Developing Students' Ability to Create Arguments through Collective Argumentation

Presenter: Shande King (Nicholas)
Location: San Diego Convention Center, 30 DE
Friday, 5 April 2019
8:00 AM – 9:00 AM
National Council of Teachers of Mathematics
Annual Meeting and Exposition

THE UNIVERSITY OF
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KNOXVILLE
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Agenda

- Introduction
- Definition of arguments
  - Why is it important?
  - Current state
- Collective argumentation approach
- Task examples
  - Try an example!
- Resources
- Conclusions
- Comments and questions

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Introduction

- 9th year in education - Mathematics and French
- PhD student in mathematics and world language education
- Research interests: interpersonal discourse of content knowledge, discourse and conceptualization of mathematical proofs and arguments, and the development of learning communities

Definition of mathematical argument

What is argumentation?

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What is a mathematical argument?

- The act of convincing oneself and others of the truth of a mathematical statement based on agreed-upon principles in a specific community.
- Verbal or written
- Sophisticated or informal
- Explanation

What comprises a mathematical argument?

- A connected sequence of assertions for or against a mathematical claim, with the following characteristics:
  - Use statements accepted as true and available without further justification
  - Employs reasoning
  - Communicated with understandable expression
- In the context of the classroom community

Why is it important?

- Conceptual understanding
    - Justification is a deeper level of mathematical processing
- Mathematicians prove!
- Engaging – math is not static!
- Standards
    - NCTM (1989, 2000): Mathematical Practice #3
    - Common Core State Standards for Mathematics (2010)

What is the current state?

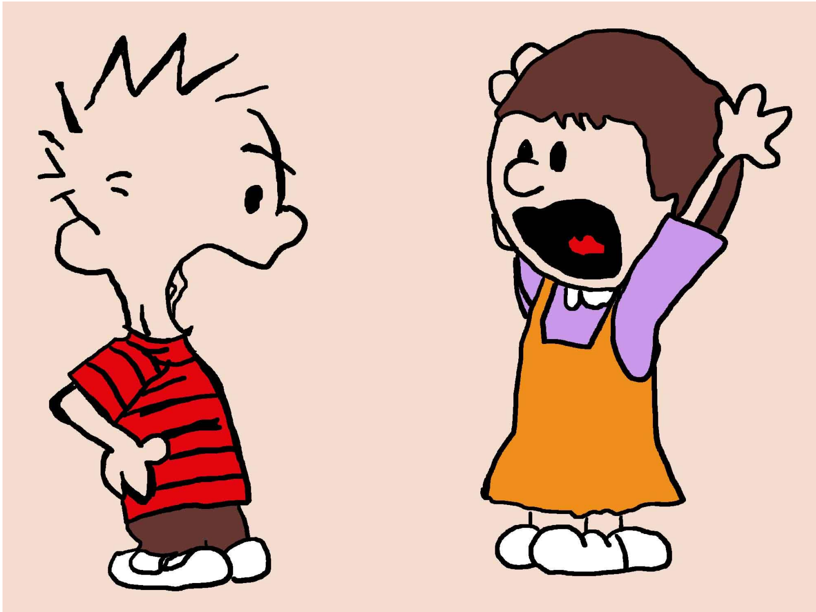
What is the current state of affairs?

- Arguments and proofs solely utilized in Geometry (if that)
- MP #3: “Create viable arguments and critique the reasoning of others.”

Collective Argumentation

What is collective argumentation?

