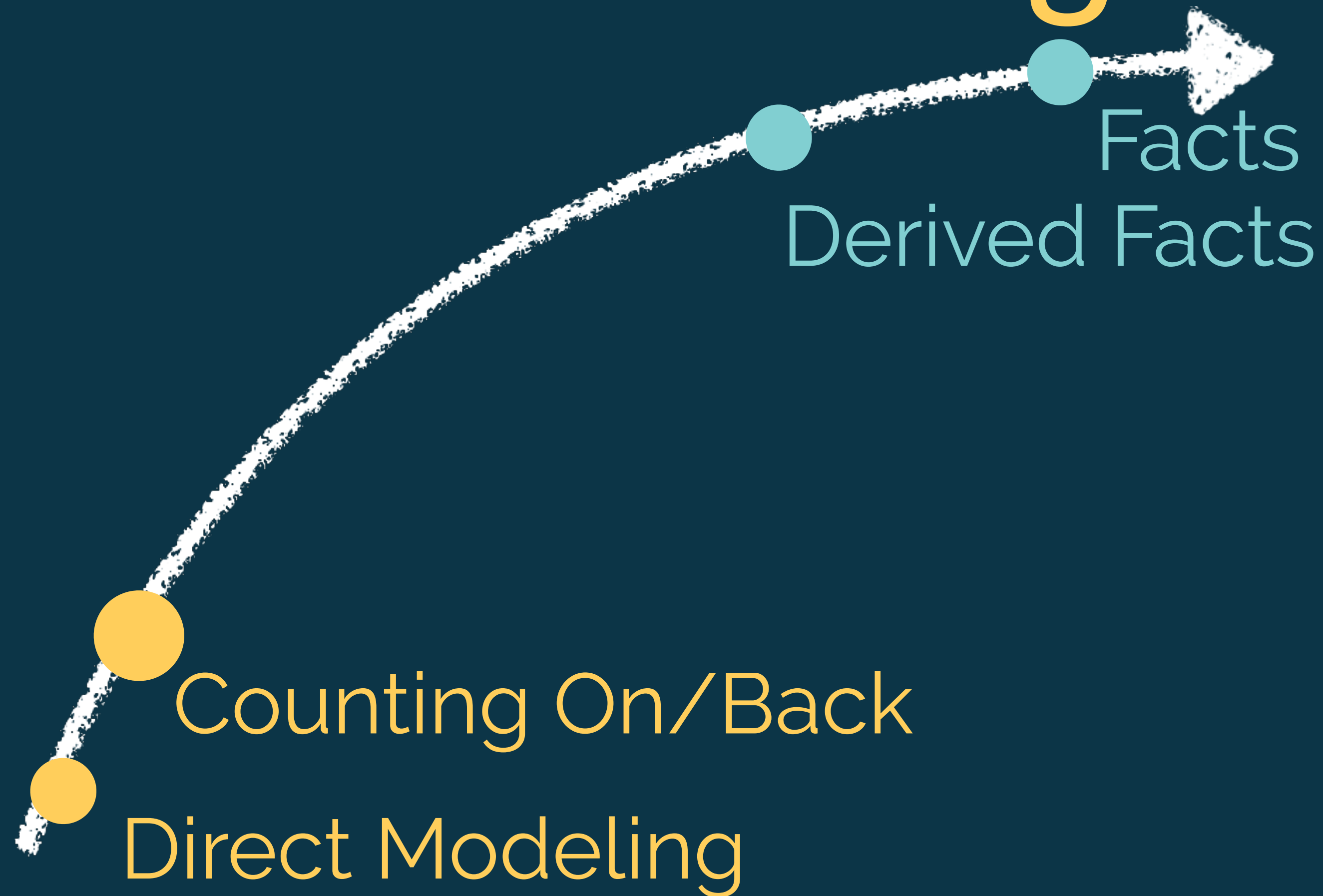


# Mental Math

16-8



# CGI Progression



**Carpenter, et al**





*3 parts to fluency*

- **Accuracy**
- **Efficiency**
- **Flexibility**

**Susan Jo Russell, 2000**



# NOT to How ~~do we~~ build strategies?

Tuesday 12:56 PM

Nothing makes u feel dumb like sitting at work googling how to do ur child's homework. 🙄😡

Solve the number sentences. Use the key to color. Once the box is colored, you do not need to color it again.

a. $5 + 2 = \underline{\quad}$	b. $7 + 2 = \underline{\quad}$	c. $2 + 3 = \underline{\quad}$
d. $3 + 3 = \underline{\quad}$	e. $7 = 1 + \underline{\quad}$	f. $2 = 1 + \underline{\quad}$
g. $\underline{\quad} = 4 + 4$	h. $8 + 2 = \underline{\quad}$	i. $3 + 4 = \underline{\quad}$
j. $\underline{\quad} = 5 + 4$	k. $10 = 1 + \underline{\quad}$	l. $10 = 5 + \underline{\quad}$

Color doubles red.

Color +1 blue.

Color +2 green.

Color doubles +1 brown.

+Addition Strategies+  $235 + 316 =$

Place Value / Decomposing the number

$$235 + 316 = 551$$
$$\begin{array}{r} 200 + 300 = 500 \\ 30 + 10 = 40 \\ 5 + 6 = 11 \\ \hline 551 \end{array}$$

\* Number Line

$$235 + 316 =$$

235 → +300 → 535 → +10 → 545 → +6 → 551

\* Vertical

$$\begin{array}{r} 235 \\ + 316 \\ \hline 551 \end{array}$$
$$\begin{array}{r} 4 \text{ } 11 \\ 551 \\ - 316 \\ \hline 235 \end{array}$$









