

Games and Tasks that Promote Deep Connections and Deep Understanding

NCTM Annual Conference 2019 San Diego, CA

K-2 Workshop

Session 242

Welcome!

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Games and Tasks that Promote Deep Connections and Deep Understanding

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K-2 Workshop

Session 242

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University of Illinois at Chicago
Learning Sciences Research Institute



Math Trailblazers

Math Trailblazers

Research and Revision Study

- Whole Number Study—UIC 2008
- Implementation Study—UIC 2006
- Fractions and Ratios—UIC 2004–2006
- Video Study—UIC 2003–2006
- Field Test Study—UIC 2006–2010
- Student Achievement Study—UIC 2009–2011
- Embedded Assessment Study—UIC 2010–2014

How can
instruction
support that
learning?

How do
students learn?

Nan and Bert Problems (pg. 2-4)

Name _____ Date _____

Nan and Bert Problems

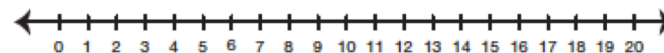
Show or tell how to solve each problem. Write a number sentence. Use cubes, a number line, or ten frames. You may also draw a picture.

Solve. Then discuss with a partner:

- Did you use the same strategy for each problem? Why?
- What strategies or representations would you expect from your students?

1. Nan and Bert went on a picnic at the lake. Nan ate 9 grapes and Bert ate 5. How many did they eat altogether?

Number sentence _____

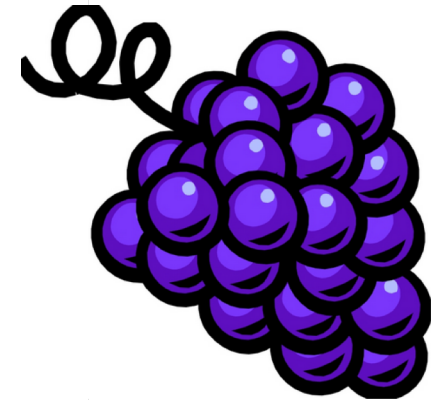


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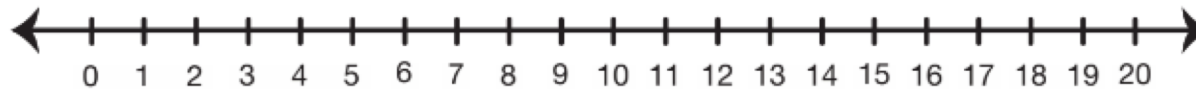


How do the tools support reasoning?

1. Nan and Bert went on a picnic at the lake. Nan ate 9 grapes and Bert ate 5. How many did they eat altogether?



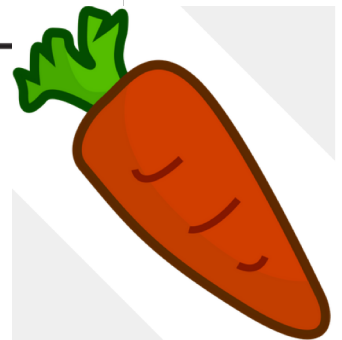
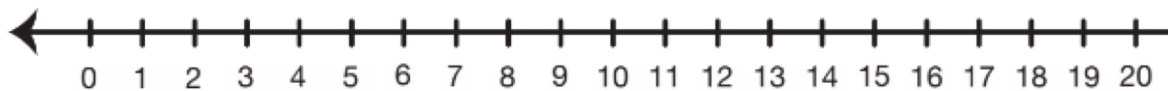
Number sentence _____

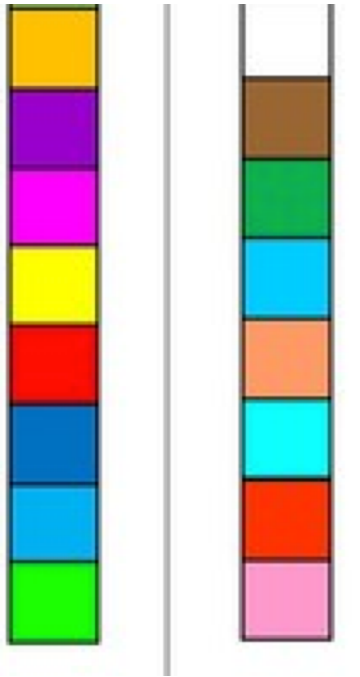


How do the tools support reasoning?

2. Grandma packed 15 baby carrots. Nan and Bert ate them all. If Nan ate 7, how many did Bert eat?

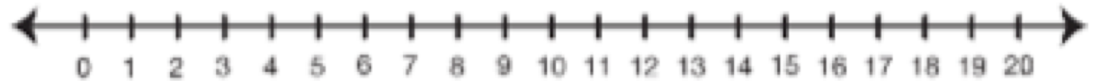
Number sentence _____






4. At the gift shop, Bert spent \$16 and Nan spent half that much. How much did Nan spend?

Number sentence _____



**How do the
tools support
reasoning?**

What does fluency look like?



Why do we need
flexible strategies
for whole number
computation?

What does fluency look like?

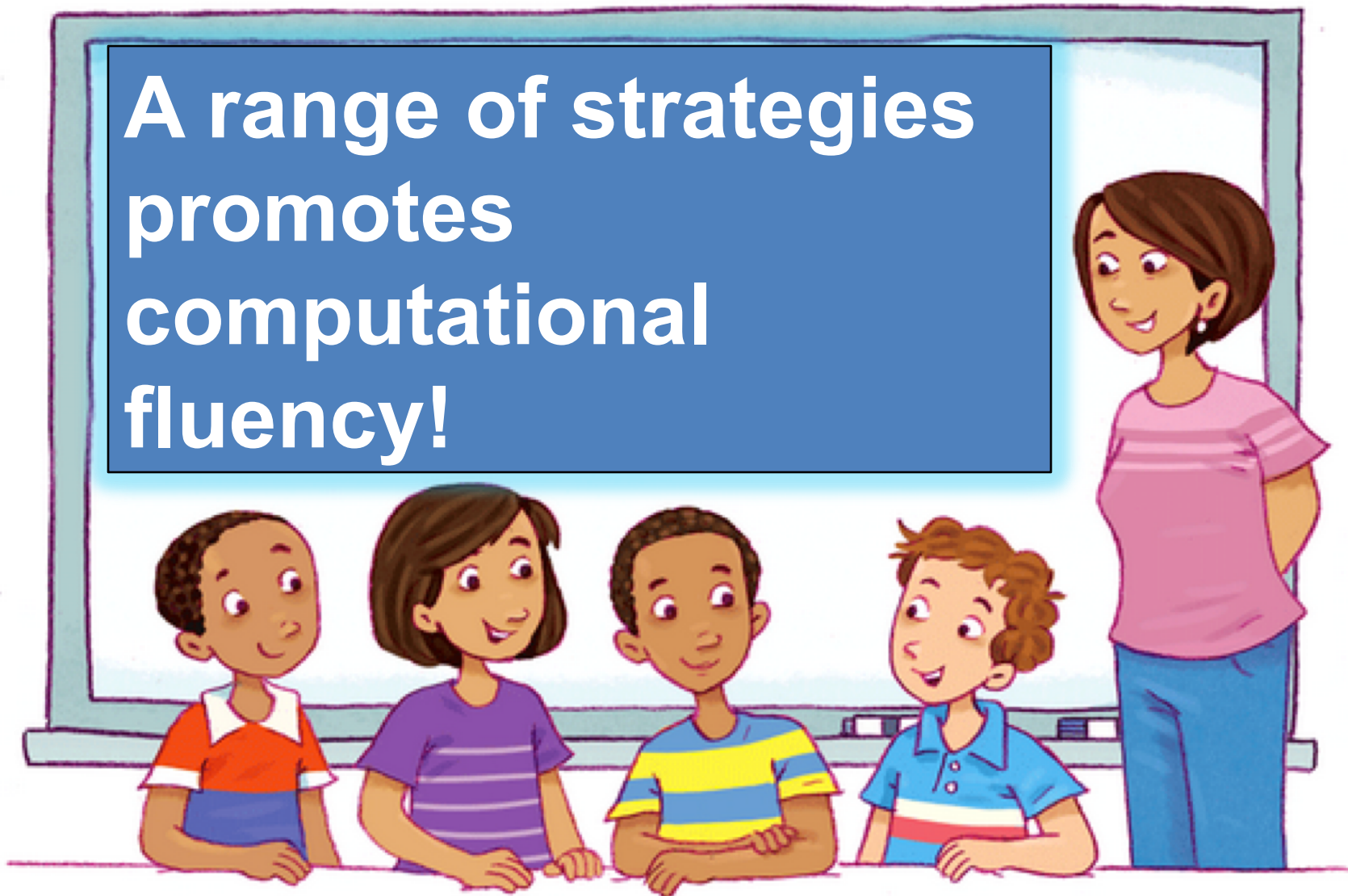
Students who are procedurally fluent can solve problems...



