Using Games to Inspire the Joy of Mathematics

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South Carolina Governor's School for Science and Mathematics

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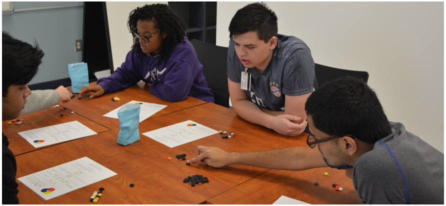
Purpose

Goals:

- Inspire students to see that mathematics is not isolated to just the classroom and can be found in a wide variety of places.
- Encourage students to formulate their own questions and develop solutions.
- Get students to work on math collaboratively

How to Use these ideas:

- Math Club
- Introduce material in certain classes
- Outreach Activities




Kroeger

Ticket To Ride: The Math

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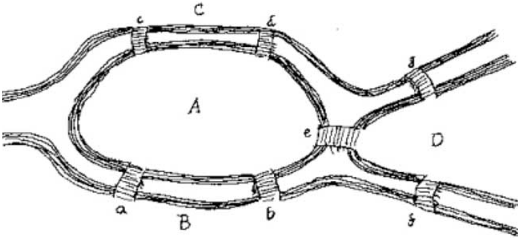
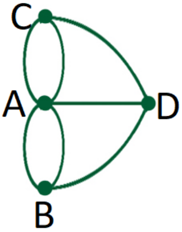
Connection to Bridges of Konigsberg:





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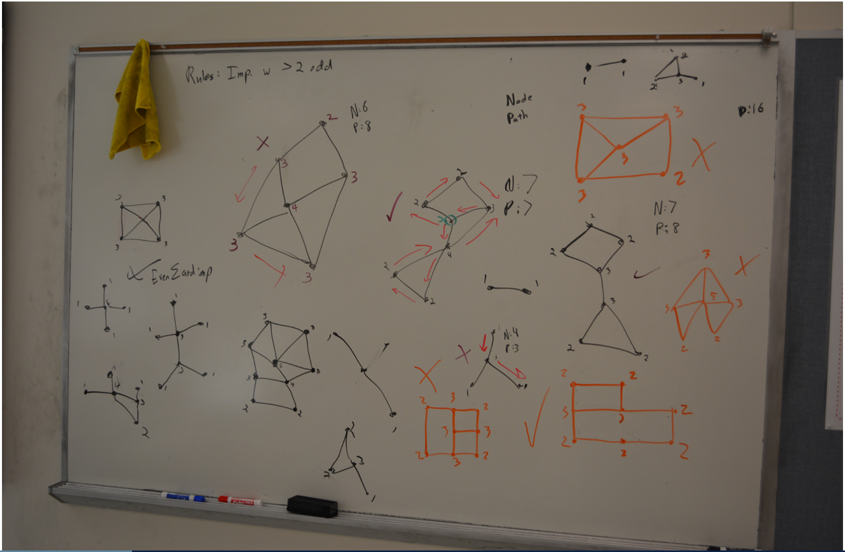

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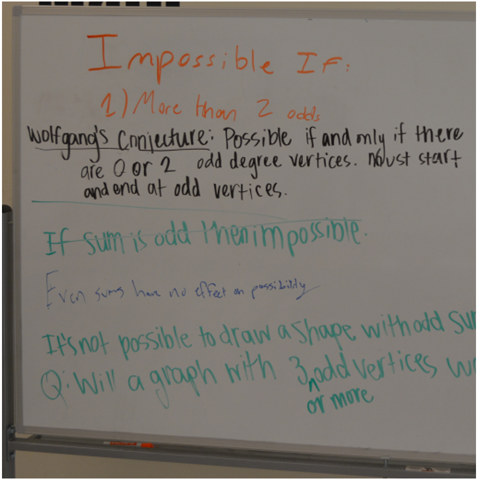

Summary:

- Encourages students to make their own conjectures and test them.
- Shows student the importance of looking at smaller cases to solve a bigger problem.
- Could be used in a discrete math class to introduce graph theory.

Instant Insanity: The Game

Set up: 4 cubes; each face of each cube has one of the four colors: red, green, yellow or blue

Objective: Stack the blocks in a column so that if you look at any side of the column you see all 4 colors.