Number Sense is the ROOT issue



# What is Number Sense?



# Number Sense is . . .

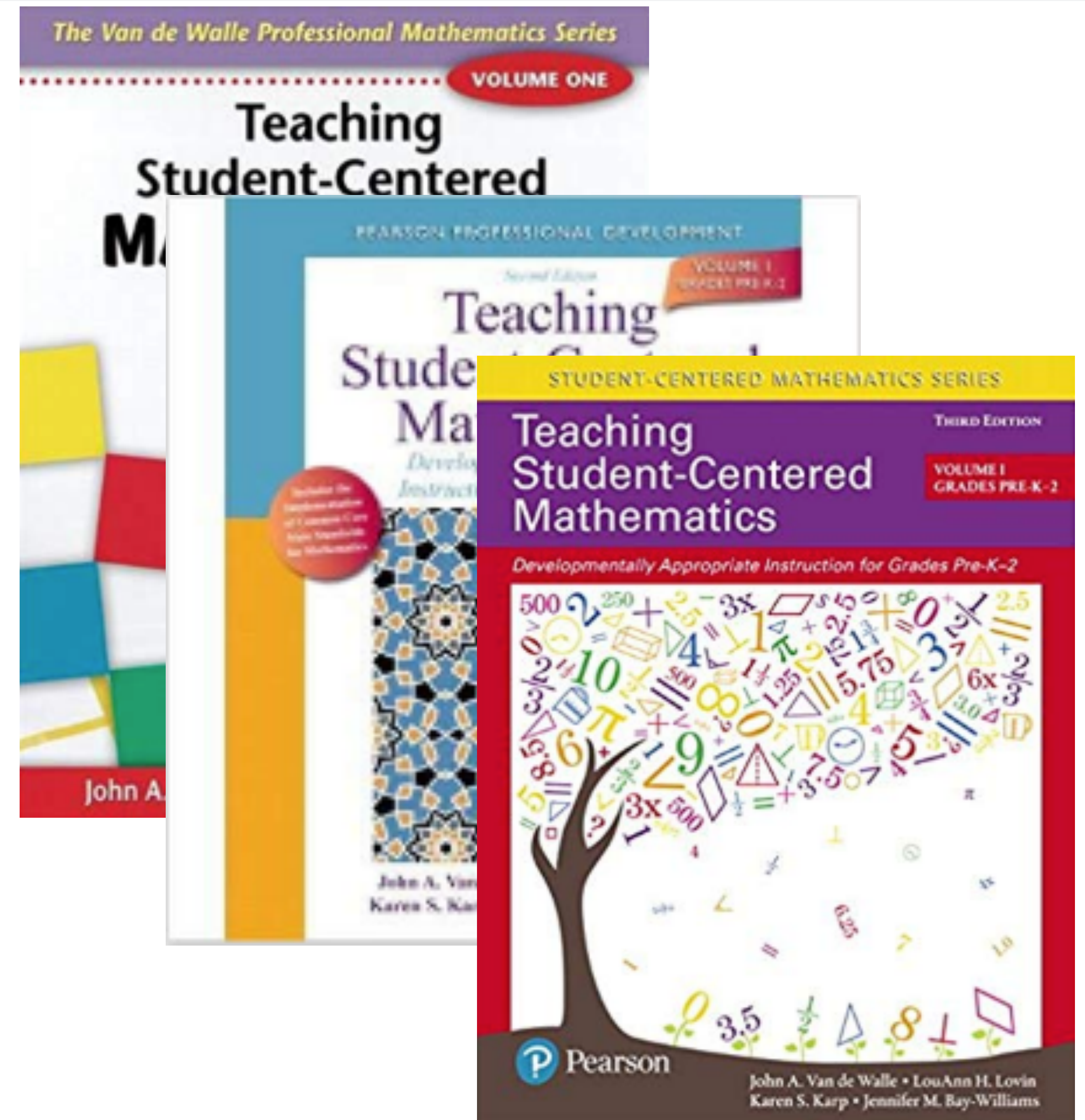
“...**good intuition about numbers and their relationships.** It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms.”

Howden, 1989



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Howden, 1989





$$9 + 7$$



01

# *Spatial Relationship*



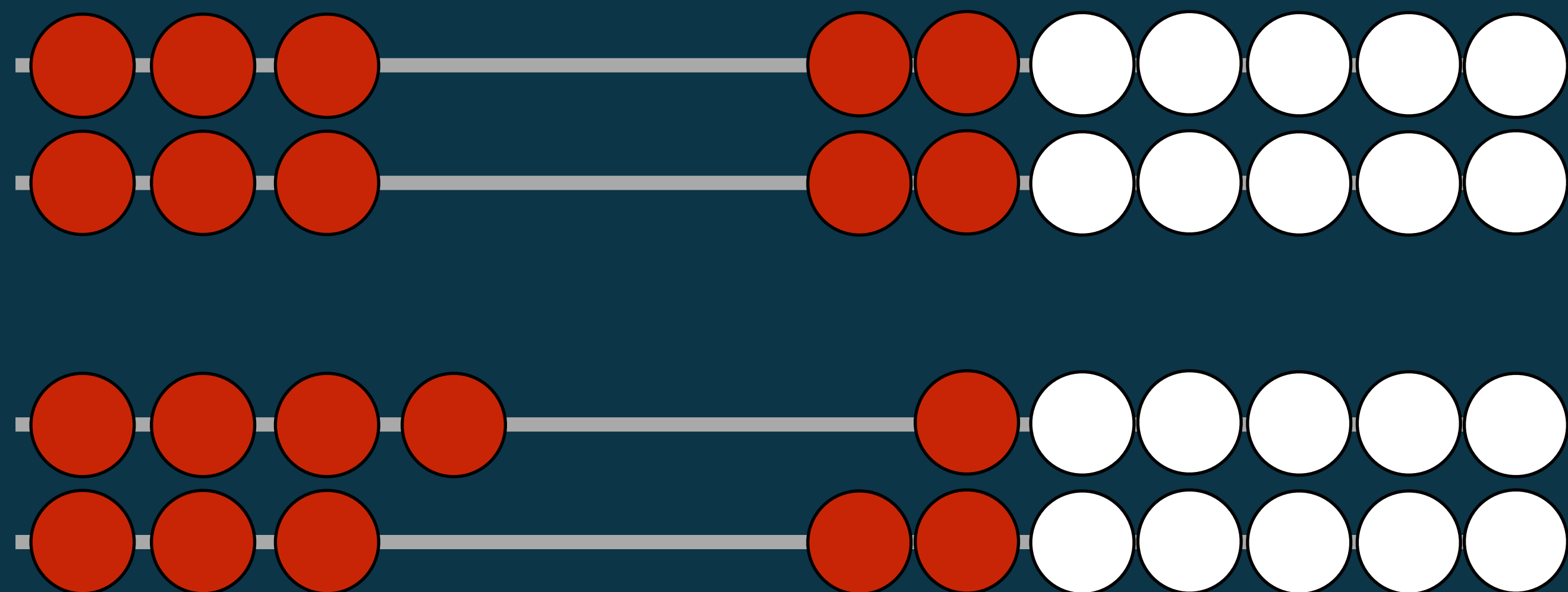
**Having a visual to go with a numeral to help  
you see relationships**