- Communal Criteria
- Open Tasks
- Facilitation of Instruction
- Role of the teacher



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Communal Criteria

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Communal Criteria

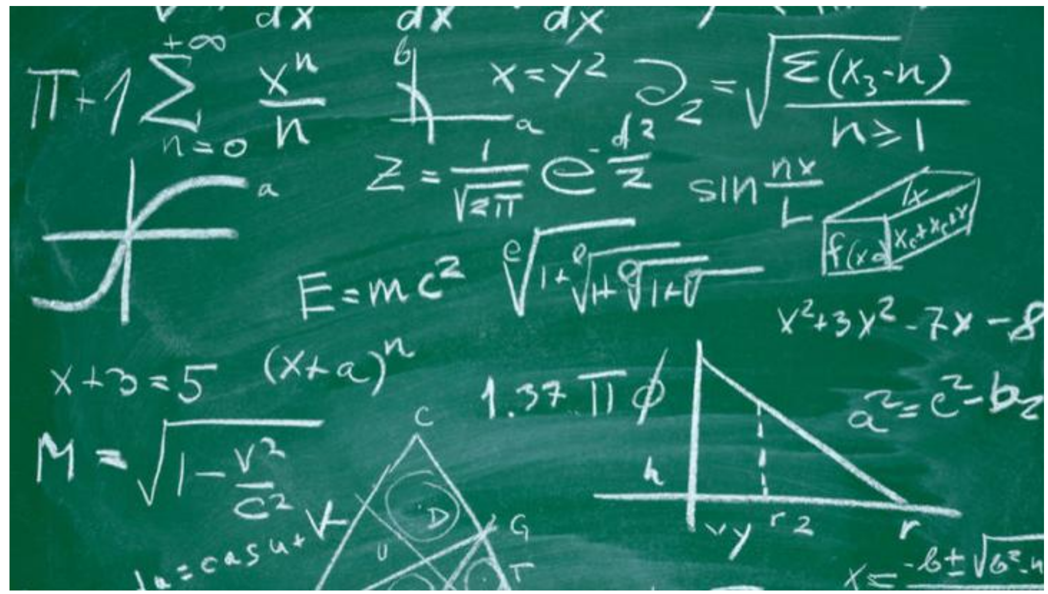
- Using facts and evidence
- Concise
- True for all cases
- Understandable

- *If necessary, go over terminology.

Open Tasks

Open Tasks

- More than one argument
- Support collaboration
- Require critical thinking and justification

