Beautiful Explanations in the Primary Math Class

NCTM - San Diego

Thursday, April 4, 2019

Solana Ray and Jessica Balli

Plan for this session:

- Build collective understanding about the big shifts in mathematics expectations and how they impact our instruction in the primary grades.
- Discuss a few of our favorite (and FREE!) instructional strategies/activities that give students opportunities to talk/make arguments/explain their thinking in math class. (SMP 3)
- Look at student work and consider teacher feedback that might help students strengthen their explanations.

Who's in the room?

What does it mean to be good at math?

Reflect on this question and consider how you would respond as a:

- Student
- Parent
- Teacher

Be ready to share.

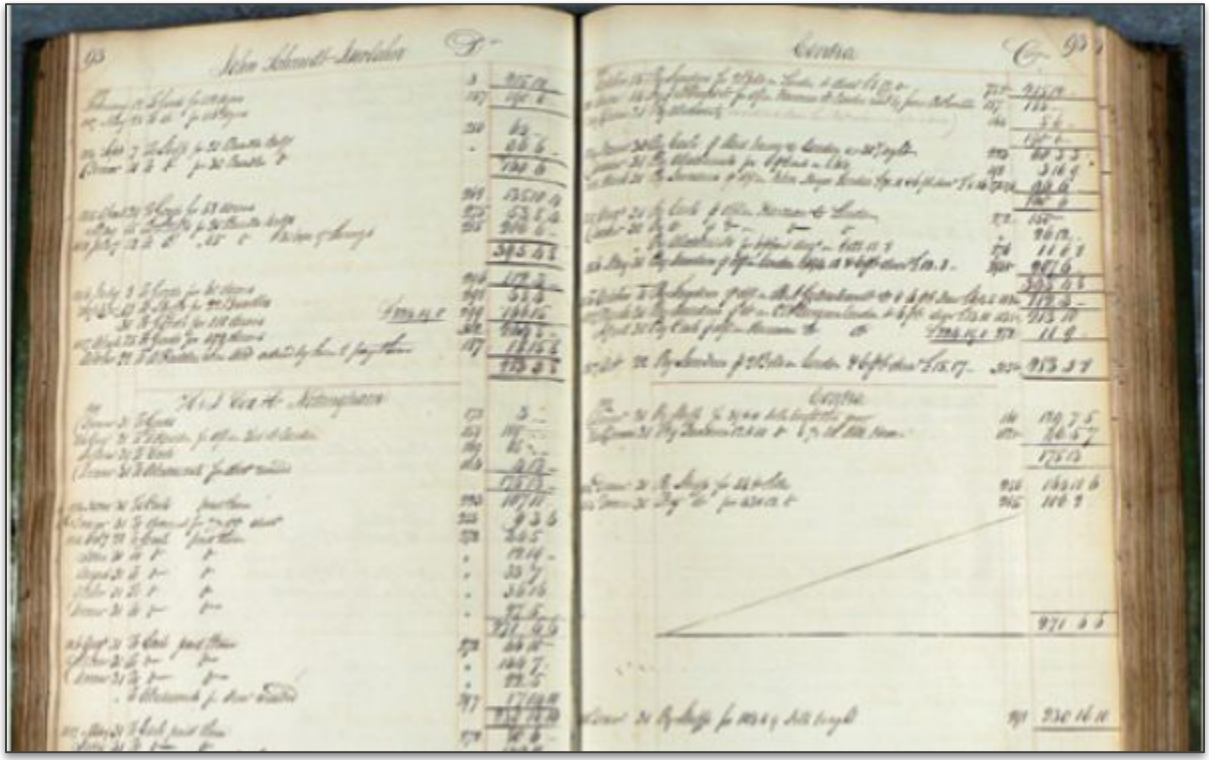
Big Shifts

Shifts in assessment, admissions and career and college expectations

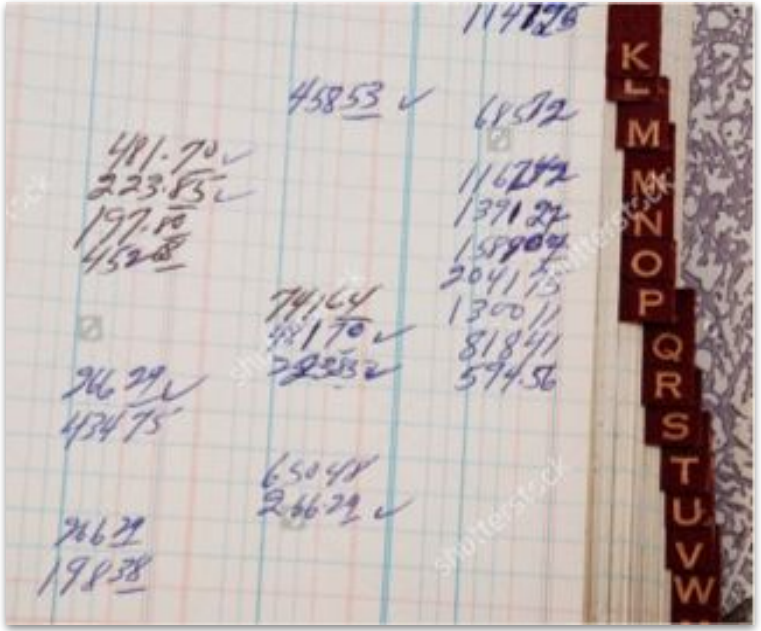
Question:

- Why did we teach mathematics the way we did?

Answer: We taught mathematics that way because we NEEDED it to be successful



But… times have changed.



Where did the shifts come from?

The changes in K-12 mathematics come from pressure from BUSINESSES and UNIVERSITIES

They want "Mathematics to be useful!"

Fortune 500: Most Valued Skills

	1970	1999
1	Writing	**Teamwork**
2	Computational Skills	**Problem Solving**
3	Reading Skills	**Interpersonal Skills**
4	Oral Communications	Oral Communications
5	Listening Skills	Listening Skills
6	Personal Career Development	Personal Career Development