Van de Walle, et al,  
2013



02

# One Two More & Less



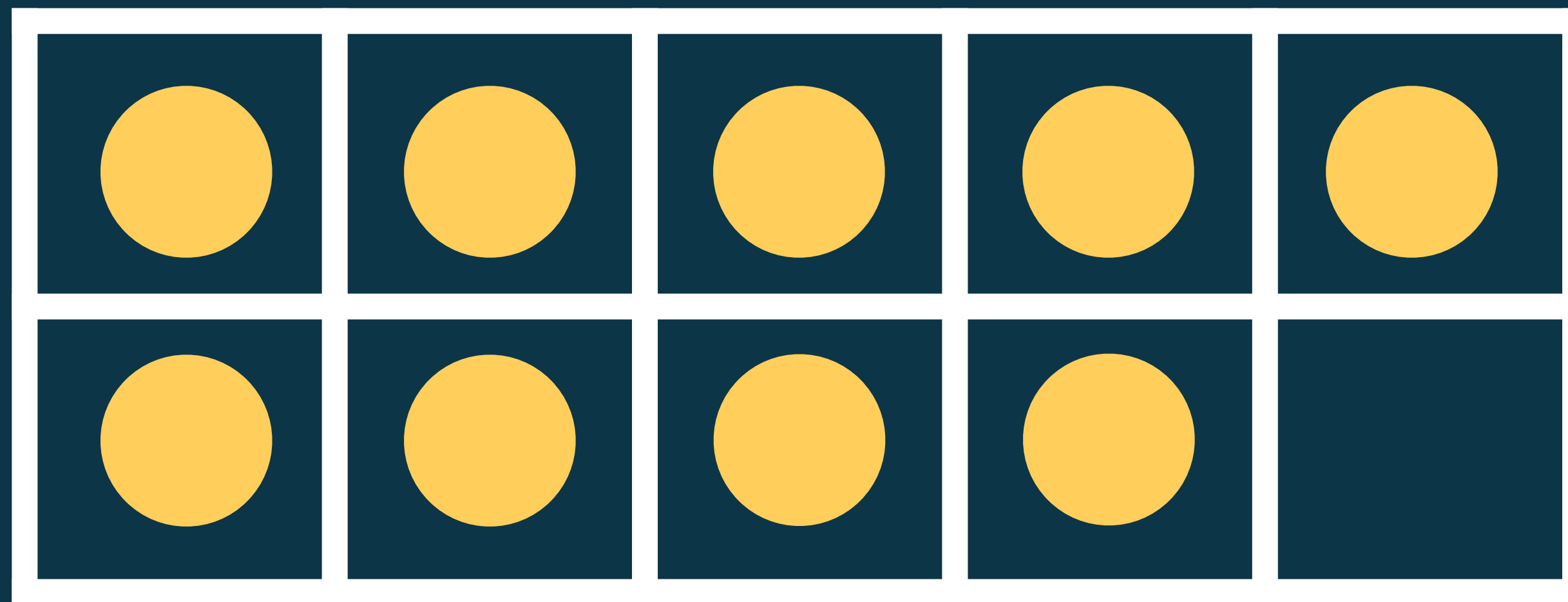
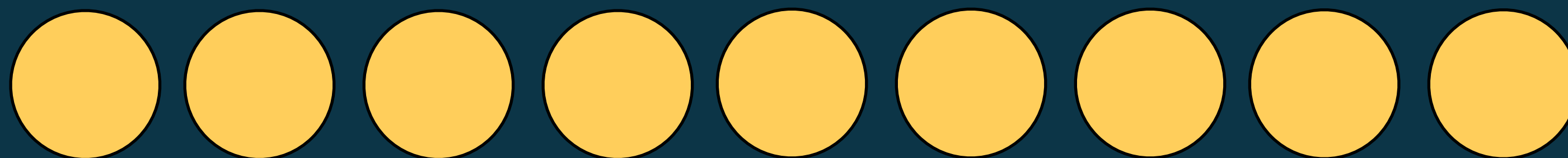
Instantly knowing the amount that is One  
or Two More & Less