Good

Bad

Instant Insanity: The Math

- How many ways are there to arrange the cubes?

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    - 24 ways to arrange a single cube

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- True or False: If you are given the colors of three pairs of opposite faces, you will always make the same cube.
- Is a solution to an Instant Insanity puzzle unique?
- Are there sets of cubes that *cannot* be solved?

Resources:`https://www.depauw.edu/files/resources/instins.pdf`
`http://faculty.etsu.edu/beelerr/insanity-supp.pdf`

Instant Insanity: Graph Theory

Create a graph from the set of blocks:

- Draw one vertex for each color
- Draw an edge between two vertices if the colors appear on opposite side of a block. Label the edge with the number from the block.
- How many edges will your graph have?

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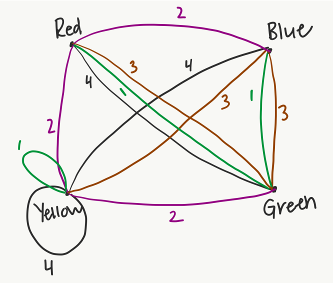

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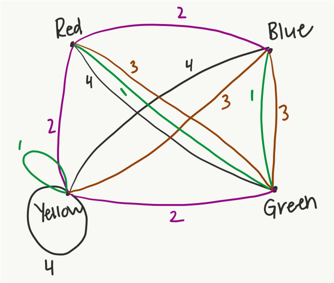

Find two subgraphs satisfying:

- Each subgraph contains all 4 vertices and one edge with each of the labels (4 edges total)
- Each vertex of subgraph has degree 2
- No edge appears in both subgraphs

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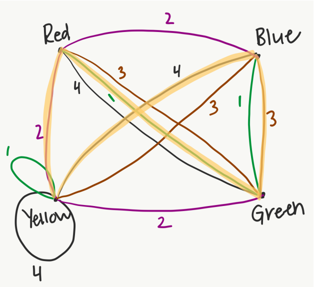

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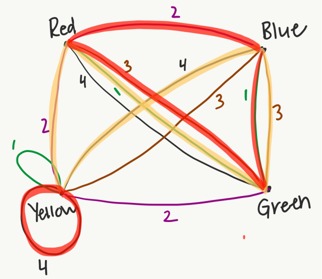

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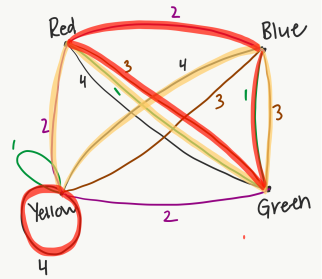


Yellow Subgraph: $Y \overset{2}{-} R \overset{1}{-} G \overset{3}{-} B \overset{4}{-} Y$

Red Subgraph: $Y \overset{4}{-} Y$ $R \overset{3}{-} G \overset{1}{-} B \overset{2}{-} R$

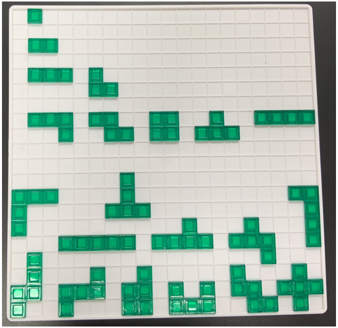
Blokus: The Game

Game Pieces: 21 playing pieces in each of 4 colors; game board

Game Rules:

- Each person starts with putting one of their pieces in a corner spot.
- Then each player takes turn places pieces of their color on the board.
- Every piece you play must touch another piece of your color, **but only at the corners!**
- Cover the most squares on the board to win.