7	Creative Thinking	Creative Thinking
8	Leadership	Leadership
9	Goal Setting / Motivation	Goal Setting / Motivation
10	**Teamwork**	Writing

[10] Cassel, R. N., and Kolstad, R. (1999). The critical job-skills requirements for the 21st century: Living and working with people. *J. Instructional Psychology*, 25(3), 176-180; Creativity in Action (1990). *Skills desired by Fortune 500 companies (in order of importance)*. Buffalo, NY: Creative Education Foundation.

Top 10 skills

in 2020	in 2015
1. Complex Problem Solving	1. Complex Problem Solving
2. Critical Thinking	2. Coordinating with Others
3. Creativity	3. People Management
4. People Management	4. Critical Thinking
5. Coordinating with Others	5. Negotiation
6. Emotional Intelligence	6. Quality Control
7. Judgment and Decision Making	7. Service Orientation
8. Service Orientation	8. Judgment and Decision Making
9. Negotiation	9. Active Listening
10. Cognitive Flexibility	10. Creativity

WORLD ECONOMIC FORUM

COMMITTED TO IMPROVING THE STATE OF THE WORLD

The Future of Jobs

Employment, Skills and Workforce Strategy for the Fourth Industrial Revolution

The Skills Companies Need Most in 2019 – And ... - LinkedIn Learning

https://learning.linkedin.com/.../top-skills/the-skills-companies-need-most-in-2019--and-...

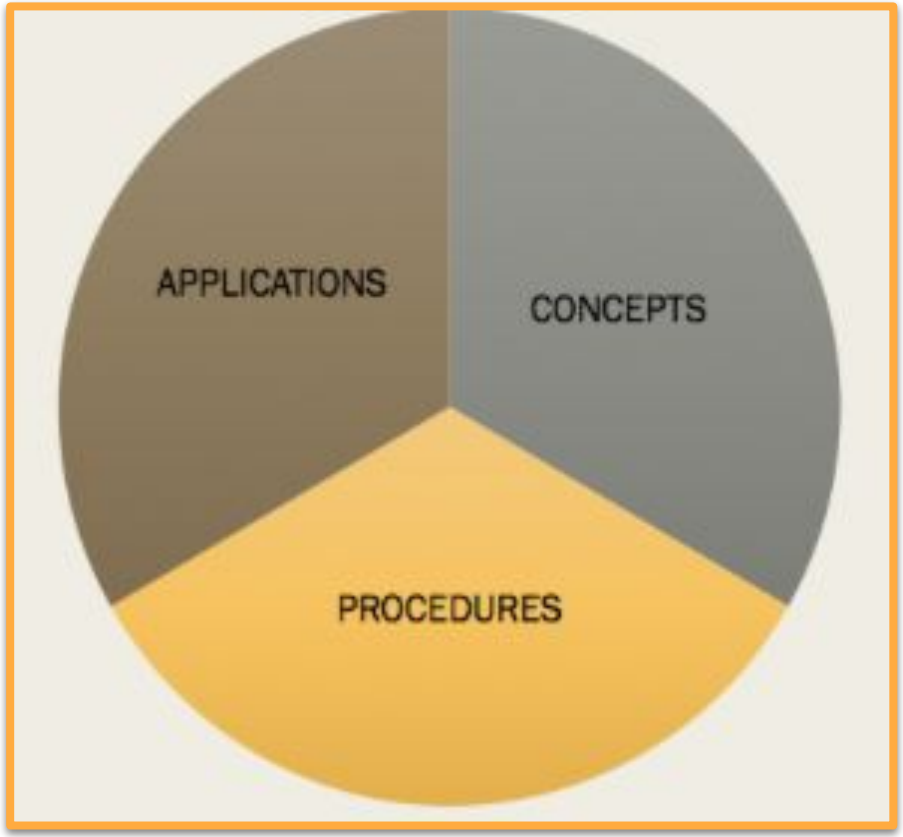
But, even candidates with exceptional hard skills need soft skills in order to stand out and succeed in any of the above roles. In fact, 57% of leaders say soft skills are more important than hard skills.