Van de Walle, et al,  
2013



03

# Benchmarks of 5 & 10



Knowing how a number relates to  
5 & 10

Van de Walle, et al,  
2013



04

# Part-Part-Whole

7

$$0 + 7$$

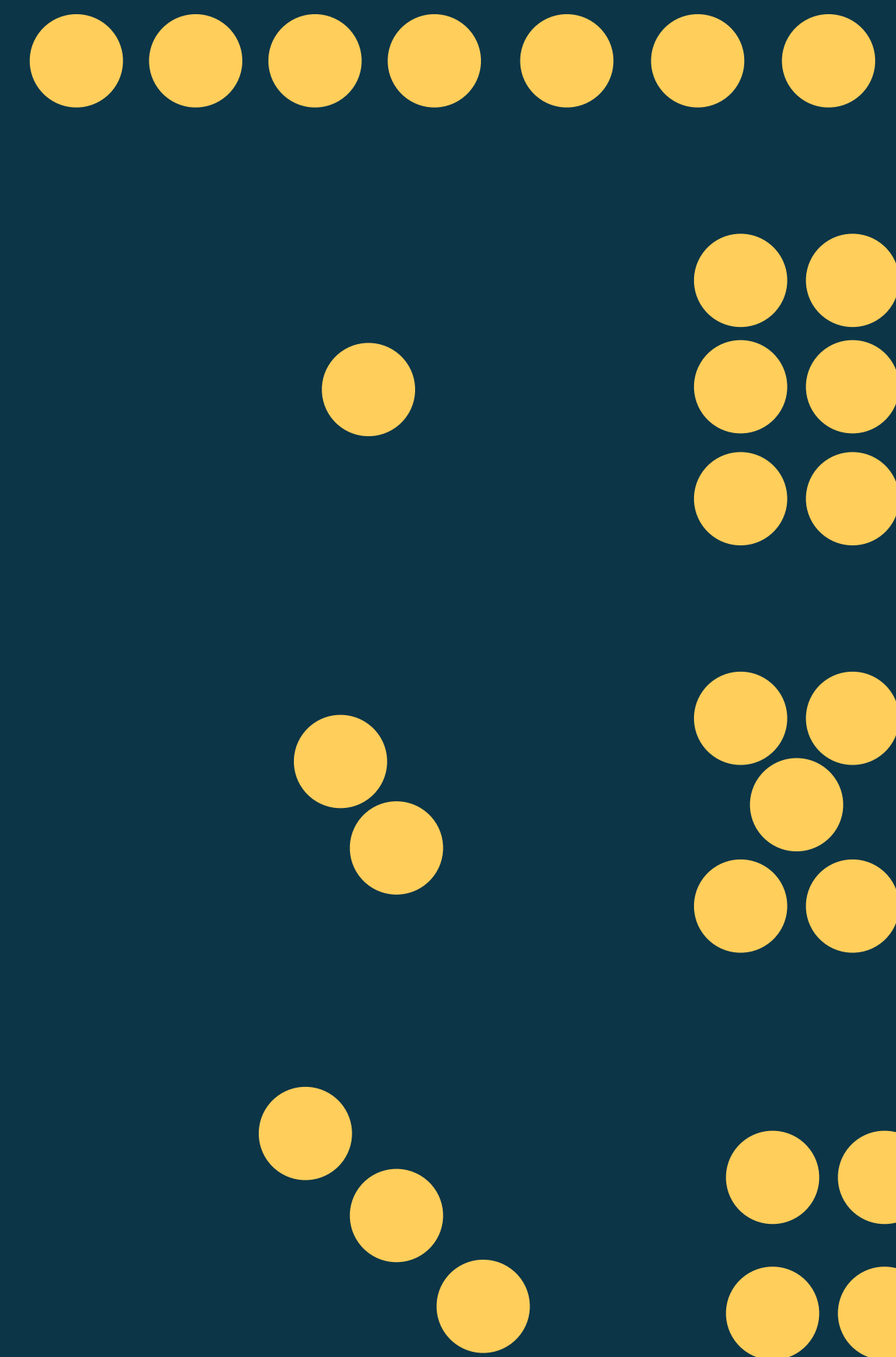
$$1 + 6$$

$$2 + 5$$

$$3 + 4$$

...

Understanding how a Whole can be broken into Parts



Van de Walle, et al,  
2013



Benchmark  
of 5 & 10

How many  
more to  
Make 10?

What is  
 $10 + 6$ ?

Benchmark  
of 5 & 10

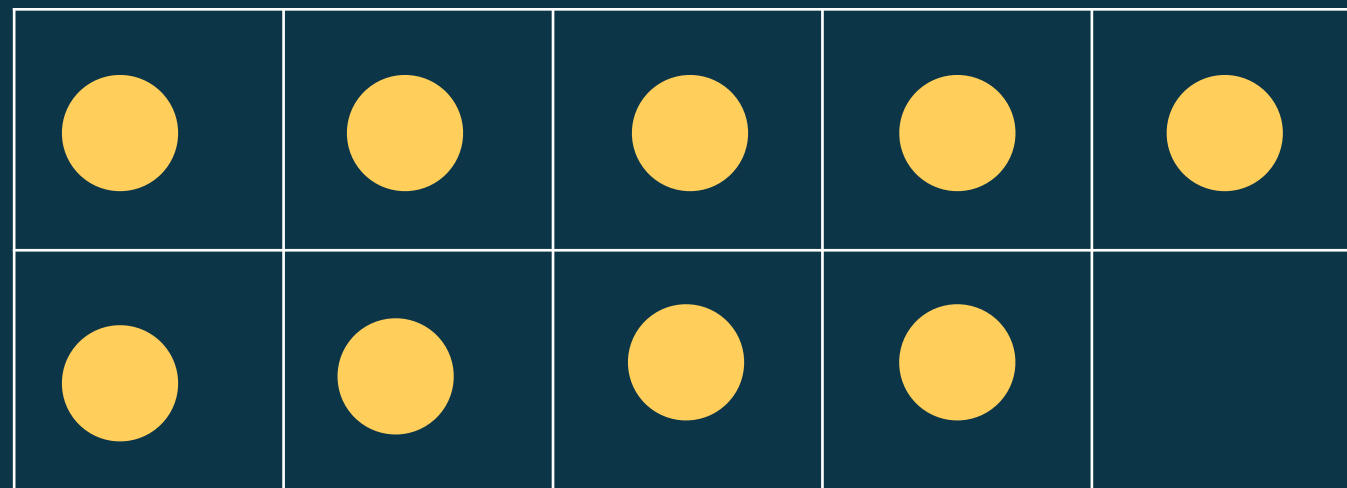
$$9 + 7$$

How  
should I  
chunk 7?

Part-Part-  
Whole

If I chunk 1  
off, what's  
left?