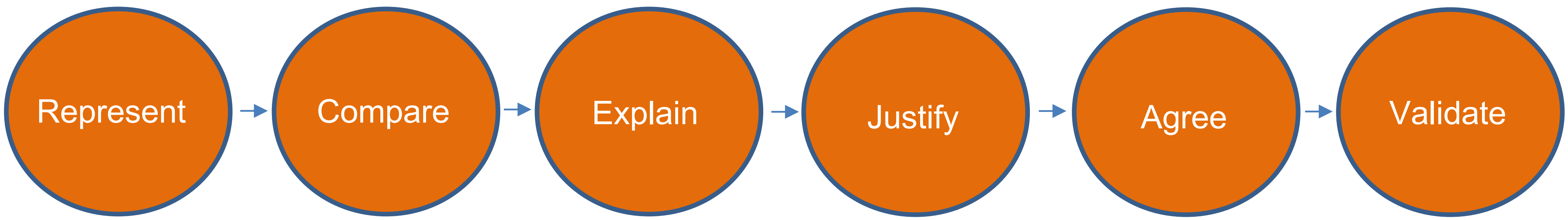


Facilitation of Instruction

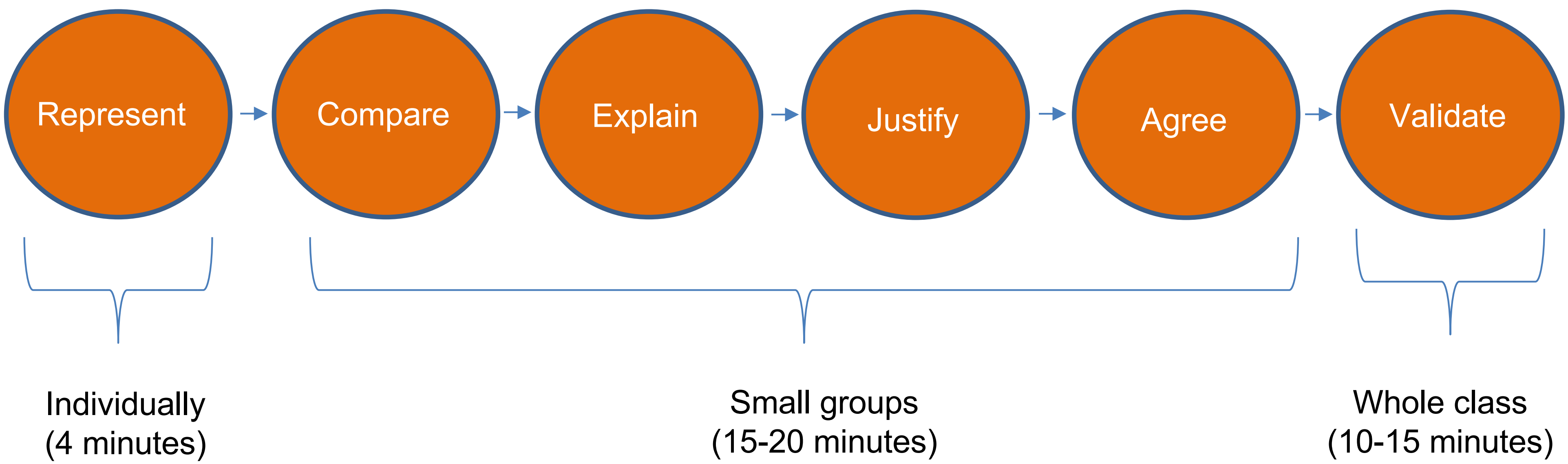
Facilitation of Instruction

- Students are presented with the task
- Students work individually (4 minutes)
- Students work in small groups of 3-5 students (15-20 minutes)
- Students validate their arguments to the whole class (10-15 minutes)

Key Word Format



Key Word Format



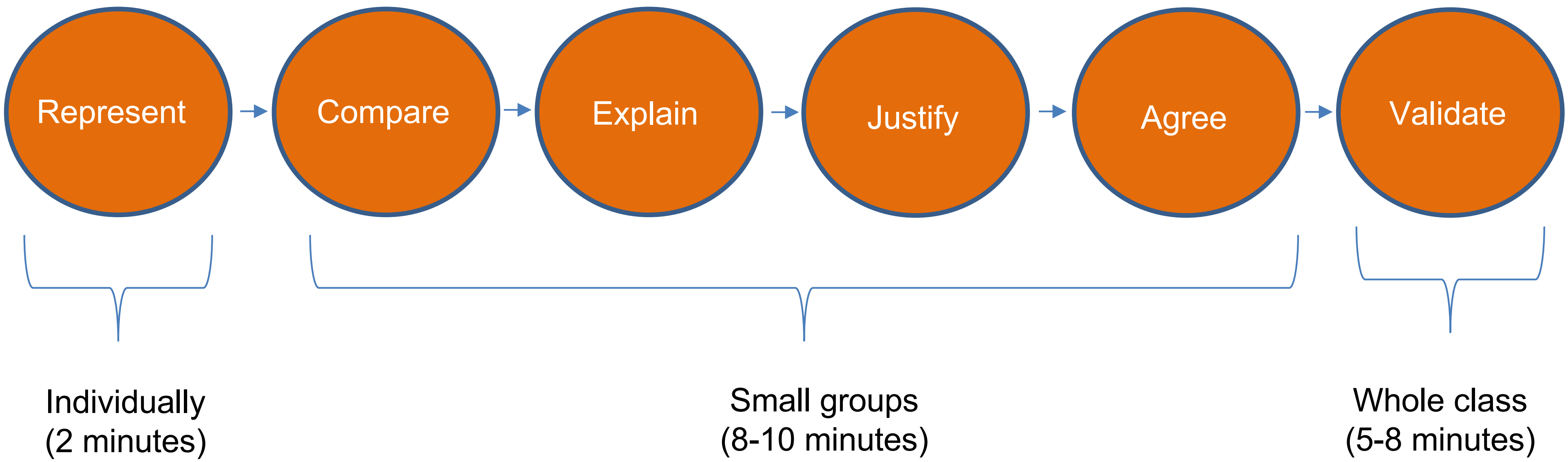
Role of the teacher

Role of the teacher

- During group work time: Question students—push them without giving hints or clues.
- Try not to show preference for different responses.
- Validate: Encourage students to critique each other respectfully. Ask critical questions to each group regardless of the sophistication of the argument.

