Making Sense of Students' Numerical Reasoning in Order to Guide Instruction

Marilyn Burns

Session 297
NCTM 2019 Annual Conference
Friday, April 5, 2019, 8:00 A.M. – 9 A.M.

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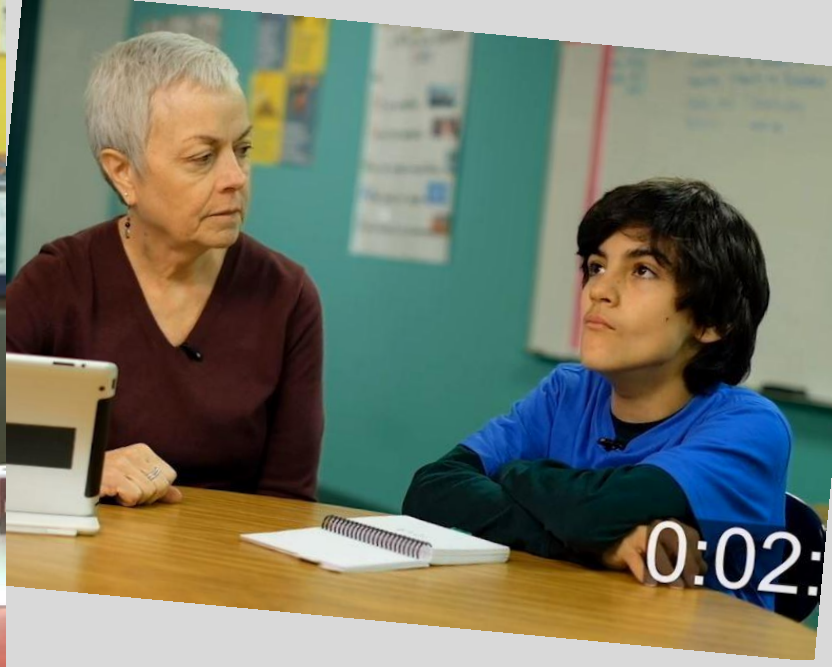
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Math Solutions celebrated its 35 birthday in February 2019.

Figure out in your head the year Math Solutions was founded?

Why Mental Math is Important

- Mental math is an important life skill.
- Learning to compute mentally is closely intertwined with developing number sense.
- Solving problems mentally pushes students to go beyond using algorithmic procedures and encourages them to reason numerically.
- Doing math mentally invites multiple strategies.

Part 1

$4 + 9$

3.1

Explanations from Students: 4 + 9

- Added 9 + 1 and then 10 + 3
- Added 4 + 6 and then 10 + 3
- Added 4 + 4, 8 + 2, and then 10 + 3
- Added 10 + 4, and reasoned that 9 + 4 is one less than 14
- Counted on from 9
- Counted on from 4
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation

Different Students . . . Different Answers

“So I know that if you add something to a 9, it’s going to be a teen number. Then minus 1 from that number and make it a teen, ‘cause 9 plus 1 equals 10, then you use the extra numbers beyond 10 and it makes, like, it made 13.”

"'Cause . . . because I took the 9 and split it into 1 and into 8, and then I put . . . and then I put 1 plus 4 equals 5 and then I put the 8 and the 5 plus 8 equals 13."

“Because 9 has a 4, I take the 5 out of the 9 and then 4 plus 4 is 8. And then I take away 2 from the 5 and then I put it with the 8, and that makes 10. And then from the 5 that leaves 3, so it’s 13.”

How Genesis reasoned

9 = 4 + 5	Decomposed 9 into 4 + 5
4 + 4 = 8	Used known fact
2 + 3 = 5	Decomposed 5 into 2 + 3
8 + 2 = 10	Used benchmark of 10
10 + 3 = 13	Added 10 + 3

Part 2

Genesis interviewed by Mallika Scott

A Sampling of an Entire Interview

Addition and Subtraction within 20A

$$12 - 6$$

"Because I took away 2 from the 6. And then I took away this 2 (pointing at the 2 in the 12) *from the 12. And then it left me 4, and 10 take away 4 is 6."*