What are the soft skills companies are looking for most in 2019? They are:

The most in-demand soft skills

1. Creativity
2. Persuasion
3. Collaboration
4. Adaptability
5. Time Management

Topping the list of desirable soft skills is creativity. And that isn't surprising – organizations everywhere need people who can innovate and conceive fresh ideas and solutions.



Rigor in Mathematics

Students need to be able to predict, explain, communicate, and be flexible in non-routine environments

Look familiar?

20 Tony had a rope 8.35 meters long. He cut off 2.6 meters. How long was the piece of rope that was left?

A 5.65 meters
B 5.75 meters
C 6.65 meters
D 6.75 meters

21

$$\begin{array}{r} 39.06 \\ \times \quad 0.3 \\ \hline \end{array}$$

A 9.708
B 9.718
C 11.608
D 11.718

24 $15.12 \div 2.4 =$

A 0.513
B 0.63
C 5.13
D 6.3

25 $35{,}705 \div 37 =$

A 89
B 843
C 925
D 965

26 At a school, there are 704 desks to place into 22 classrooms. If the same number of desks is

California STAR test released items 5th Grade

SAT (2016)

It's About the Real World

Instead of testing you on every math topic there is, the SAT asks you to use the math that you'll rely on most in all sorts of situations. Questions on the Math Test are designed to mirror the problem solving and modeling you'll do in:

- College math, science, and social science courses
- The jobs that you hold
- Your personal life

For instance, to answer some questions you'll need to use several steps—because in the real world a single calculation is rarely enough to get the job done.

SAT (2016)

SAT | PSAT/NMSQT | PSAT 10 | Calculator: Permitted | Heart of Algebra

Aaron is staying at a hotel that charges $99.95 per night plus tax for a room. A tax of 8% is applied to the room rate, and an additional onetime untaxed fee of $5.00 is charged by the hotel. Which of the following represents Aaron's total charge, in dollars, for staying x nights?