One/Two  
More & Less



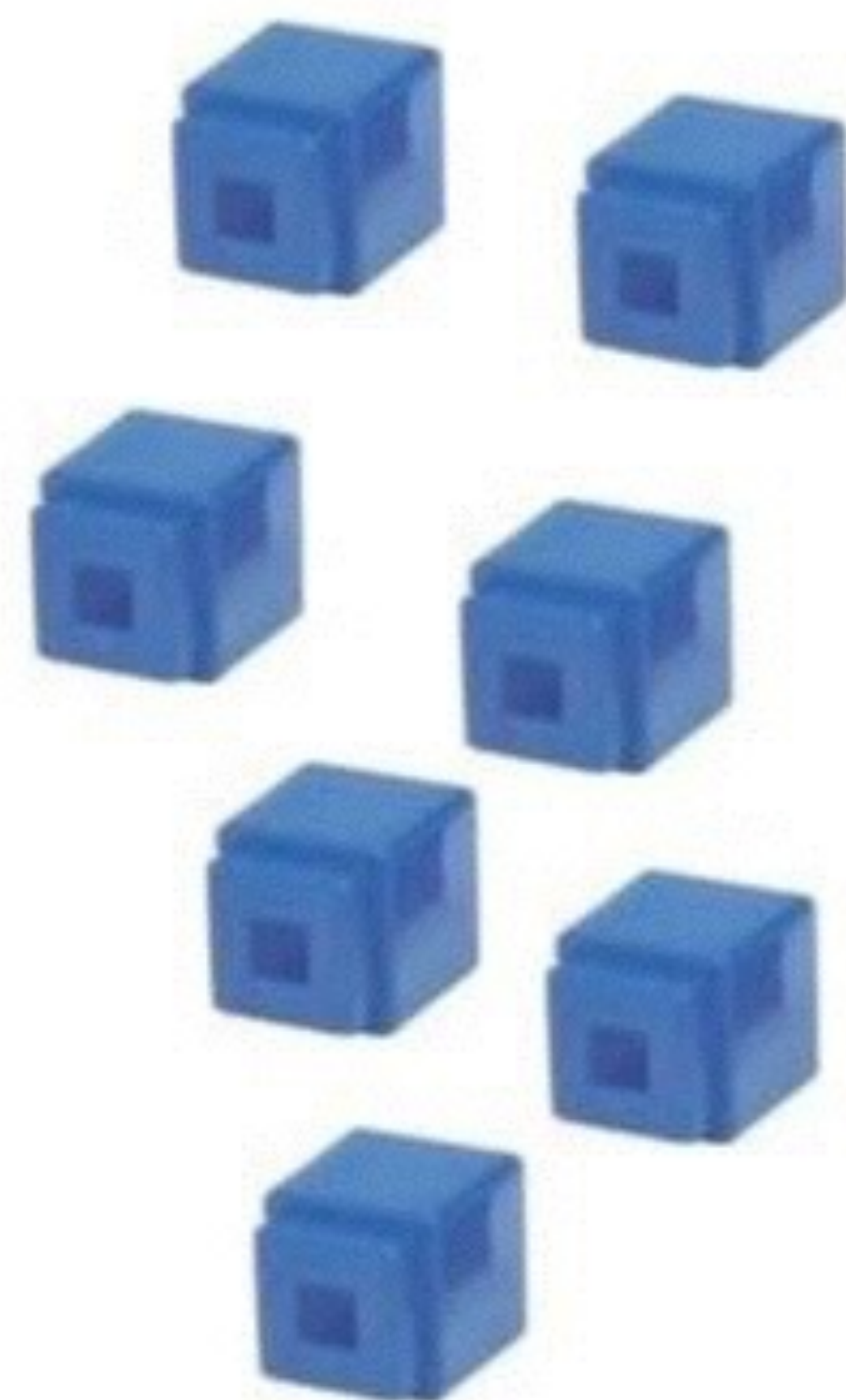
Spatial  
Relationships









various








				
				



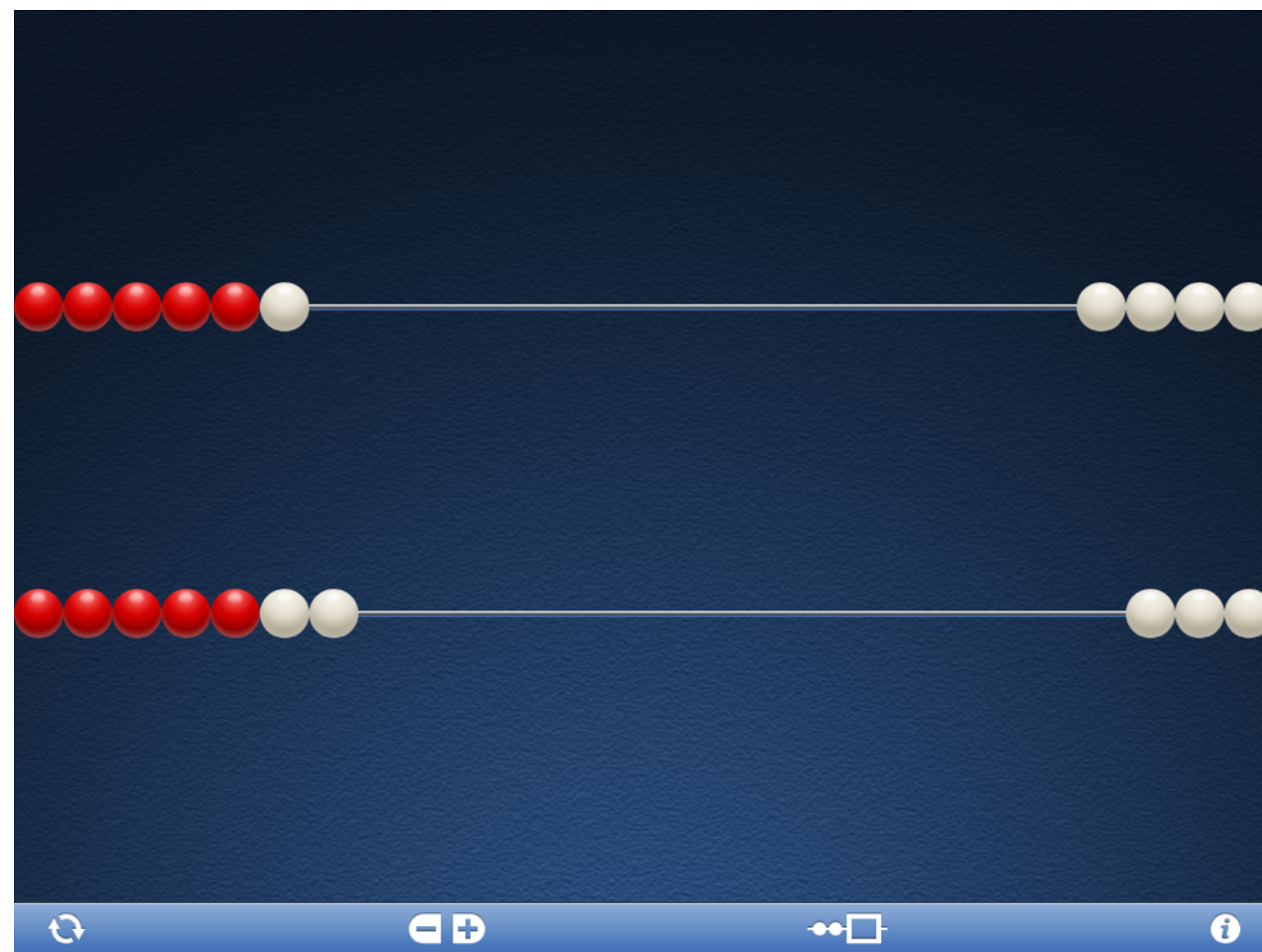










				
				