-National Research Council

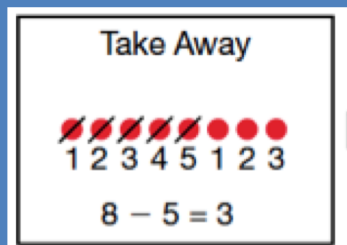
What does fluency look like?

A range of strategies
promotes
computational
fluency!

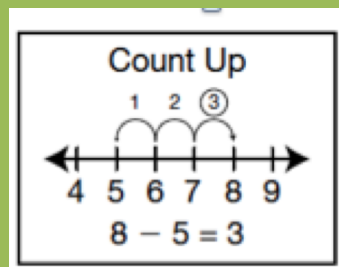


Phases to Develop Fluency

Direct Modeling

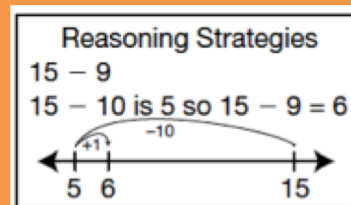


Counting Strategies



Reasoning From Known Facts

I know $3 + 5 = 8$
so, $8 - 5 = 3$



Fluency

Flexibly

Accurately

Efficiently

Appropriately

Stages of Conceptual Development

Direct Modeling

Counting Strategies

Reasoning from Known Facts

Counting All

● ● ● ● ● ● ● ●
1 2 3 4 5 6 7 8

$$5 + 3 = 8$$



Counting On

● ● ● ● ● ● ● ●
 5 6 7 8

$$5 + 3 = 8$$



Reasoning from Known Facts

$$9 + 6 = 10 + 5 = 15$$

Stages of Conceptual Development

Direct Modeling

Counting Strategies

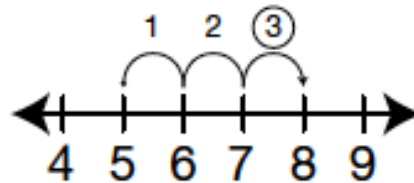
Reasoning from Known Facts

Take Away



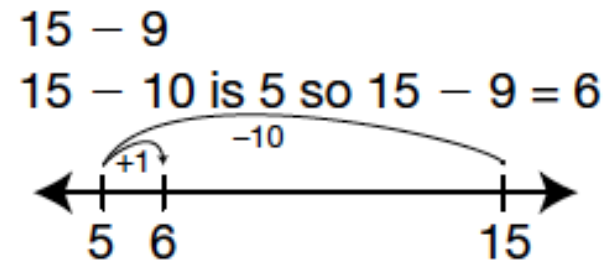
$$8 - 5 = 3$$

Count Up



$$8 - 5 = 3$$

Reasoning Strategies



Stages of Conceptual Development

Sorting Activity

Direct Modeling

**Counting
Strategies**

**Reasoning from
Known Facts**

**Look at student work.
What phase of reasoning is evident?**

Obstacles to Stages of Conceptual Development

Direct Modeling

**Counting
Strategies**



**Reasoning from
Known Facts**

Obstacles to Stages of Conceptual Development



- Too many strategies.
- Found a favorite and won't move.
- Not sure which to use when.
- Strategy is a procedure rather than a way to support reasoning or understanding.

Connect to Representations

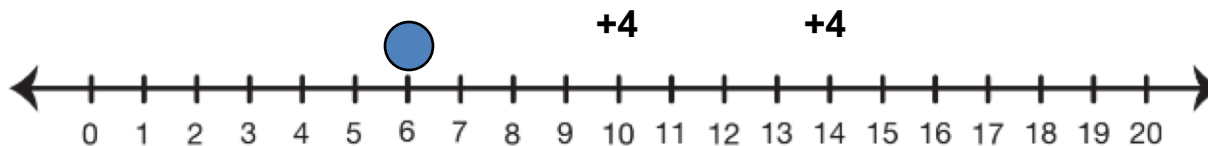
*“To find one’s way around the mathematical terrain, it is important to see **how the various representations connect with each other**, how they are similar, and how they are different. The degree of students’ conceptual understanding is **related to the richness and extent of the connections they have made**.”*

- National Research Council, 2001

Connect to Representations

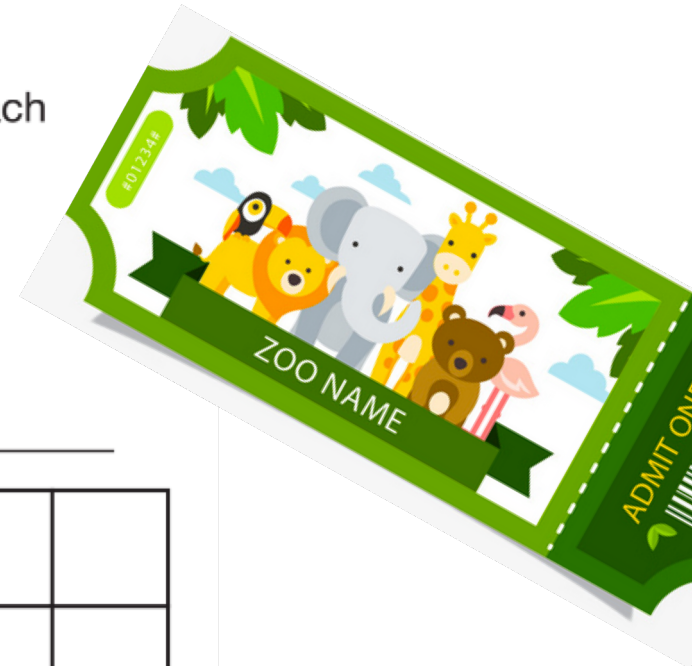
3. Grandma's ticket to the zoo cost \$6. Nan and Bert's each cost \$4. How much did all 3 tickets cost?