Explanations: 12 – 6

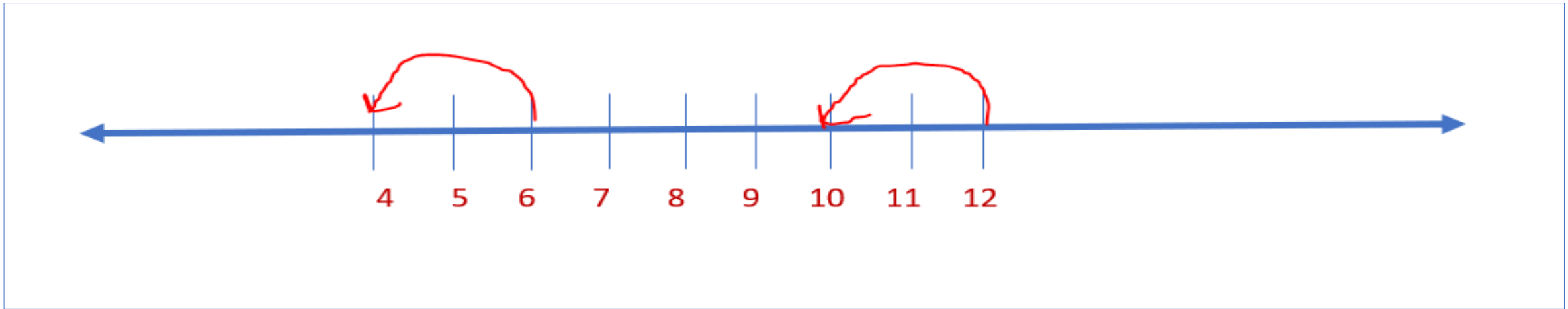
- Added 6 + 6
- Counted back from 12
- Subtracted 12 – 2 and then 10 – 4
- Subtracted 6 – 2, 12 – 2, and then 10 – 4
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation

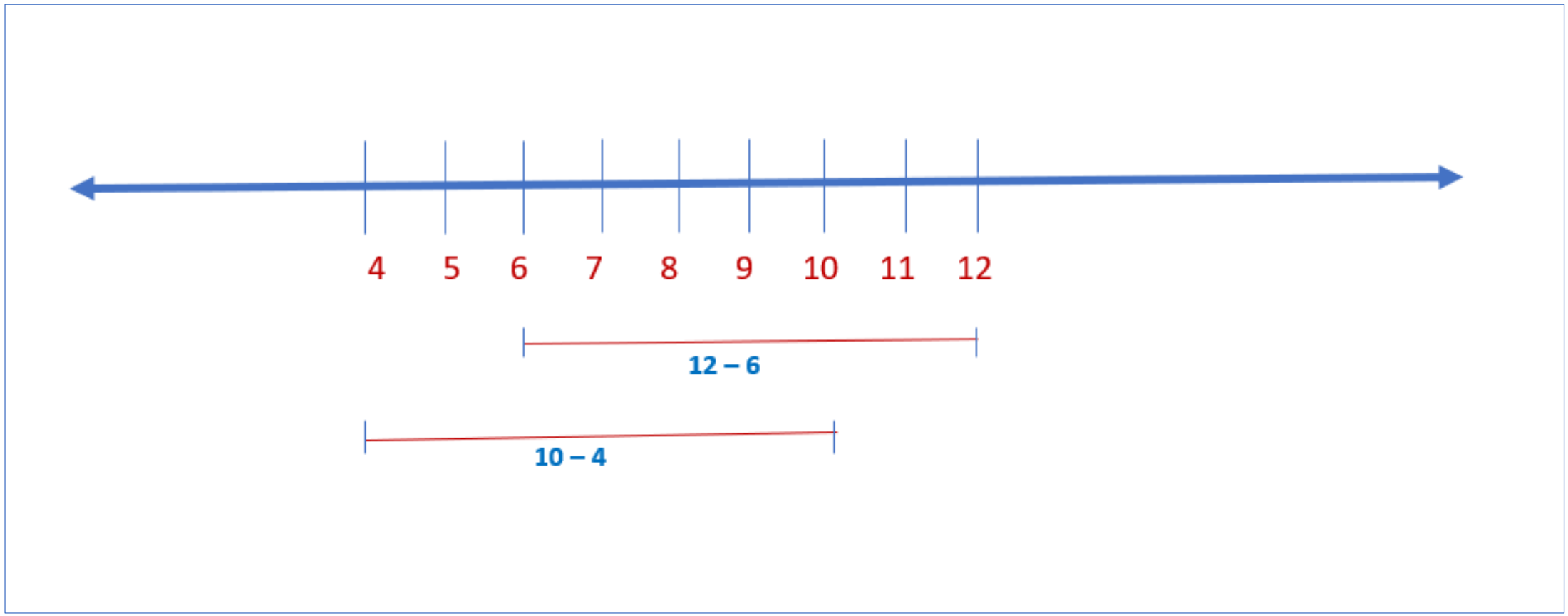
How Genesis reasoned

$6 - 2 = 4$

$12 - 2 = 10$

$10 - 4 = 6$




4
5
6
7
8
9
10
11
12
12 – 6
10 – 4

$$7 + 8$$

“Because I take away the 2 and the 3 from the 7 and 8, and then it’s 5 plus 5 equals 10, and 3 plus 2 equals 5, and then it’s 15.”

Explanations: 7 + 8

- Made 10 (7 + 3, 8 + 2, or 5 + 5) and then added 5
- Used doubles (7 + 7 + 1 or 8 + 8 –1)
- Counted on from 7 or 8
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation

11 – 9

Explanations: 11 – 9

- Counted on from 9 or added 9 + 2
- Explained that 9 is 2 away from 11 or counted back from 11 to 9
- Used 10 + 1 or 10 – 9 to solve 11 – 9
- Counted back 9 from 11
- Changed 11 – 9 to 10 – 8
- Subtracted 11 – 1 and then 10 – 1
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation

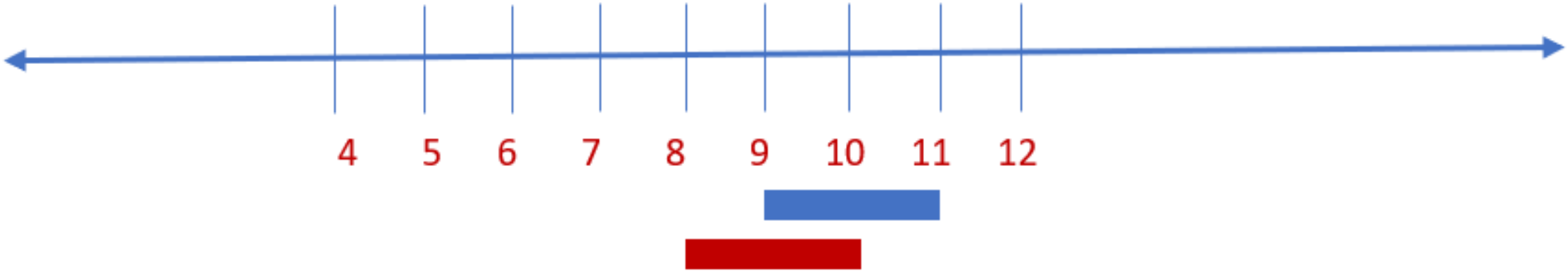
How Genesis reasoned

$9 - 1 = 8$

$11 - 1 = 10$

$10 - 8 = 2$

Why does this work?



I baked 20 oatmeal cookies.
There were 12 cookies left.

How many cookies were eaten?

Explanations: I baked 20 cookies. There were 12 left.

- Added (e.g.,12 + 3 and then 15 + 5)
- Subtracted without using standard algorithm (e.g., 20 – 10 and then 10 – 2)
- Counted on from 12
- Counted back from 20
- Used standard algorithm to subtract
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation

What did Genesis do?

$2 + 8 = 10$

$10 - 8 = 2$

So, $20 - 8 = 12$

Her go-to strategy is to use the benchmark of 10. She breaks 10 into 2 + 8 and, therefore, knows that 10 – 8 equals 2. Then she reasons that 20 – 8 equals 12.

Why does this work?