Best with exactly 4 players



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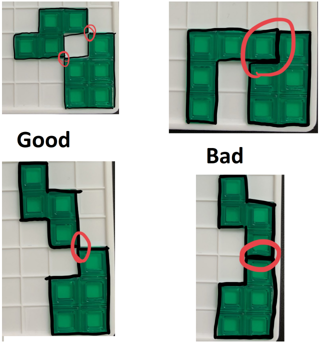

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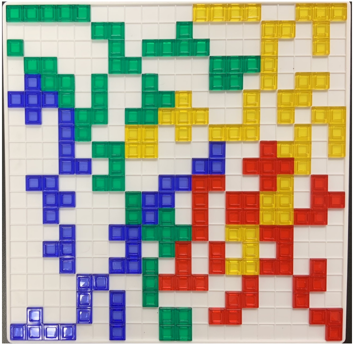

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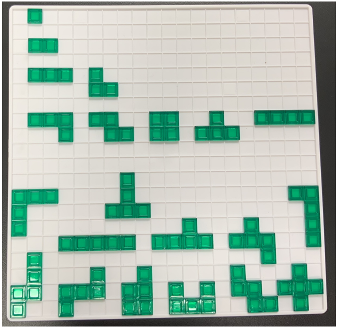

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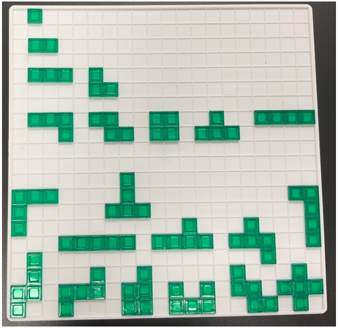
- Does each set of colored tiles in Blokus include *all* possible polyominoes with 1, 2, 3, 4, and 5 squares?
- How many different polyominoes are there with 6 squares? With 7 squares? Can you find a general pattern?



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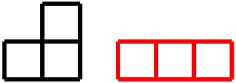
Tiling Questions:

- Can we tile a 2x3 grid with trominoes? How many ways can we do so?



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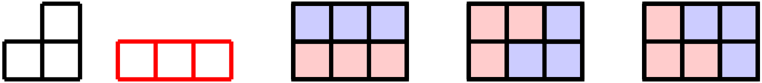

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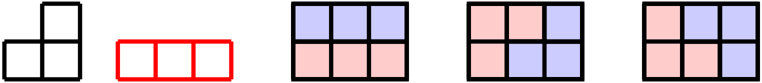

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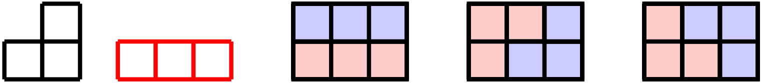
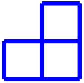
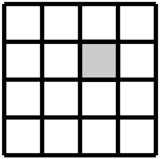


- Pick a certain shaped polyomino. Can you tile an 11×11 grid with that shape?
- Consider a 4×4 grid with one square removed. Using only L-shaped trominoes, can we tile this grid? What about with a $2^n \times 2^n$ grid? (Golomb's Tromino Puzzle)

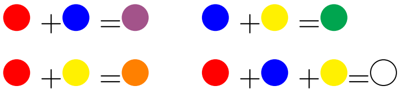




Al-Jabar: The Game

Game Pieces: 10 pieces of each: white, red, yellow, blue, orange, green, and purple; 30 black pieces

Color Mixing Rules: The four defining color rules are as follows



Behavior of black pieces:

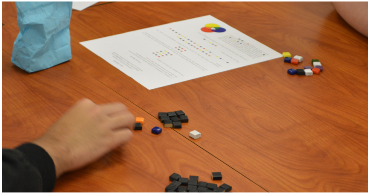
https://al-jabargames.com/

Al-Jabar: The Game Rules

2-4 Players

Goal: Finish with the fewest number of pieces in your hand.

- Black pieces go in a separate pile
- Each player dealt 13 game pieces from the 70 colored pieces.
- Place one colored tile and one white tile in center



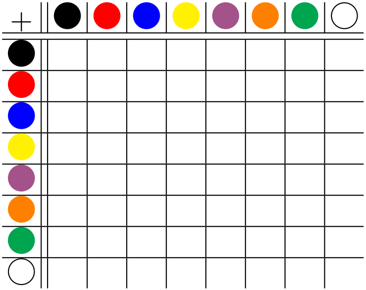
- A turn consists of a player exchanging a combination of 1, 2, or 3 pieces form their hand with a set of pieces from center having an equal color sum.
- Pairs of like colors that appear in the center are traded for a black piece and every player (except the one who caused the double) is required to take a black piece.