Number sentence _____



6

14



Connect to Representations

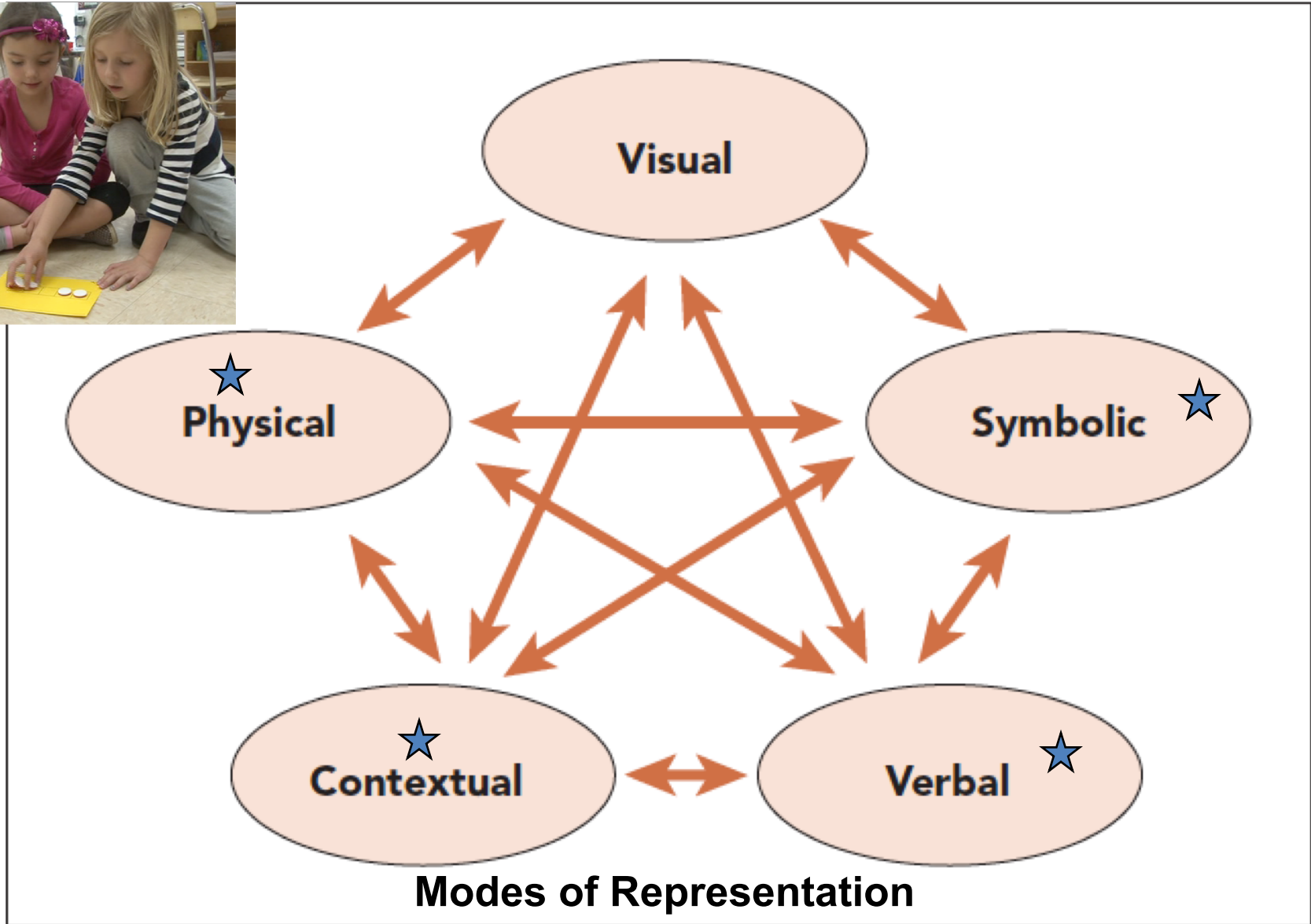


Five frogs on the log.
Three jump off.

There are 2
frogs on the log.

3 minus 5 is 2!

Connect to Representations



Stages of Conceptual Development

Direct Modeling

**Counting
Strategies**

**Reasoning from
Known Facts**

**What reasoning stages are
supported by this activity?**

Stages of Conceptual Development

Direct Modeling



Counting Strategies

Reasoning from Known Facts

What do the tasks look like?

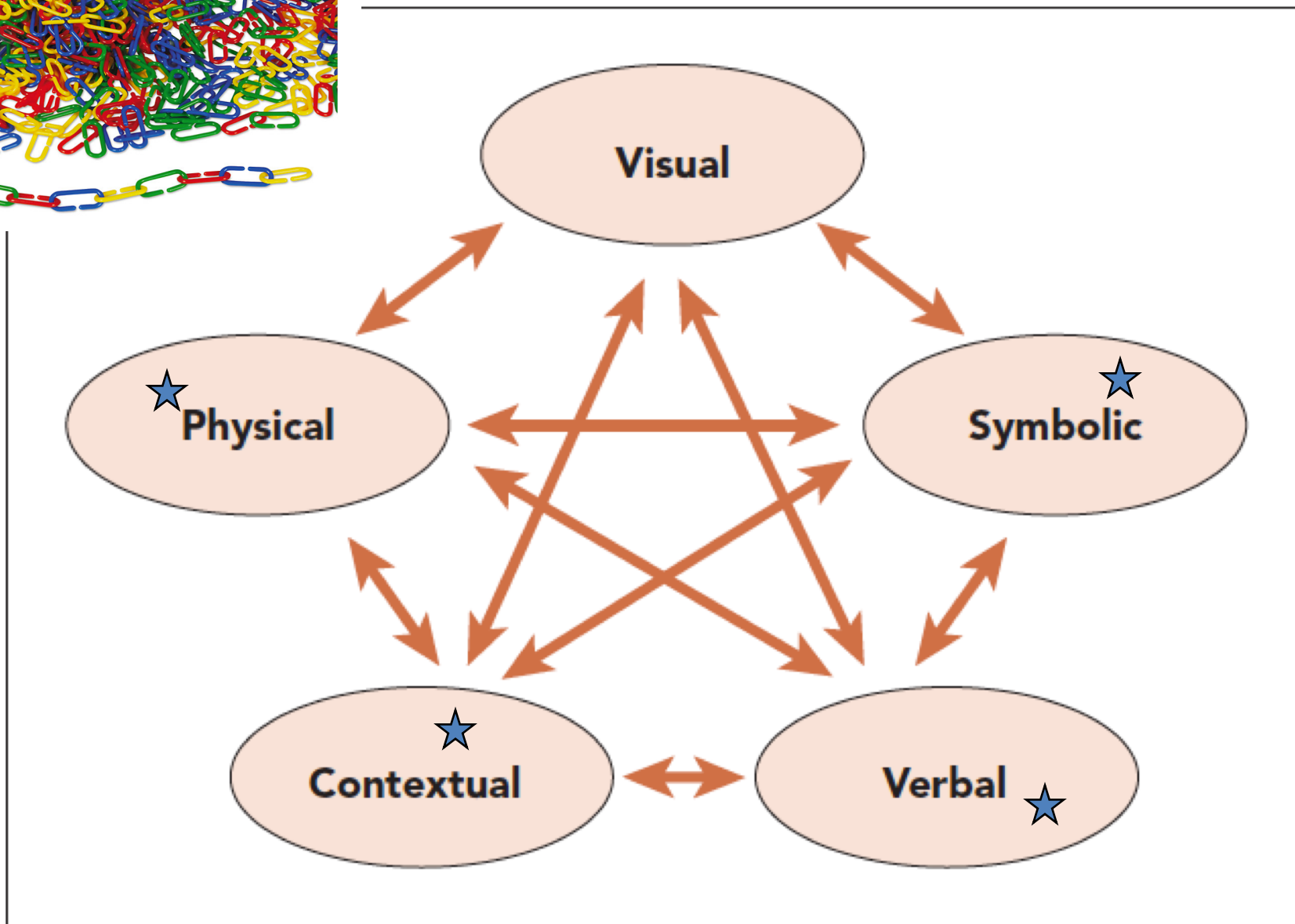
“Discussions that focus on **cognitively challenging mathematical tasks**, namely those that **promote thinking, reasoning, and problem solving**, are a primary mechanism for promoting conceptual understanding of mathematics.”

-Smith, Hughes, Engle & Stein, 2009, p. 549

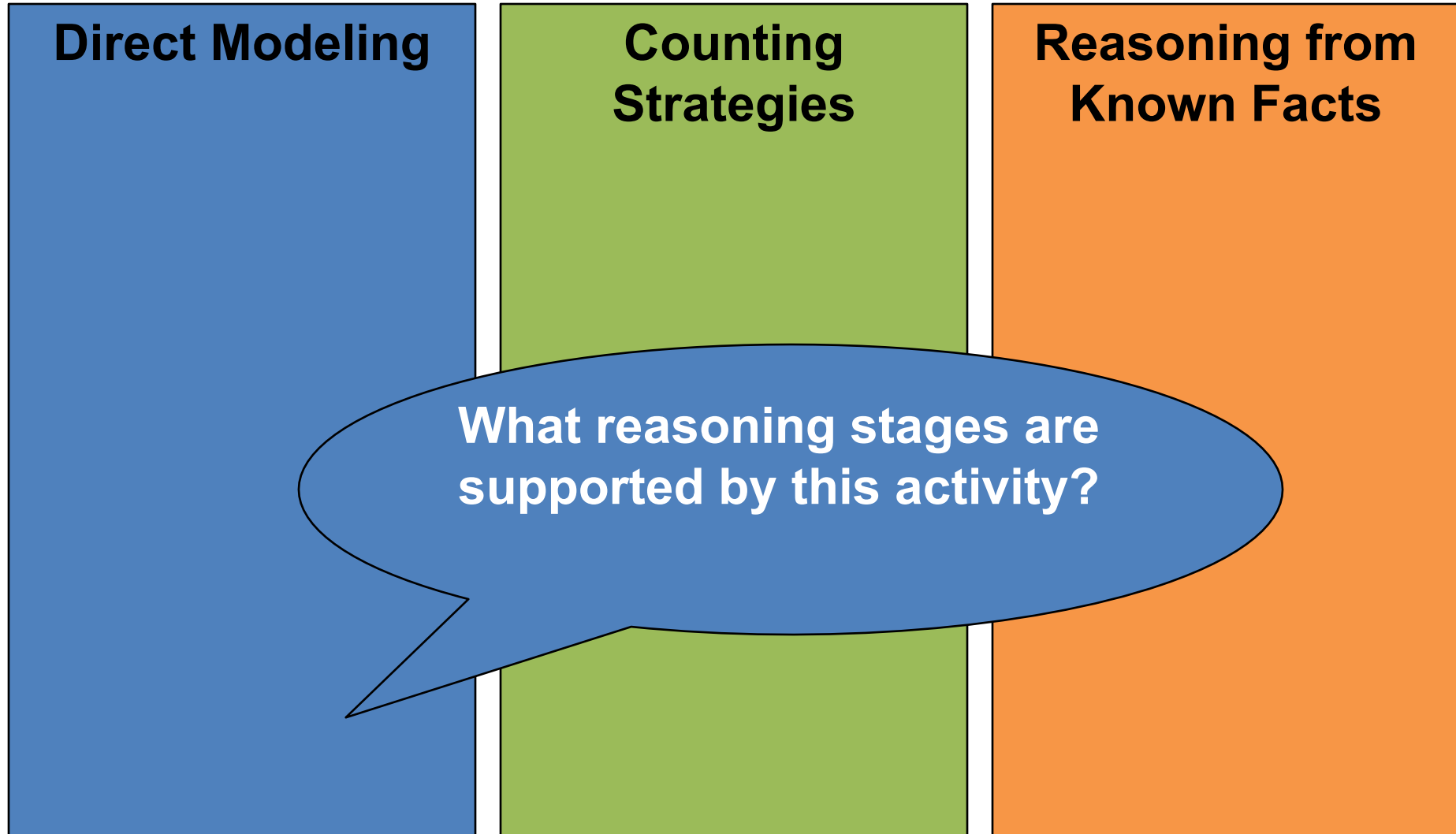
What connections are being made?