Task Example

Timing


Represent
Compare
Explain
Justify
Agree
Validate
Individually
(2 minutes)
Small groups
(8-10 minutes)
Whole class
(5-8 minutes)

Task

- **Claim**: The sum of three consecutive integers is divisible by three.

- Is this conjecture true? Write an argument for why or why not.

Volunteers

- Individual
- Small Groups
- Validate based on criteria

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Example Arguments

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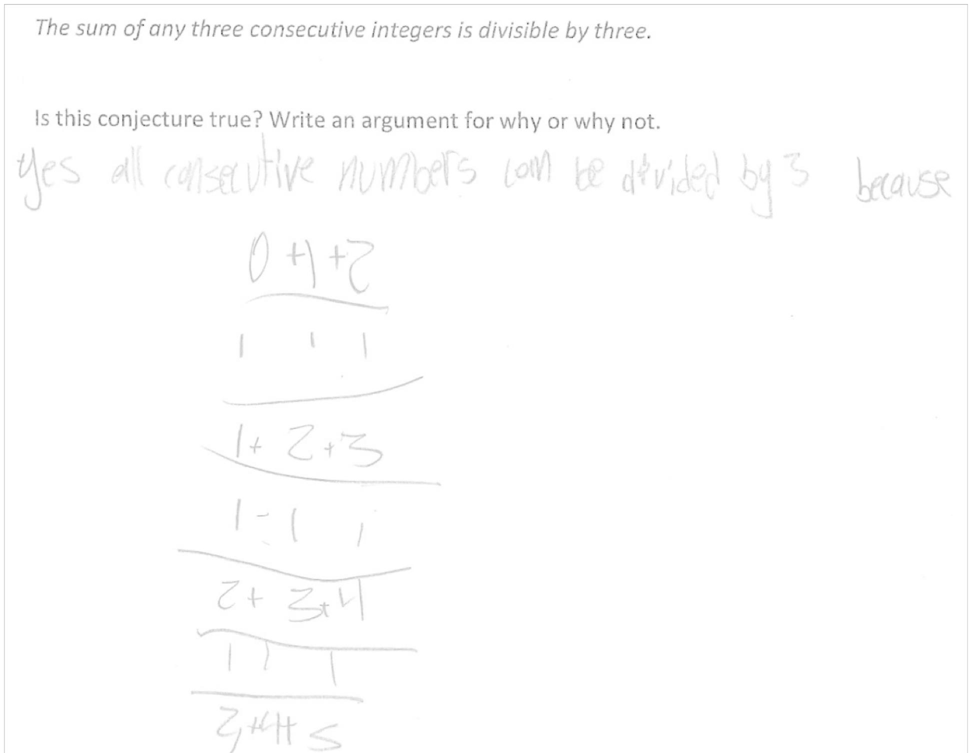
Example Argument #1


The sum of any three consecutive integers is divisible by three.

Is this conjecture true? Write an argument for why or why not.

yes all consecutive numbers can be divided by 3 because

0 +1 +2

1+ 2+3

2+ 3+4

3+4+5

"So, you start with zero, one, two as your base one. And then what you do to bring it up to one, two, three is you add one to each of the integers. So, when you're adding one to each of the integers, you're really just adding three. So, that's why it's divisible by three. And then, that's why it's divisible by three every single time."

"Because if you have three consecutive numbers, if you go up one for each of them, you just add three. And if the base one does...is divisible by three that means all the ones after that will also be divisible by three. And in this instance the base one is divisible by three."