Select an Answer

A. $(99.5 + 0.08x) + 5$

B. $1.08(99.95x) + 5$

C. $1.08(99.95x + 5)$

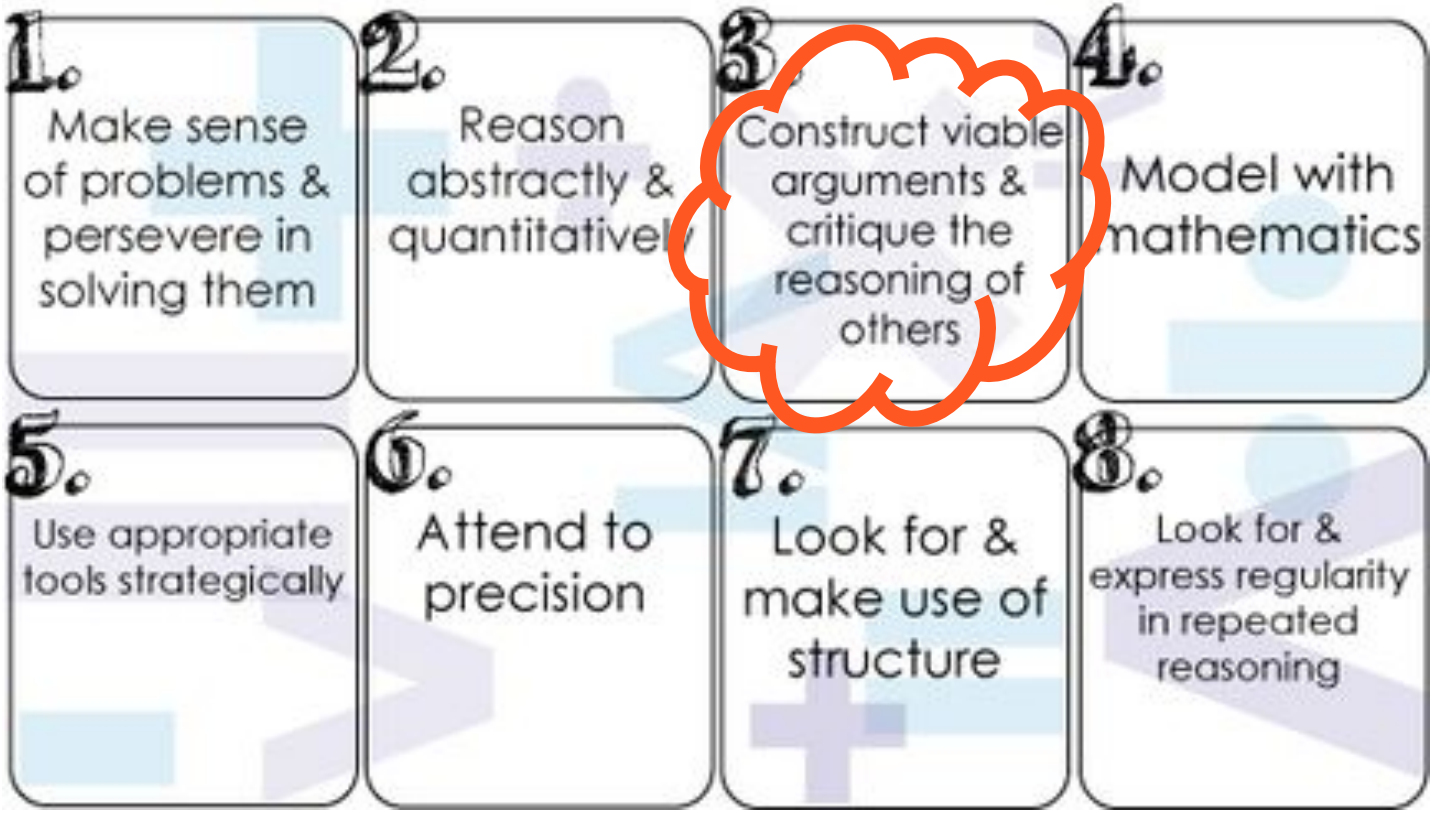
D. $1.08(99.95 + 5)x$

So, what do these big shifts mean for the primary classroom?

Beautiful Mathematical Explanations in the Primary Grades

How do we get there?