"100 Link chain"

Segment 2



Stages of Conceptual Development



Stages of Conceptual Development

Direct Modeling



Counting Strategies

Reasoning from Known Facts



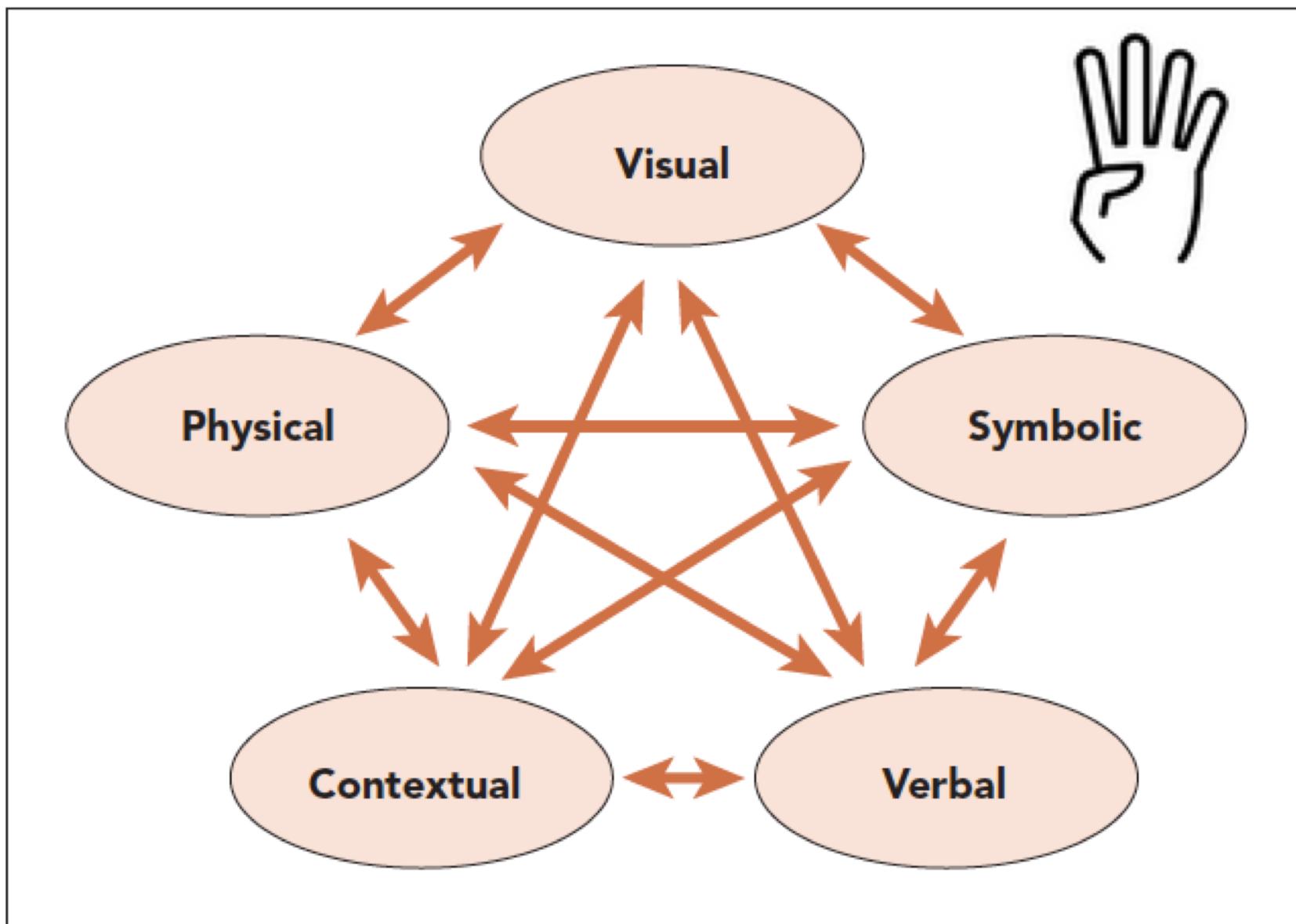
What connections are being made?