Example Argument #2

"Take any three consecutive numbers. You can add one to the first number and subtract one from the last number. The sum will be equal to the original because we are essentially adding zero. The result will be three numbers that are equal. Adding three numbers that are equal creates a sum that is divisible by three."

$$\begin{array}{ccccc} 1 & + & 2 & + & 3 \\ \underline{+1} & & & & \underline{-1} \\ 2 & + & 2 & + & 2 \end{array}$$

Example Argument #3

"The conjecture is true because I tried several examples, and it worked out. I even tried examples of negative numbers and it worked. So, we think the conjecture is true because it worked for all of the examples we tried."

"For instance, $-11 + (-10) + (-9) = -30$, which is divisible by 3."

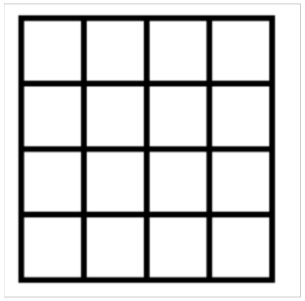
Example Argument #4

“The conjecture is true. We even found that it works for any odd number of consecutive integers. Take 5 consecutive integers: 5+6+7+8+9=35, which is divisible by 5. This works for 7 and 9 too, and we think it will work for any odd number of consecutive integers.”

Task Example

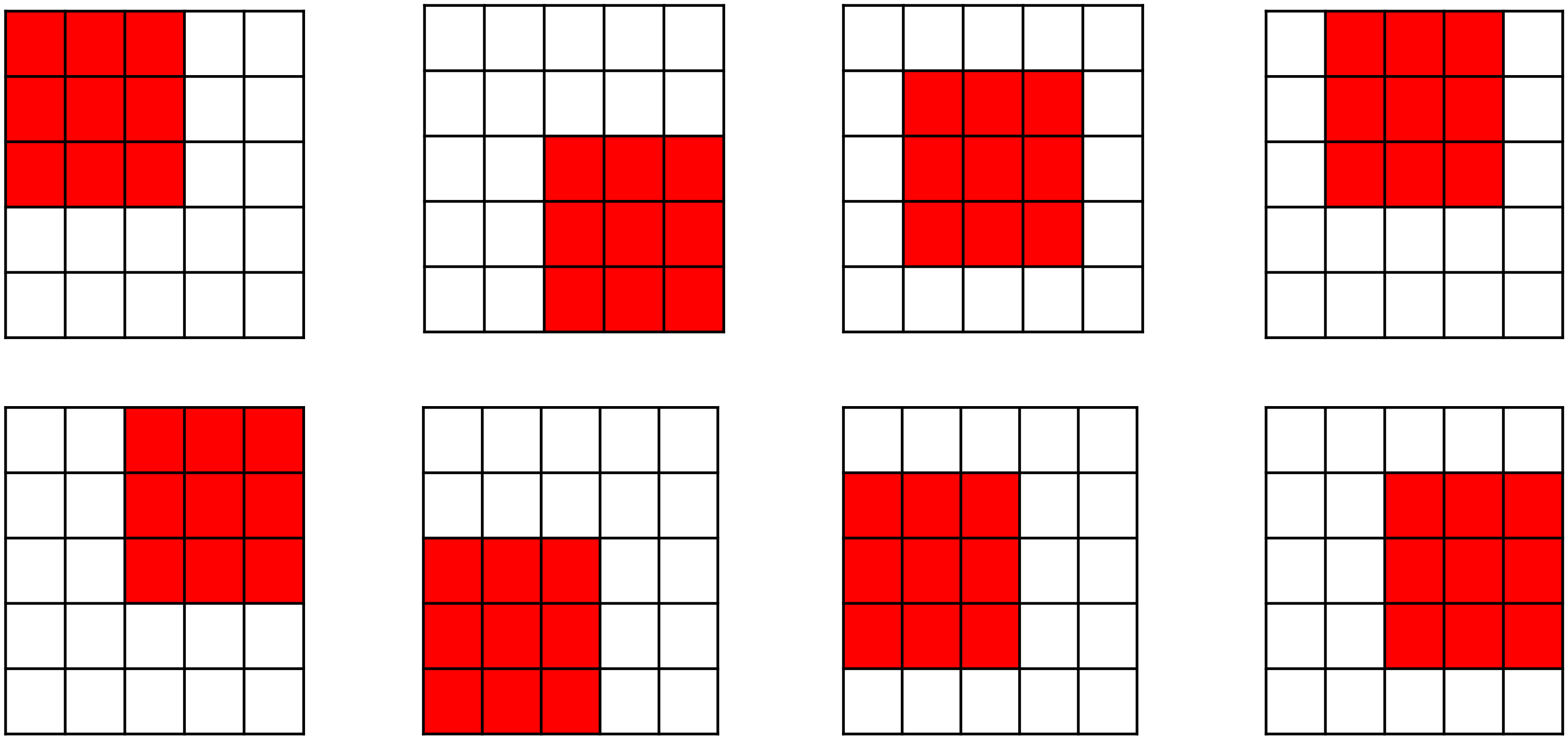
The Squares Problem

- Find the number of 3-by-3 squares there are in the 4-by-4 square below.



- How many different 3-by- 3 squares are there in a 5-by- 5 square?
- How many different 3-by- 3 squares are there in a 6-by- 6 square?
- How many different 3-by- 3 squares are there in a 10-by- 10 square?
- How many different 3-by- 3 squares are there in any size square?

Example Argument #1

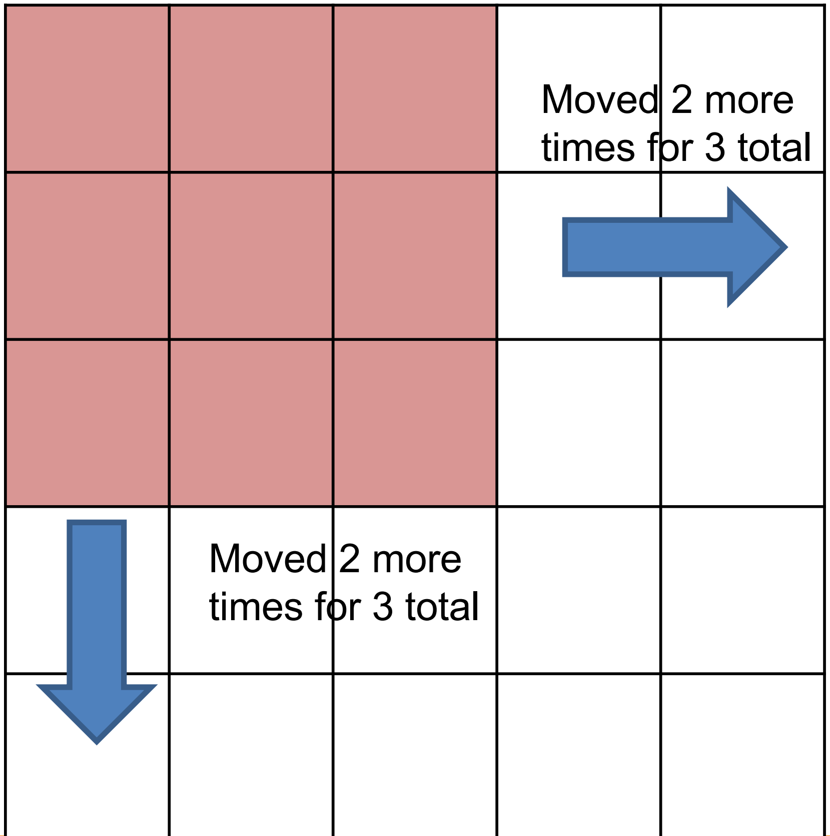


Example Argument #2

x	y
4-by-4	4
5-by-5	9
6-by-6	16
7-by-7	25
x-by-x	$(x - 2)^2$

"I noticed that the y values are squares and then I realized that the squares are two less than x and I got $(x - 2)^2$."