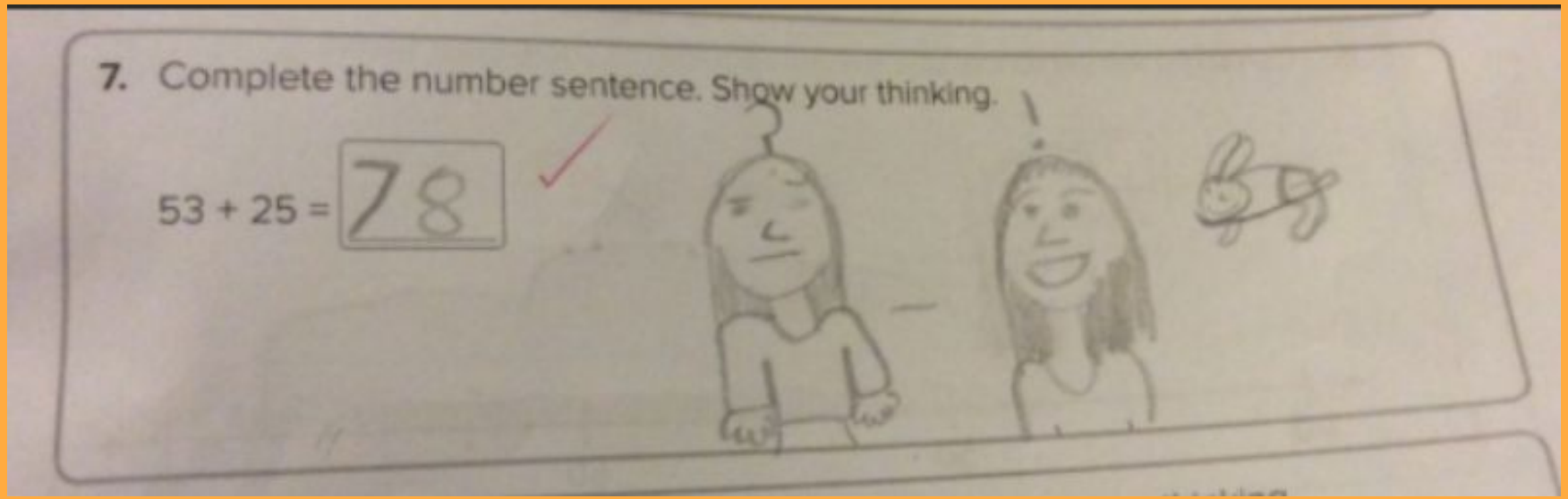
Standards for Mathematical Practice



How to identify an Aspen tree.

How do we get better at it?

1 Ask questions worth explaining.



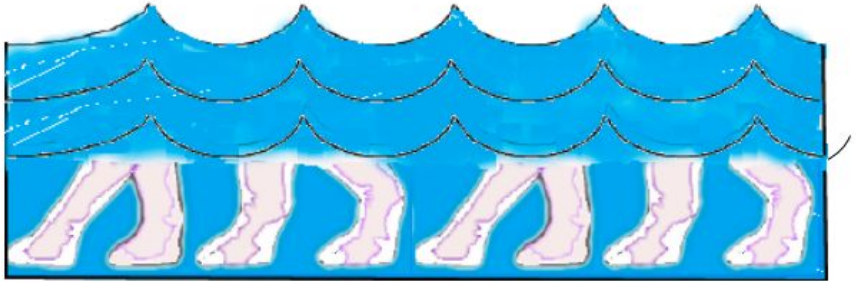
2

Give students opportunities to practice.

Digging Dinosaurs

Level A:

You are swimming under water in a lake and you see dinosaur feet in the water. You don't want to go to the surface in case they are not friendly dinosaurs. Below is a picture of what you see.



How many dinosaurs are standing in the lake?
Explain how you know. Use words and mathematical language to explain your solution.

Inside Mathematics
Problem of the Month

3 Push for better explanations.

What does it look like in K-2?

INSTRUCTIONAL STRATEGIES

(A few of our favorites)

- All of these can (but don't have to) be connected to specific math content you're teaching
- They give students opportunities to practice making arguments
- They encourage students to learn from and listen to each other
- They provide access for a variety of learners

Which One Doesn't Belong?

By Christopher Danielson

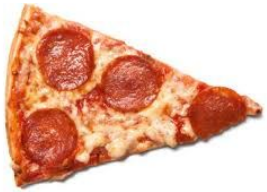
"Danielson's book reveals the wonder and freedom of expression that many children don't often experience in mathematics. ***A single, simple question puts children in a position to speak mathematically even at early ages. Ask students of all ages 'Which one doesn't belong?' and revel in the reasoning and conversation that results."*** - Dan Meyer