D Partition Ten

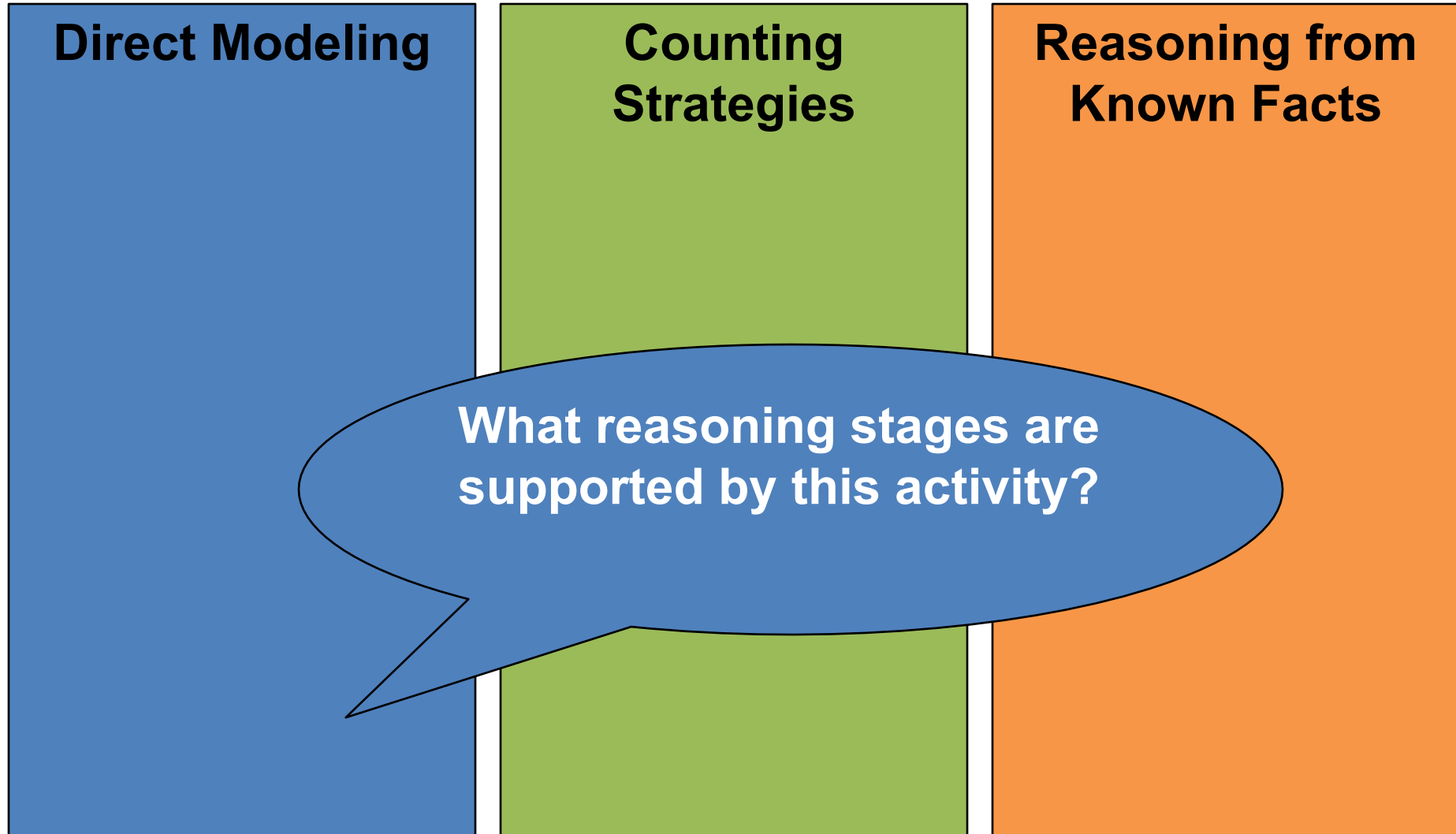


Stand up and find a partner.

1. Face your partner.
2. Player 1 puts both arms straight up in the air.
3. Once in the air, Player 1 uses his or her fingers to show a number between 0 and 10.
4. Player 2 puts his or her arms down and displays the number needed to make ten. For example, Player 1 shows 4 fingers and Player 2 shows 6 fingers because $4 + 6 = 10$.
5. On the next turn, Player 2 leads.



Stages of Conceptual Development



Stages of Conceptual Development

Direct Modeling



Counting Strategies



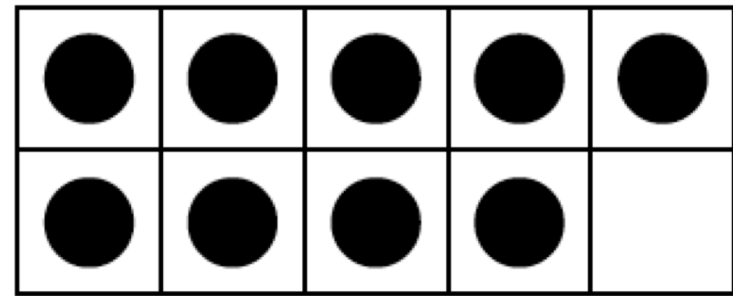
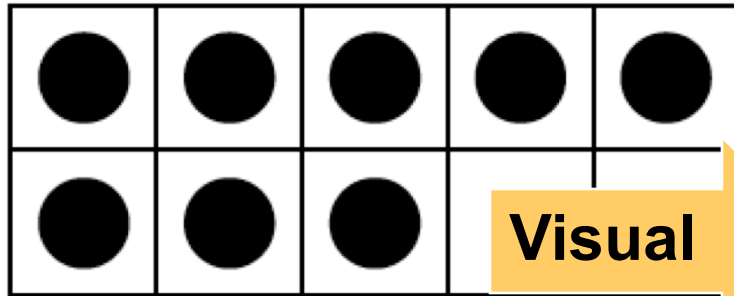
Reasoning from Known Facts



Connect to Representation: Math Talk

$$8 + 9 = 8 + 2 + 7$$

$$8 + 9 = 9 + 1 + 7$$



Visual

$$8 + 9 = 5 + 5 + 3$$

Symbolic

$$8 + 9 = 8 + 8 + 1$$

Verbal

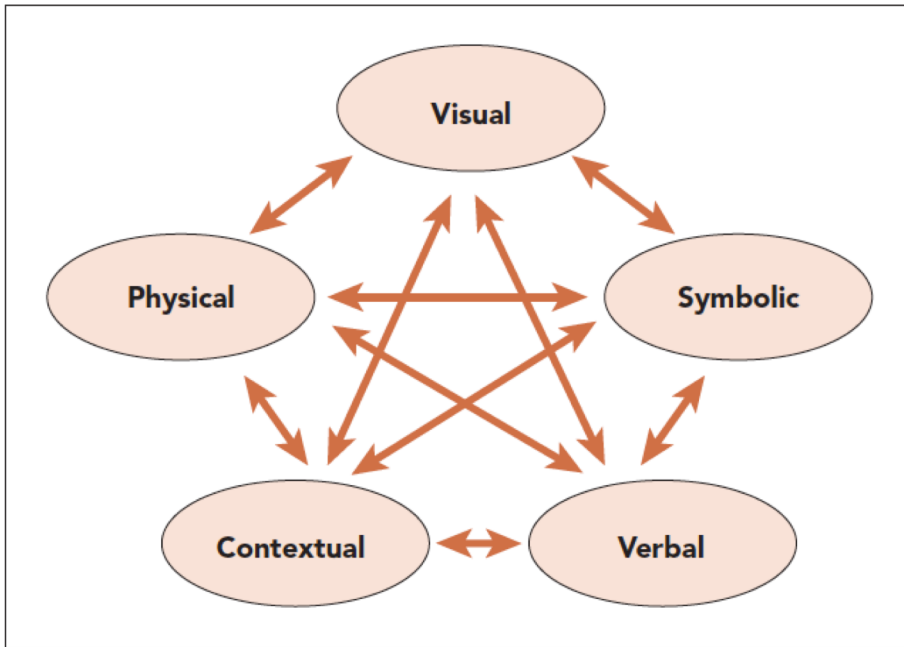
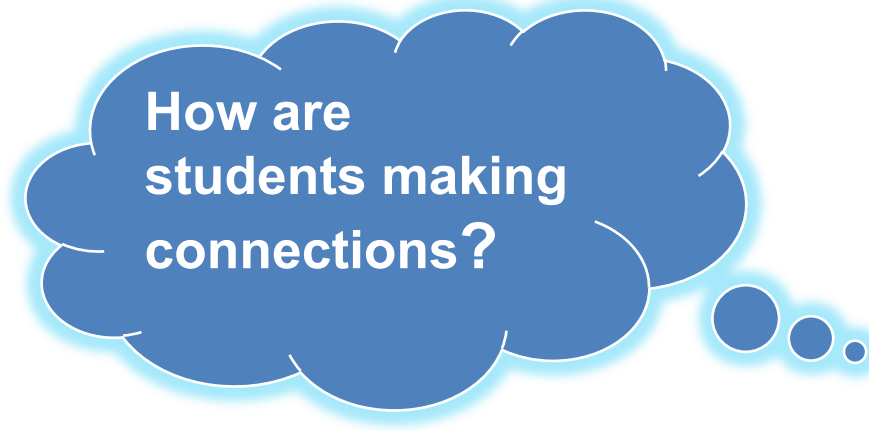
Analyze Kinds of Connections

Play a game with someone from your grade level:

- ***Towers of Ten Game*** Grade K (p. 5)
- ***Doubles, Doubles +1, Doubles -1*** Grade 1 (p. 6)
- ***Not More Than 100*** Grade 2 (p. 9)



Connect Representations



Connect to Reasoning Development

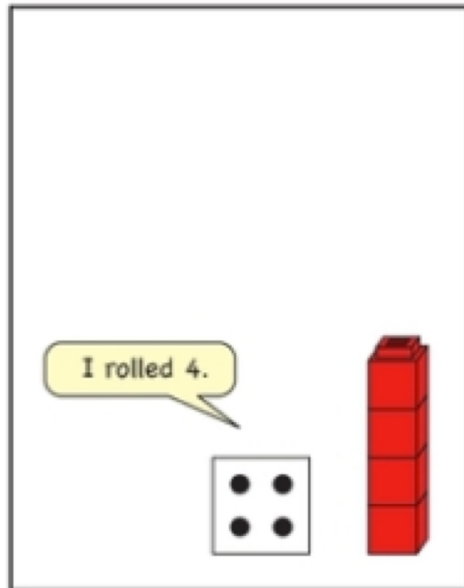
Direct Modeling

Counting Strategies

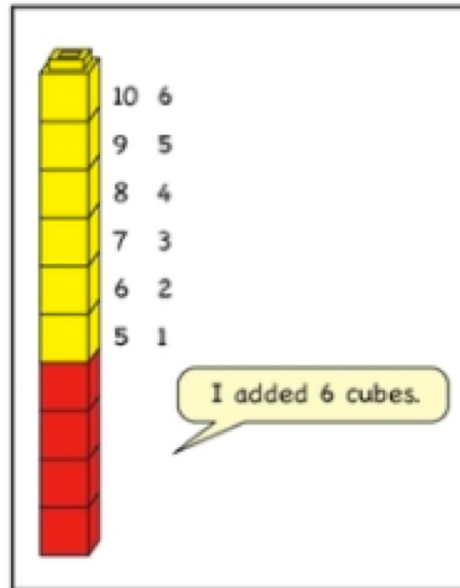
Reasoning from Known Facts

Stages of Conceptual Development

Direct Modeling



Counting Strategies



Reasoning from Known Facts

Stages of Conceptual Development

Direct Modeling

Counting Strategies

Reasoning from Known Facts

Name _____ Date _____

Doubles, Doubles +1, Doubles -1

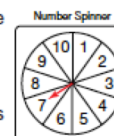
The object of this game is to write number sentences to show doubles, doubles +1, or doubles -1 and to predict whether the sum will be odd or even. This is a game for two players.