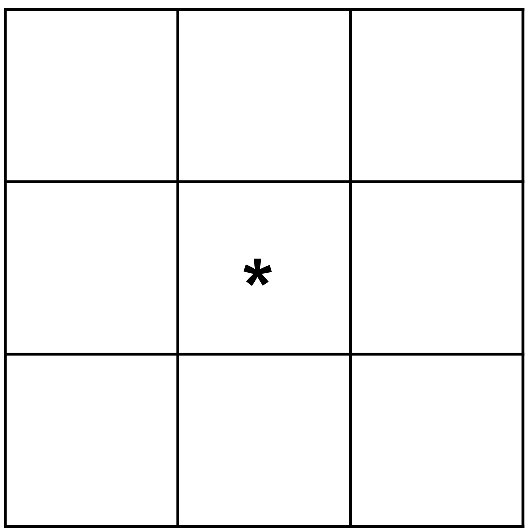
Example Argument #3



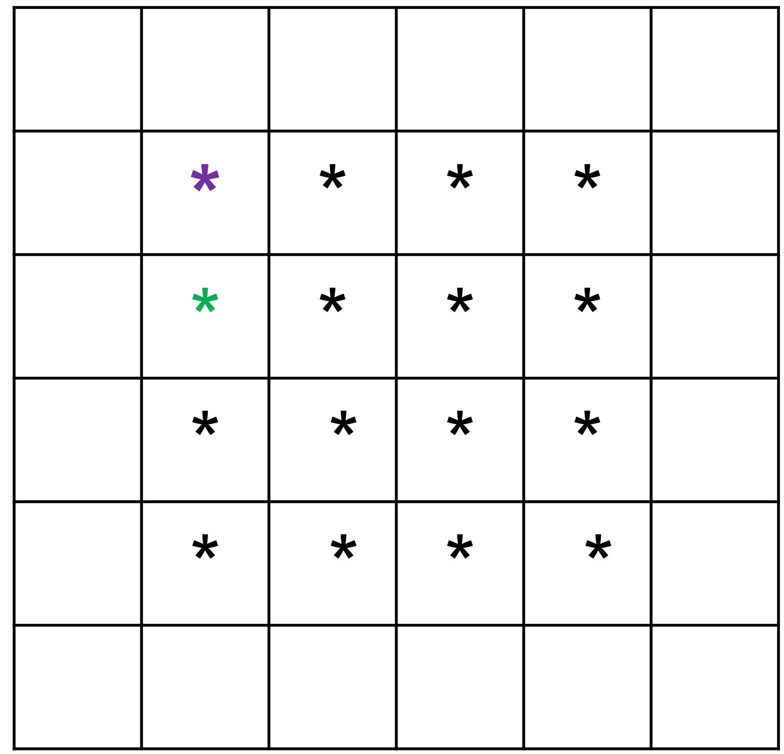
"I started in the top left corner and moved the 3-by-3 square to the right 2 times. Then I moved the 3-by-3 square down and again move it 2 times to the right. Finally, I moved it down again and again moved it 2 times to the right. For an n-by-n square, you move the 3-by-3."

"n – 2 times to the right and down n – 2 times."

Example Argument #4



I used the middle square (*) of the 3-by-3 to count the total in any size larger square. For a 6-by-6, I placed the 3-by-3 in the top left corner which is the * (purple one). Then I shifted the 3-by-3 square down and got the * (green one). As I moved the 3-by-3 down, I see that middle of the 3-by-3 will never touch the bottom row or the first column. This is the same for the right column and top row. Therefore, using this counting method any size square (n-by-n) will leave a border and so the number of possible 3-by-3 squares will be $(n – 2)^2$ where n is a counting number greater than 2.



Resources

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- https://www.illustrativemathematics.org/curriculum for 6-8
- https://playwithyourmath.com
- Dan Meyer’s Three-Act Math Tasks
- http://map.mathshell.org
- NCTM website (problems of the week, etc.)
- Reading academic articles

Conclusions

Conclusions

- Argumentation **can** be done at all levels!
- Create and utilize the classroom community.
- Use open tasks accessible to the community.
- Allow for students to **represent**, **compare**, **explain**, **justify**, **agree**, and **validate** (key word method).
- Encourage collaboration and creativity!

Questions? Contact me!

Today’s presentation & materials:

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