Al-Jabar: The Math

Questions to explore:

- Does it matter which order we "add" the colors in? i.e., does Red+Blue = Blue+Red? (Commutative)
- Does it matter how we "group" the colors? i.e., does (Blue+Red)+Green = Blue+(Red+Green)? (Associative)

Fill out the following table for adding pairs of colors.



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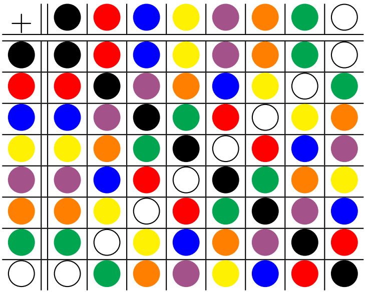

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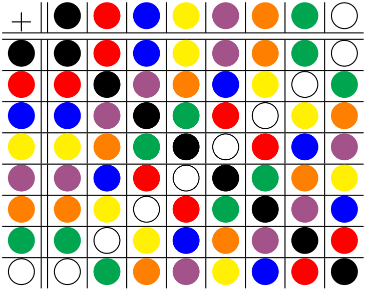
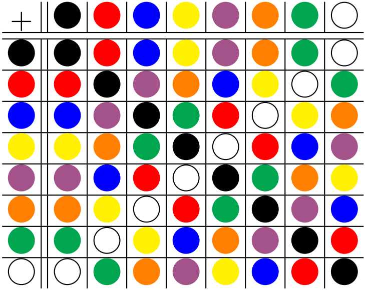


What do you notice about the table? What questions do you have?

Al-Jabar: The Math

A **group** is a set with an operation that satisfies 4 properties:

- **Closure**: Combining two elements of the set gives an element from the set.
- **Identity**: There is an element that when combined with any other element gives the other element back.
- **Inverses**: Every element has an inverse so that the original element combined with its inverse gives the identity.
- **Associativity**: The operation is associative.



Questions:

- Why is the set of colors with "addition" a group?
- What are other examples of groups?

Spot It!: The Game

- Deck of cards with 8 symbols on each card.
- Each pair of cards has *exactly* one pair of symbols in common.
- Several variants on the game all rely on finding common pairs of symbols.

Spot It!: The Math

Questions to Ask:

- Are all possible cards include in the Spot It! deck?
- How many different symbols are used in the Spot It! deck?
- Could you create your own set of cards using 3 symbols? How many cards would you need to do so? How many symbols will be on each card?

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Finite Projective Planes

1. Any two points (symbols) are on exactly one line (card).
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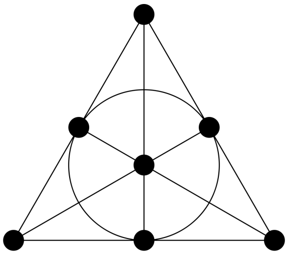

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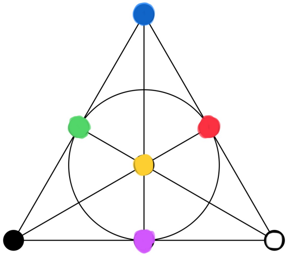

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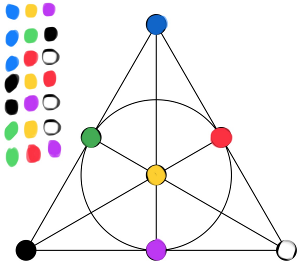
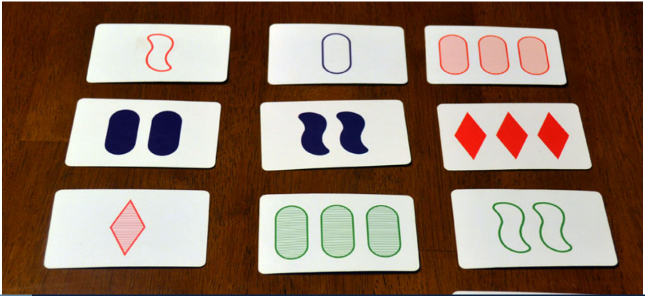


SET: The Game

Deck of cards; Each card has symbols with the following characteristics:

- Either 1, 2, or 3 symbols
- Symbols are Red, Green, or Purple
- Symbols are diamonds, ovals, or squiggles
- Symbols are filled in, shaded, or empty

A SET is a group of 3 cards where each of the characteristics is all the same or all different.