Materials:

- Number Spinner (1–10) and Doubles Spinner
- Clear plastic spinner or a pencil and paper clip
- 25 connecting cubes
- Doubles, Doubles +1, Doubles -1 Recording Sheet

Directions:

1. Player 1 spins the Number Spinner and the Doubles Spinner. For example, Player 1 spins a 7 and Doubles -1.
 - Use the spins to write a number sentence and predict whether the sum is odd or even. Find the sum and circle whether the sum is even or odd. Record your work on the recording sheet.



Number Sentence	Sum	Even or Odd?	Prediction Correct
$7 + 6 =$	13	Even	Odd

2. Player 2 checks Player 1's work and places a ✓ in Player 1's last column if the prediction is correct.
3. Player 2 now spins, makes a prediction, and completes the next row on the recording sheet.
4. Players continue to take turns for five rounds.
5. The player with most correct predictions wins.

Stages of Conceptual Development

Direct Modeling

Counting Strategies

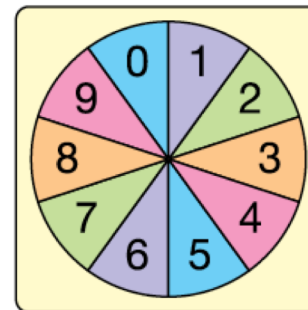
Reasoning from Known Facts

Name _____ Date _____

Sample Recording Sheet

Spin	Tens	Ones	Number Sentence
1	5	0	$50 + 0 = 50$
2		6	$50 + 6 = 56$
3	2	0	$56 + 20 = 76$
4		9	$76 + 9 = 85$
5	1	0	$85 + 10 = 95$

Not More Than 100 Spinner



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Stages of Conceptual Development

Direct Modeling

**Counting
Strategies**

**Reasoning from
Known Facts**

Solve $79 + 28$.



Connect to Students' Thinking

**I thought about
coins . . .**

**$80 + 28 = 108$
1 less than 108
is 107.**

**Are all students
in the same
place?**



[Adding: $79 + 28$]
I think um...I made 79 circles and 28
squares...

Ask Questions to Make Connections

(p. 13)

Direct Modeling

Tell me a story for this problem. Ask connection questions: What does + mean in your story? What does the [3] mean?

Show your story [on a number line, with a picture, by acting it out, by acting it out with objects].

Provide the problem in a context.

Counting Strategies

Show or tell me how you would solve the problem.

Try to solve it with [a number line, counters].

See how many different ways you can solve this problem.

Try to count a different way. counting up, counting back, counting on

I see you solved $[4 + 3]$. Find a way to solve $[14 + 3]$.

Reasoning from Known Facts

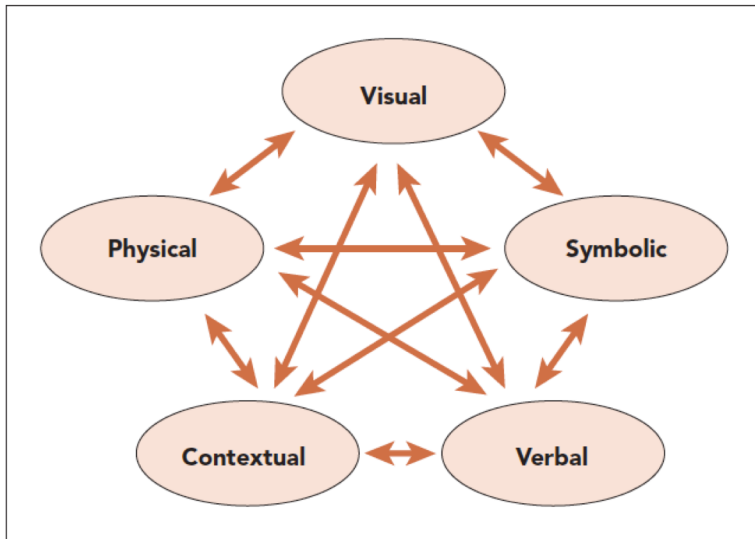
What friendly fact might help?

Try to solve it with a different friendly fact. How could [ten, a double] help?

Try to use [ten frames, a rekenrek].

I see you know $[6 + 4]$. How can that help you solve $[8 + 4]$?

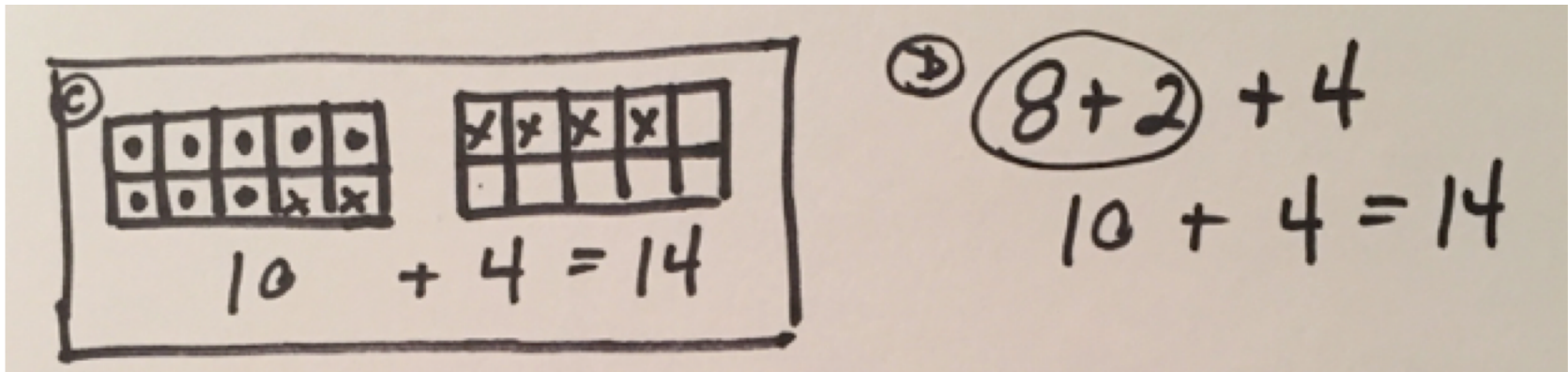
What addition fact might help with this subtraction problem?



Connect Representations



Connect to Reasoning Development



Connect Strategies

Connect to Strategies (p. 14)

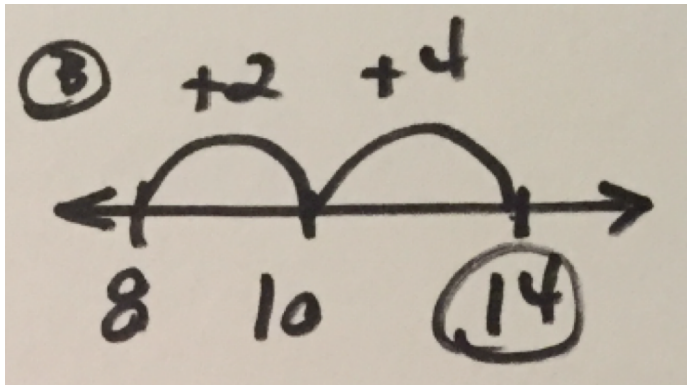
Look at the solutions to $8 + 6$.