				
				





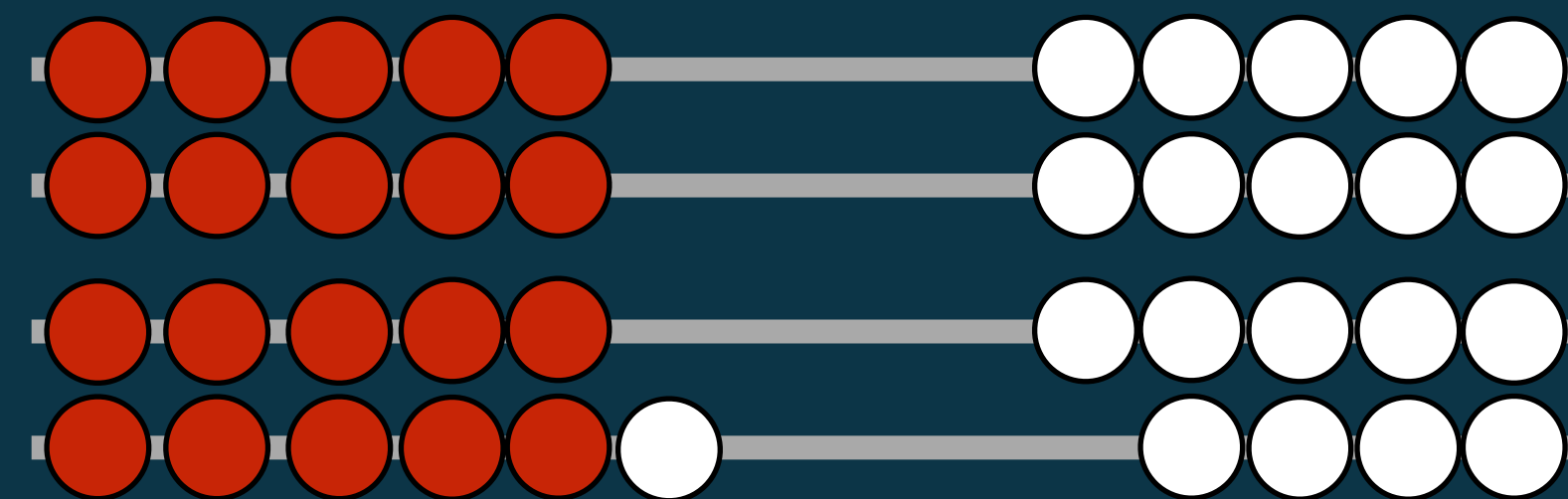
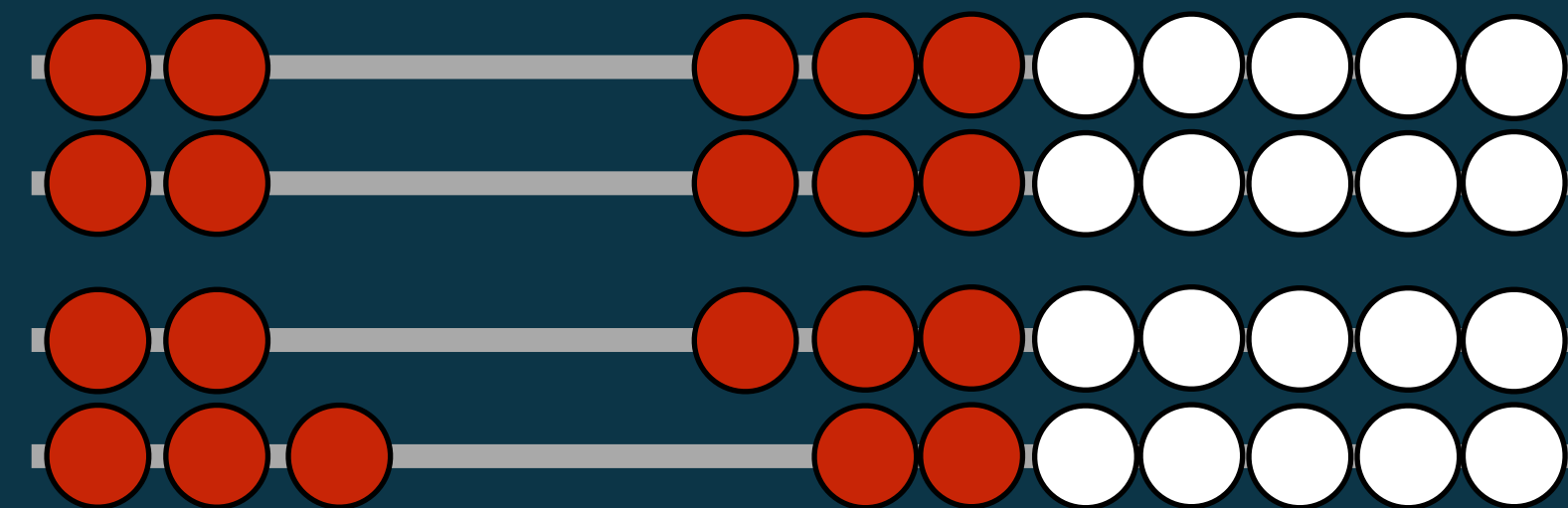
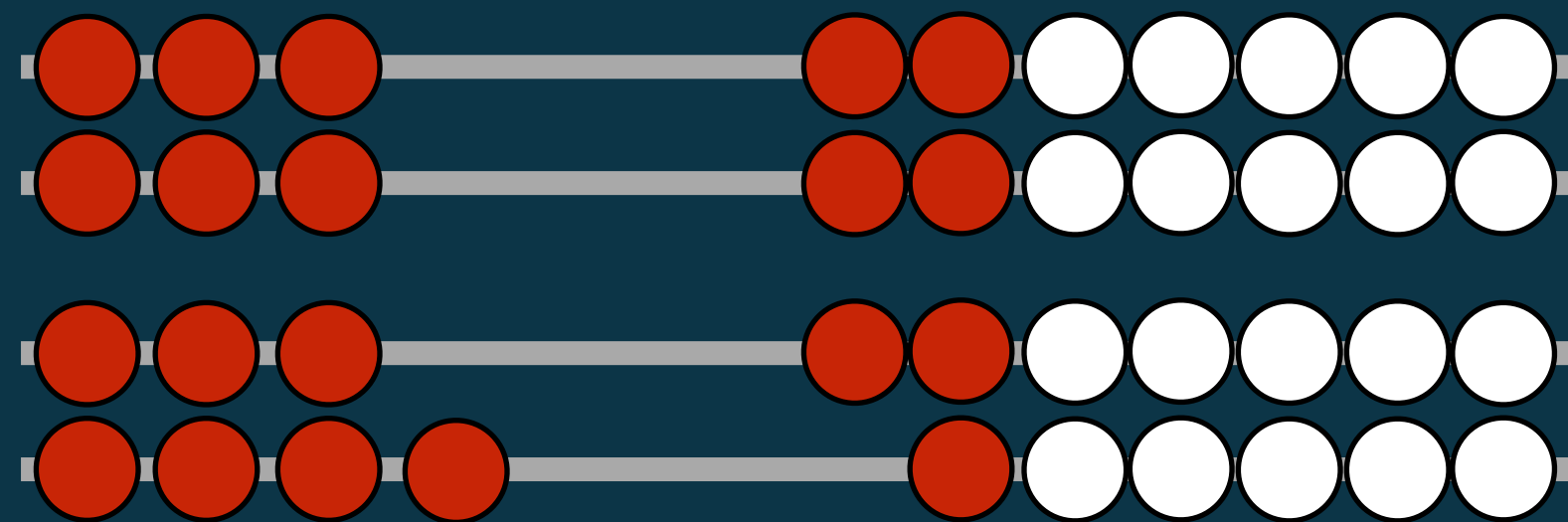