Rules for WODB

- Look at the 4 images. 
- Think to yourself 

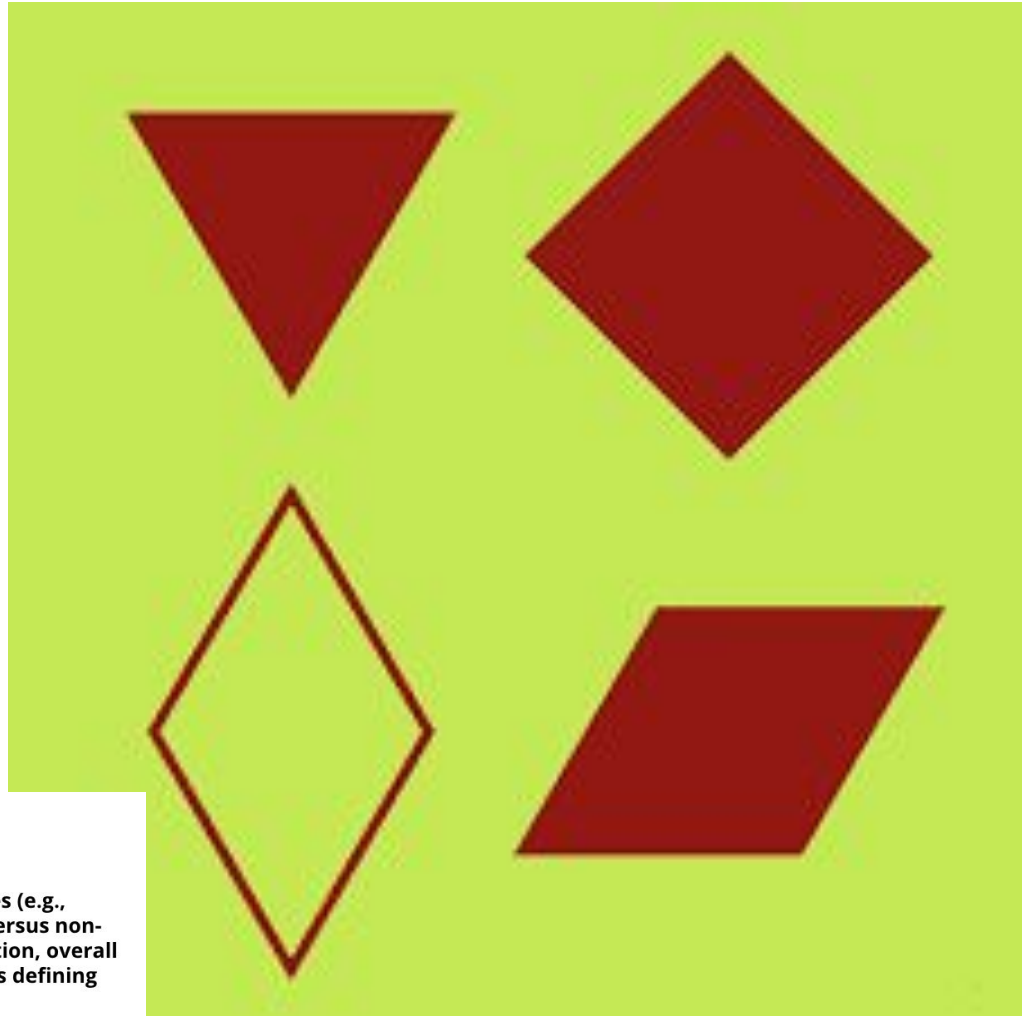
  “Which one seems different from the others and why?” Make sure you test your theory to make sure that your claim holds true.
- State your claim and share your reasoning. 
- Listen to the claims of others.  What are they paying attention to? Do you agree?


STRAWBERRY
& BANANA



“Because all of their answers are right answers, students naturally shift their focus to justifications and arguments based on the shapes’ geometric properties.”