- Which used the same representation?
- Which used the same strategy?

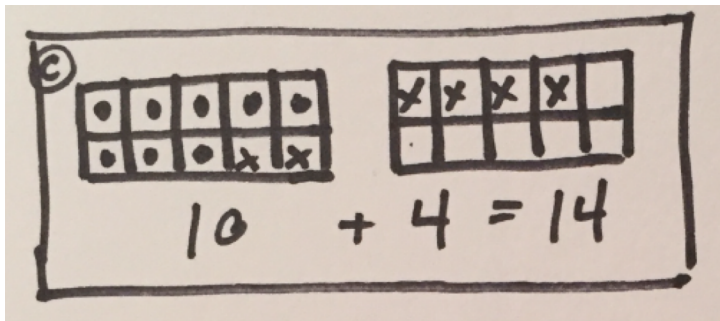
Which of these students thought about the solution in the same way?

Connect to Strategies (p. 14)

Student B



Student C



Student D

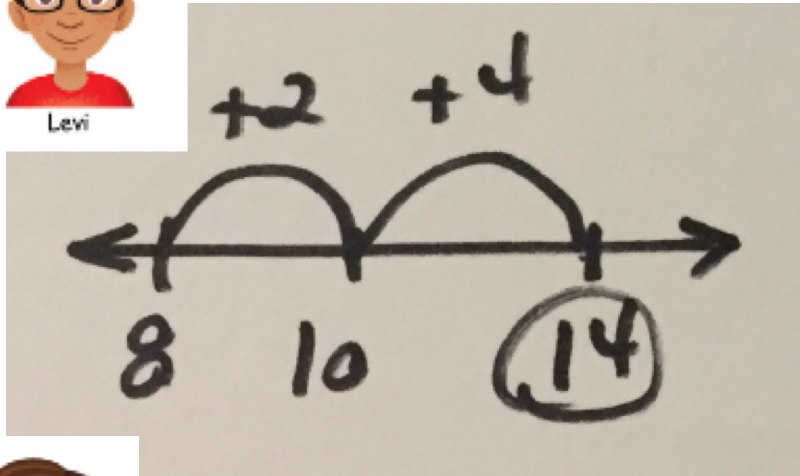
Handwritten equations for Student D. The first equation is $(8 + 2) + 4$, where $8 + 2$ is circled. The second equation is $10 + 4 = 14$. A small circled 'D' is in the top left corner.

- Did these three students use the same strategy?
- I notice the +2 in Student B's solution. Why?
- Does student C add 2 to 8? How is that shown?
- How is the +2 shown in Student D's solution?

Connect to Strategies



Levi



$$8 + 6 =$$

I notice that both of these students counted on.

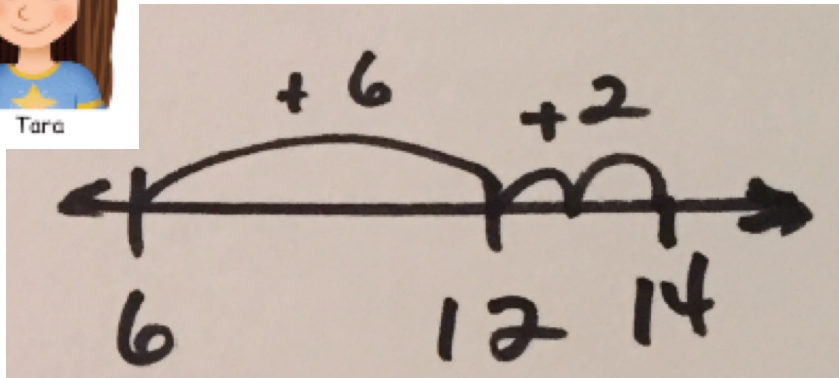
Levi started at 8 and
Tara started at 6.

Can they do that?

Why does that work?



Tara



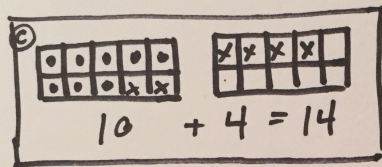
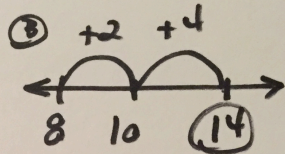
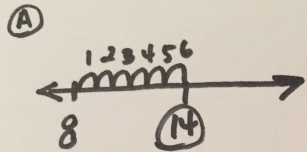
Connect to Strategies and Representations (pg. 14)

Your turn. Work with a partner.

Notice which ones go together.

Practice asking questions that help students connect the strategies and representations.

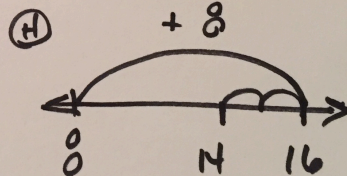
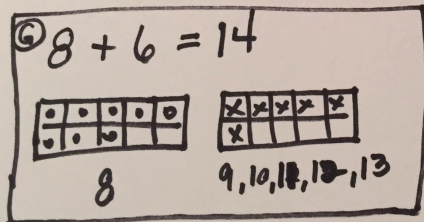
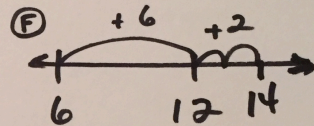
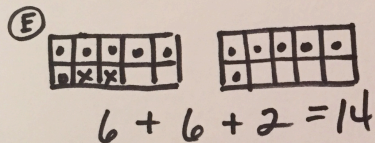
$$8 + 6$$



(D)

$$8 + 2 + 4$$

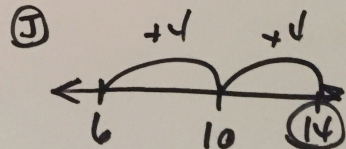
$$10 + 4 = 14$$



(I)

$$8 + 8 - 2$$

$$16 - 2 = 14$$

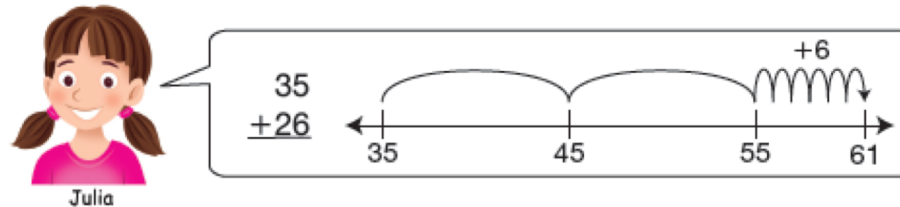


Connect to Strategies

How Did They Do It

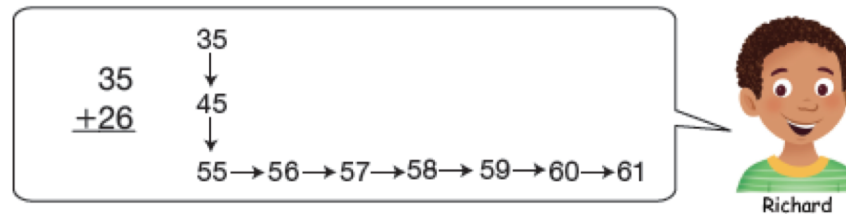
Check to see how each student solved $35 + 26$.

1. Julia showed her work like this:



Explain how she solved the problem.

2. Richard showed his work like this:



Explain how he solved the problem.

3. Show another way to solve $35 + 26$.

Connect to Strategies

1. Jason solved the following problem. Does his answer make sense? Why or why not?



Jason

$$\begin{array}{r} 97 \\ + 86 \\ \hline 1713 \end{array}$$

Connect to Context Demands

Sort the problems by strategy.

$$2 - 1$$

$$2 - 2$$

$$\underline{6} - 5$$

$$8 - 7$$

$$7 - 5$$

$$7 - \underline{6}$$

$$3 - 1$$

Connect to Context Demands

$$9 + 5$$

$$8 + 7$$

$$8 + \underline{6}$$

$$8 + 8$$

Would you use the same strategy?

$$9 + 2$$



Connect to Student's Thinking

How can you use one fact to solve another?