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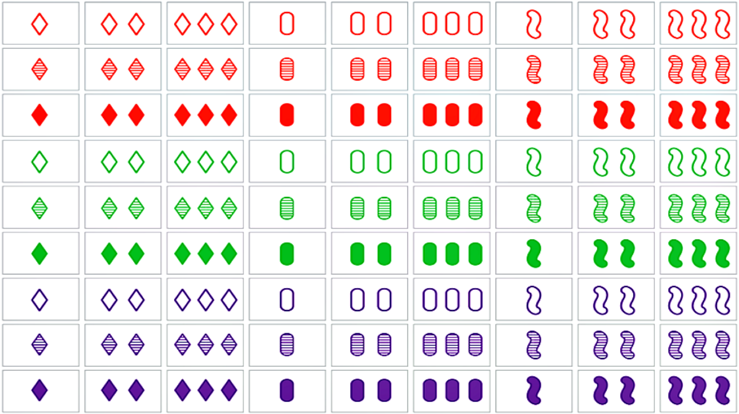

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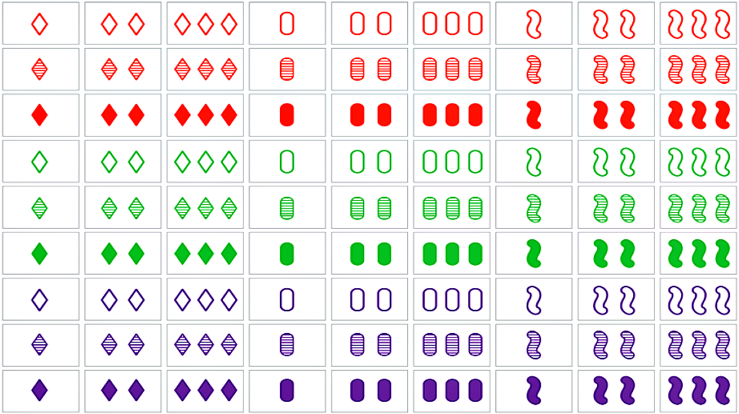

List elements in the set Red $\cap$ Oval

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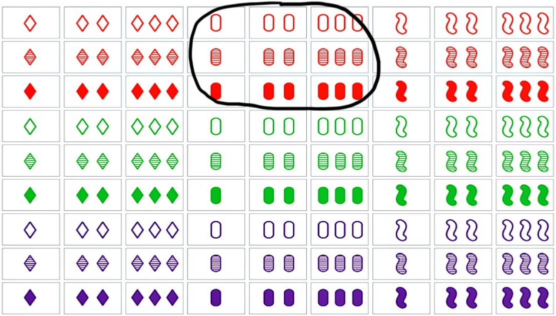

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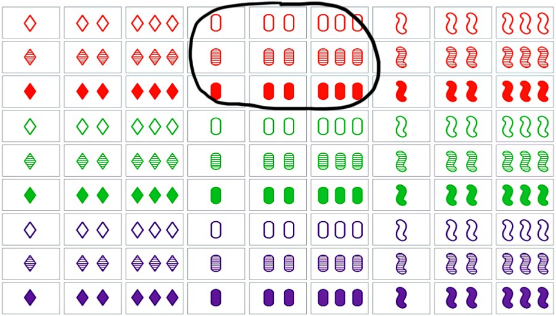

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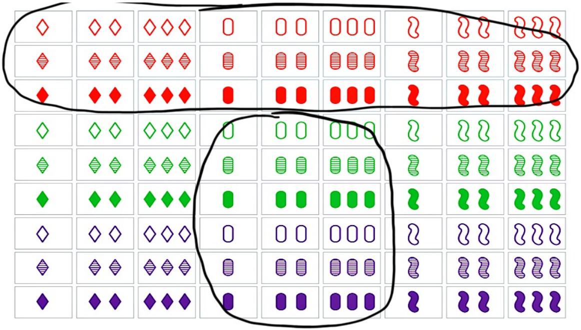

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Thank you!

Contact me: Nicole Kroeger, SC Governor's School for Science and Mathematics, kroeger@gssm.k12.sc.us