# Number Talks/Number Strings





# Number Sense is . . .

“...good intuition about numbers and their relationships. It **develops gradually** as a result of **exploring numbers, visualizing them** in a variety of contexts, and **relating them** in ways that are not limited by traditional algorithms.”

Howden, 1989



**Number Sense**

**Can't Be** *Taught, It's Caught*



# STRATEGIES

**Can't Be** *Taught,* **They're** *Caught*



$$9 + 7$$

$$9 + 6$$

$$9 + 8$$

$$19 + 7$$

$$39 + 7$$

$$59 + 17$$

$$299 + 17$$

$$3998 + 176$$

$$3.99 + 0.17$$

$$3\frac{1}{2} + 5\frac{3}{4}$$



## Why Structure Matters



$$9 + 7$$

$$19 + 25$$

$$299 + 346$$



$$\begin{array}{r}
 82 \\
 3 \overline{) 246} \\
 \underline{6} \\
 21 \\
 \underline{21} \\
 0
 \end{array}$$

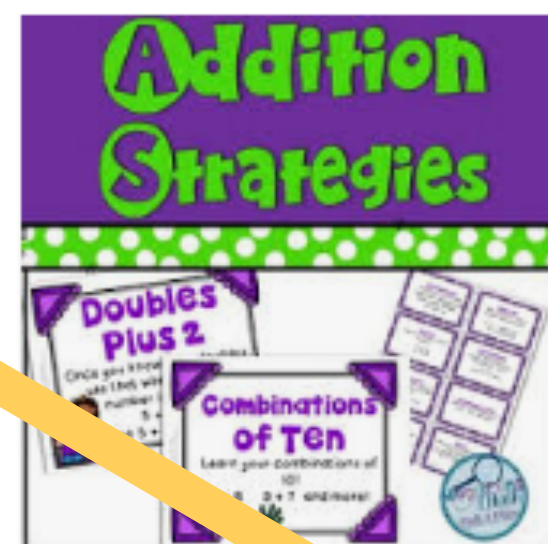
$$\begin{array}{r}
 60 \\
 4 \overline{) 264} \\
 \underline{4} \\
 260 \\
 \underline{40} \\
 220 \\
 \underline{20} \\
 200 \\
 \underline{20} \\
 200
 \end{array}$$

$$\begin{array}{r}
 5 \\
 5 \overline{) 25} \\
 \underline{5} \\
 0 \\
 5 \overline{) 25} \\
 \underline{5} \\
 0 \\
 5 \overline{) 25} \\
 \underline{5} \\
 0
 \end{array}$$

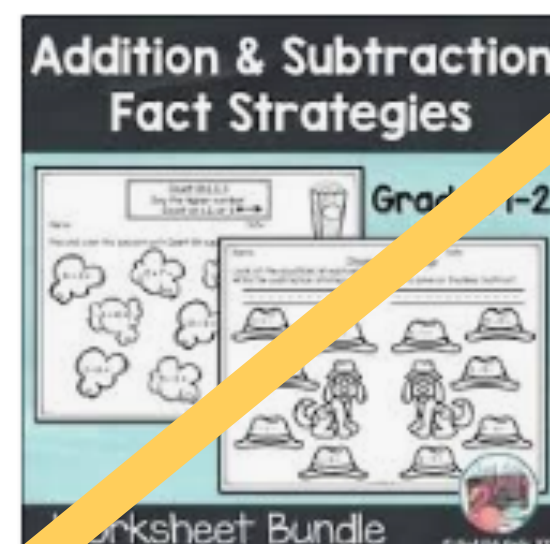




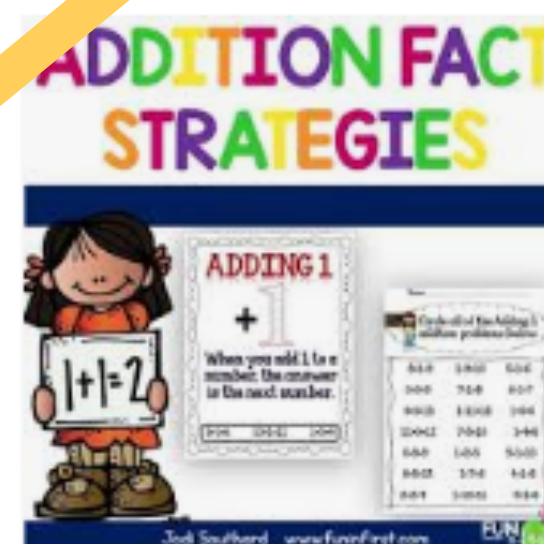
Addition and Subtraction Fact Strat...  
bridges1.mathlearningcenter.org