-Christopher Danielson



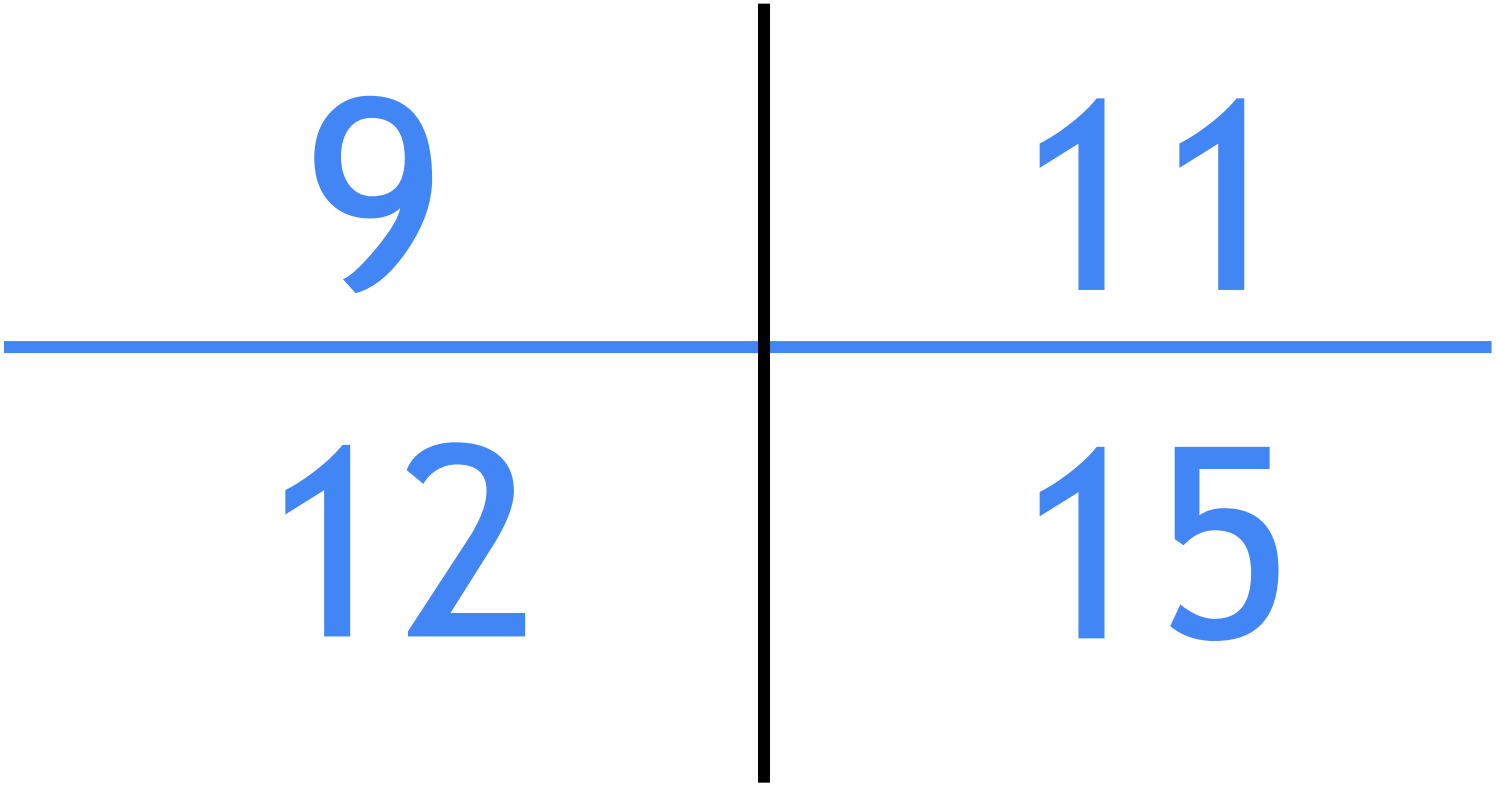
1.G.A.1

Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size) ; build and draw shapes to possess defining attributes.

“The_____ doesn’t belong because_____.”

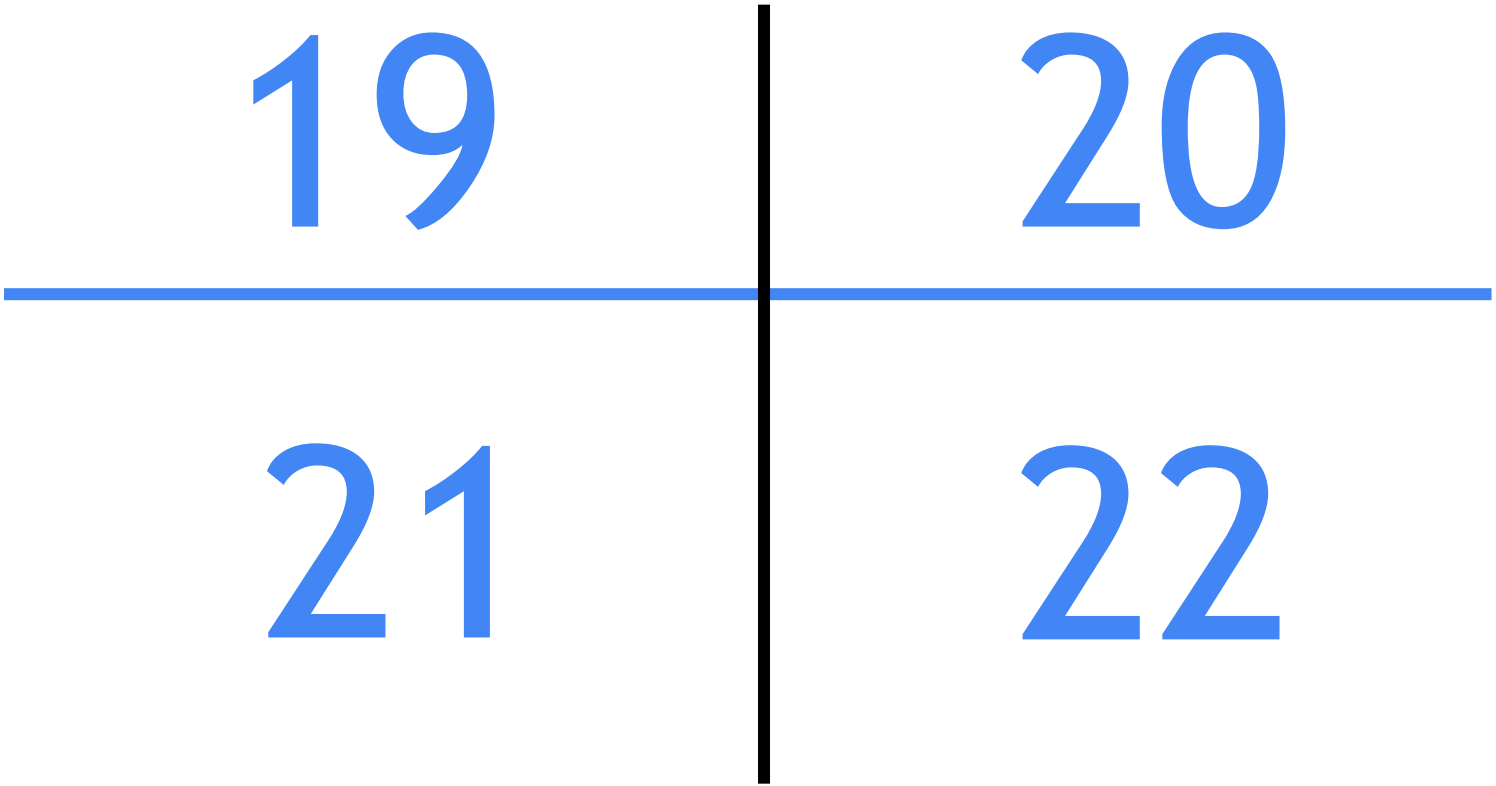
“When ____ said that _____ did not belong, he/she was focusing on _____.”