$4 - 4$

$4 - 3$

$10 - 7$

$9 - 7$



Yolanda



F. (Skip) Fennell

@SkipFennell

Follow



Students with number sense know **WHEN!**
When to compute, When to use mental math,
When to estimate, When to use data sources
to inform, When they must work quickly, and
When they should take their time. The
intersection of experience, opportunity, **AND,**
confidence gets them to **WHEN.**



10:42 AM - 20 Feb 2018

26 Retweets 54 Likes



↻ 26



54



Connect to Context Demands

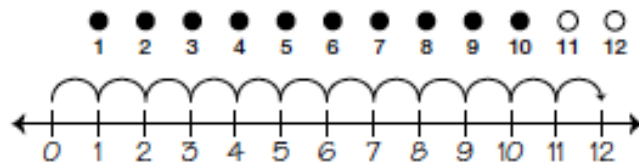
My Addition Strategies Menu for Larger Numbers

Counting All	Making Ten
Counting On	Using Ten
Another Strategy _____	Using Doubles

Addition Strategies Menu for the Facts

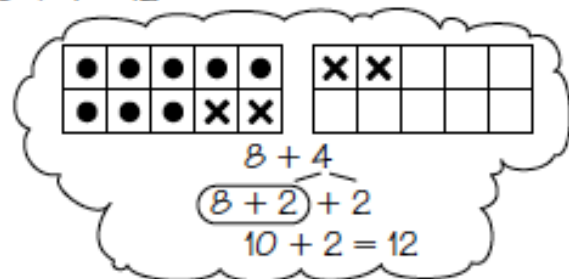
Counting All

$$10 + 2 = 12$$



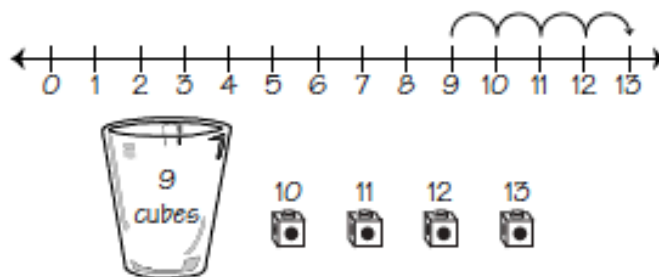
Making Ten

$$8 + 4 = 12$$



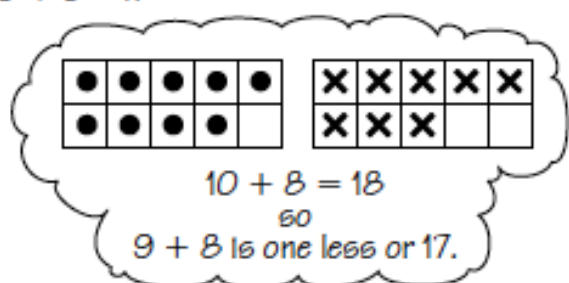
Counting On

$$9 + 4 = 13$$



Using Ten

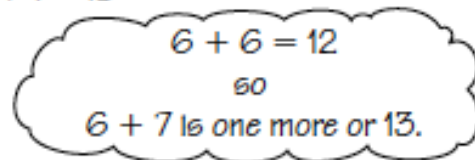
$$9 + 8 = 17$$



Another Strategy _____

Using Doubles

$$6 + 7 = 13$$



Name _____ Date _____

Addition Strategies Menu

Finding Friendly Numbers

$$138 + 29$$

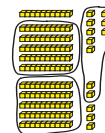
$140 + 30 = 170$
170 is a reasonable estimate.



Levi

Using Base-Ten Pieces

$$\begin{array}{r} 68 \\ + 55 \\ \hline 123 \end{array}$$



Trade 11 skinnies and 13 bits for
1 flat, 2 skinnies, and 3 bits

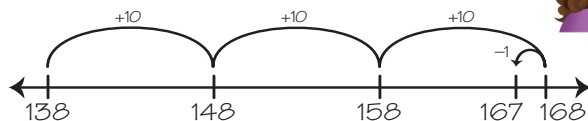


Peter

Counting On

$$138 + 29$$

$$138 + 30 - 1 = 167$$



Yolanda

137	138	139	140
147	148	149	150
157	158	159	160
167	168	169	170

138
↓
148
↓
158
↓
168
← 167

Using Expanded Form

$$\begin{array}{r} 68 = 60 + 8 \\ + 55 = 50 + 5 \\ \hline 110 + 13 = 123 \end{array}$$



Tara

Using All-Partials

$$\begin{array}{r} 68 \\ + 55 \\ \hline 110 \\ + 13 \\ \hline 123 \end{array}$$



Josh

Using the Compact Method

$$\begin{array}{r} 1 \\ 68 \\ + 55 \\ \hline 123 \end{array}$$



Julia

Use Menus to Prompt

Try a method you hardly ever choose.

Show Tanya's method using a number line instead.

Which strategy do you think is best for this?

Carlos is stuck. . .What strategy do you think will help him?

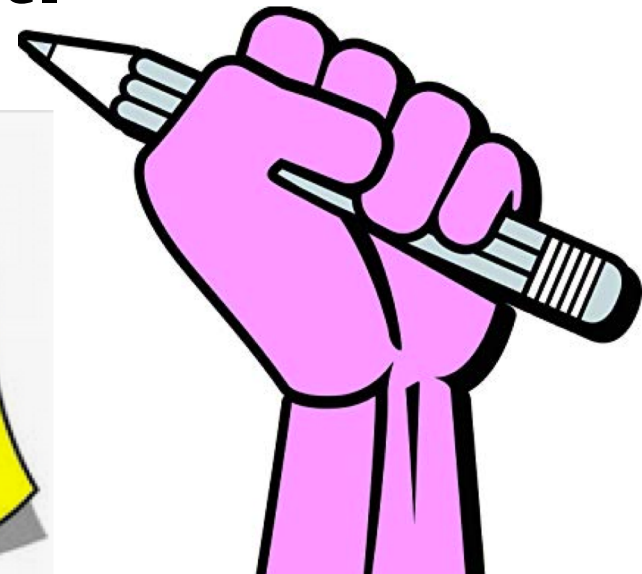
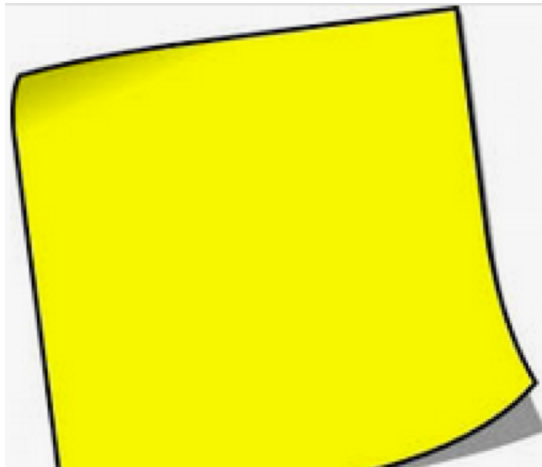
Is your strategy similar to . . . ?

To move students through the stages toward fluency, we can

- Patiently help students **construct number sense** by encouraging them to invent, share, and refine informal strategies.
- Encourage students to **look for patterns and relationships** and to use these discoveries **to construct reasoning strategies**.
- Encourage children **to build on what they already know**.
- **Assign rich tasks and practice wisely** to discover patterns and relationships and to make reasoning strategies more automatic – not on overly structured worksheets or by drilling isolated facts.
- Encourage students to **flexibly use a variety of strategies**.
- Provide opportunity to **engage in meaningful dialog** about learning: share, discuss, compare strategies.

Call to Action

- **Jot down a take away from today on a sticky note.**
- **Share with someone in the room.**
- **Post it on your way out.**



Keep Connecting!

Keep in touch.

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(A) $74 + 38 = 112$

$4 + 8 = 12$
 $70 + 30 = 100$
 $100 + 12 = 112$

(B) 74

$$\begin{array}{r} +38 \\ \hline 12 + 100 = 112 \end{array}$$

74
 $+38$
 $\hline 100$
 12
 112

(C) $74 + 38 = 112$

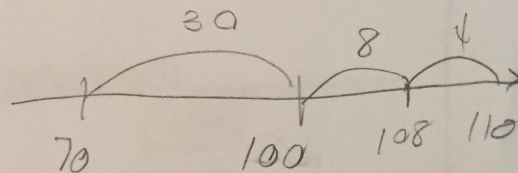
$74 + 30 = 104$
 $4 + 8 = 12$
 $104 + 8 = 112$

(D) $74 + 38 = 112$

$70 + 30 = 100$
 $100 + 8 = 108$
 $108 + 4 = 112$

(F) $74 + 38 = 112$

112



Design your own based on something you are seeing in your classroom.