Addition Fact Strategies Post...  
teacherspayteachers.com



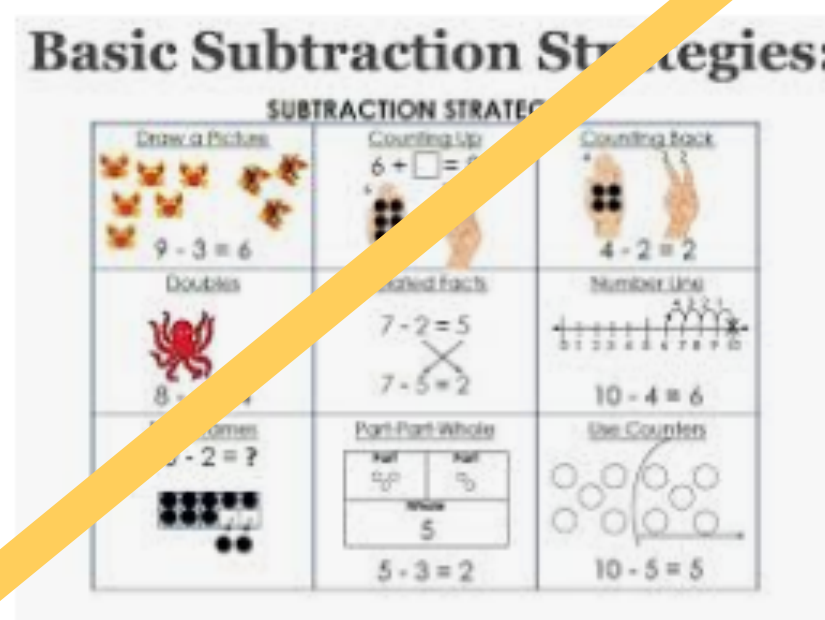
Subtraction Fact Strategies ...  
teacherspayteachers.com



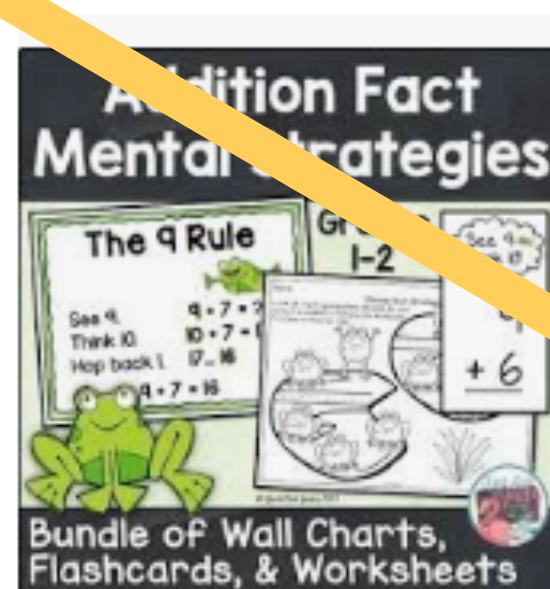
Addition Fact Strategies an...  
teacherspayteachers.com

GAME	ADDITION MENTAL MATH STRATEGY
BOOM	PLUS ZERO
CRUNCH	COUNT UP +4
CARDOH	COUNT UP +3
ZOOM	COUNT UP +8
KAPOW	MIXED STRATEGIES
SHAZAM	TEN ADDITION FLIP
CRASH	DOUBLES
RANG	DOUBLES +1
SUPER SPEED MATCH	TENS MATHEMATICS
ZAP	PLUS TEN
POW	PLUS NINE

Math Fact Fluency Addition ...  
teacherspayteachers.com



Subtraction Strategies - Maxwell's ...  
teacherspayteachers.com



Fact Strategies Bundle of A...  
teacherspayteachers.com

Addition Fact Strategies		
Strategy	Strategy Description	Van de Walle
Counting On	Start with the first number and count on to the second number.	Vol. 1 pp. 100-101 Vol. 2 pp. 19-20
Make with 10	Start with one of the addends and count on to 10, then add the other addend.	Vol. 1 pp. 102-103 Vol. 2 pp. 21-22
Doubles	Adding two of the same number together.	Vol. 1 pp. 104-105 Vol. 2 pp. 23-24
Doubles + 1	Adding a double to one more.	Vol. 1 pp. 106-107 Vol. 2 pp. 25-26
Combinations of Ten	Adding two numbers to find a sum of 10.	Vol. 1 pp. 108-109 Vol. 2 pp. 27-28
Make Ten	Adding two numbers to find a sum of 10, then adding the remaining number.	Vol. 1 pp. 110-111 Vol. 2 pp. 29-30
Doubles + 2	Adding a double to two more.	Vol. 1 pp. 112-113 Vol. 2 pp. 31-32
Two Apart	Adding two numbers that are two apart.	Vol. 1 pp. 114-115 Vol. 2 pp. 33-34

Subtraction Fact Strategies		
Strategy	Strategy Description	Van de Walle
Think	Using the addition fact to solve the subtraction problem.	Vol. 1 pp. 116-117 Vol. 2 pp. 35-36
Fact Families	Using the relationship between addition and subtraction to solve the problem.	Vol. 1 pp. 118-119 Vol. 2 pp. 37-38
Build Up Through Ten	Adding to the first number to reach 10, then subtracting the second number.	Vol. 1 pp. 120-121 Vol. 2 pp. 39-40
Back From Through Ten	Subtracting from the first number to reach 10, then adding the second number.	Vol. 1 pp. 122-123 Vol. 2 pp. 41-42

Fact Strategies Wichita Pub...  
teacherspayteachers.com

Stop Teaching Strategies...  
Start Teaching “Sense”



# *Helping Kids CATCH Strategies*

**Focus on helping  
kids develop  
number relationships**





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#NCTMSD19