Strategies for Mentally Adding & Subtracting (numbers within 10, 20, 100, 1000)

1. Decomposes numbers within 10
2. Counts on or back
3. Uses known facts (e.g., doubles)
4. Uses benchmark numbers (e.g.,10, 25, multiples of 10 or 25)
5. Uses the inverse relationship between addition and subtraction
6. Decomposes numbers into their place value parts
7. Solves missing addend problems
8. Interprets and solves contextual problems that involve addition or subtraction

Part 3

Another Problem, Six Strategies

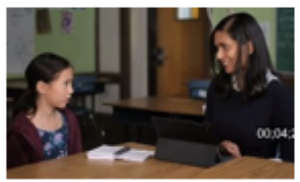
7 x 6

Explanations: 7 x 6

- Skip-counted
- Used a known fact (e.g., 6 x 6 = 36, so 7 x 6 = 42)
- Added (e.g., 6 + 6, then 12 + 6, etc.)
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation

7 x 3 = 21, 7 x 3 = 21, 21 +21 = 42

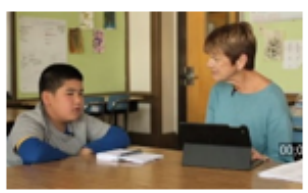
7 x 5 = 35, 35 + 7 = 42



Skip count by 6 or 7

6 x 5 = 30, 6 x 2 = 12, 30 + 12 = 42

7 x 7 = 49, 49 – 7 = 42



6 x 6 = 36, 36 + 6 = 42

“I’m now convinced that listening to students, one on one, provides access into how they reason in a way that’s unique and invaluable.”

--Marilyn Burns

Our teaching goal is not to *cover* the Standards . . .

but to *uncover* them.

Do The Math
Created by Marilyn Burns

Transforming Our School's Math Instruction with Marilyn Burns and *Do The Math*

Math educator Sara Liebert explains the positive impact Marilyn Burns and her instructional support have had on Title 1 students in San Francisco, California.

John Muir Elementary School in San Francisco, California, is a Title 1 elementary school. I work at John Muir as the Instructional Reform Facilitator, focusing on supporting teachers in math, and I teach a fifth-grade math class daily. This was my first year in the dual role; for the previous 10 years I was a classroom teacher for grades 3–5.

I had the opportunity to meet Marilyn Burns when she observed a guest teacher in my fifth-grade classroom during the spring of 2015. She reached out to me after she observed the lesson in my classroom and asked me, "What

centers, but I had never been able to keep up with the organizational component that they required. (For information about math menus, two resources from Marilyn were helpful. One is an article she wrote for **Educational Leadership**. The other is her more recent **blog post**. I have found that the math menu provides time for students to engage with concepts and skills through games and lesson extension activities. Also, it provides me with a way to differentiate instruction and better meet all of my students' needs.)

A Professional Breakthrough: Understanding "Why," "What," and "How

After 10 years of classroom teaching, I felt that this year I was finally teaching math successfully. The structure of *Do The Math* gave me much

Lisa Alley, a second-grade teacher with 28 years of classroom experience, discussed what she appreciated about the *Do The Math* lessons.

Justin Stoddard, a third-grade teacher with 11 years of classroom experience, also reflected on what he appreciated about *Do The Math*.

Kristin Matthews, a second-grade teacher with 9 years of classroom experience, commented on the features of *Do The Math* that she found particularly helpful.

Lisa Cowles, a second-grade teacher with 10 years of classroom experience, was skeptical at first, but then was surprised at her experience using the lessons.

Rashida Carter, a first-year teacher teaching fourth grade, especially liked the timeliness of these assessments.

Joe Mannarrino, a first-year teacher teaching fifth grade, had a different view of the scripted lessons.

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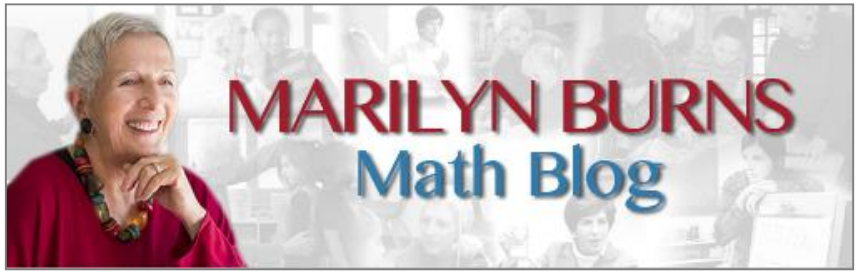
Math Solutions.
FOUNDED BY MARILYN BURNS



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Math Reasoning Inventory™

Coming 2020!

Listening to Learn
MATH REASONING INTERVIEWS