With a partner, come up with a claim for why each of these numbers “doesn’t belong”. Be sure to have a convincing reason to back it up!



First Grade Responses

- “The 9 doesn’t belong because it is the only one that doesn’t have another number, like since it is below 10.”
- “The 9 is the only one you can count on your fingers. The other ones you need to use your friends fingers too.”
- “The 12 doesn’t because it is the only one you can skip count by twos to get there.” (similar reasoning with 5s and 15)
- “The 11 because it is the only one with the same number in the front and in the back.”
- “The 15 because it is the only one that is facing to the right.”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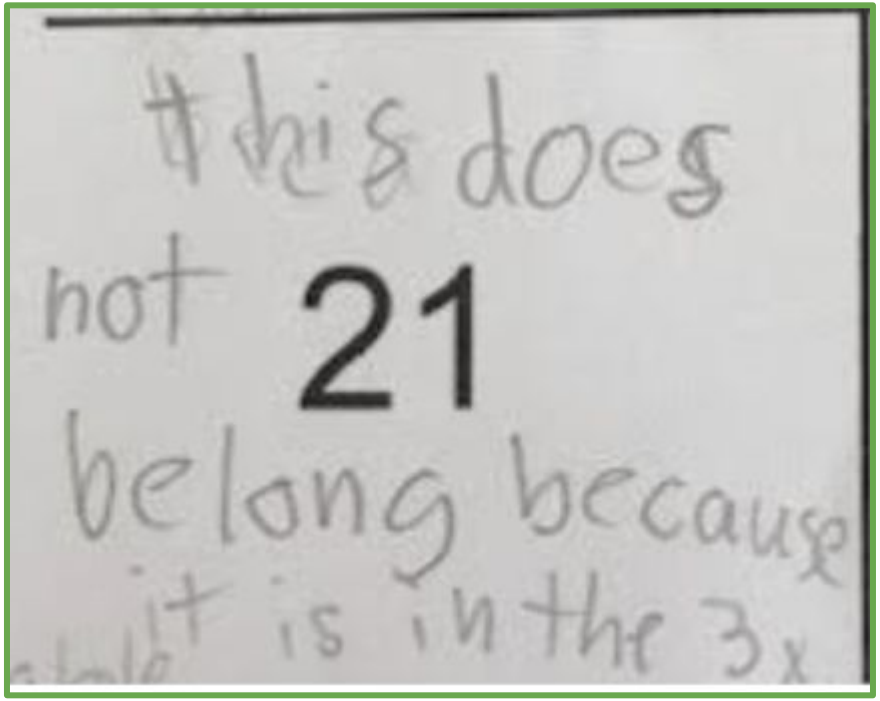
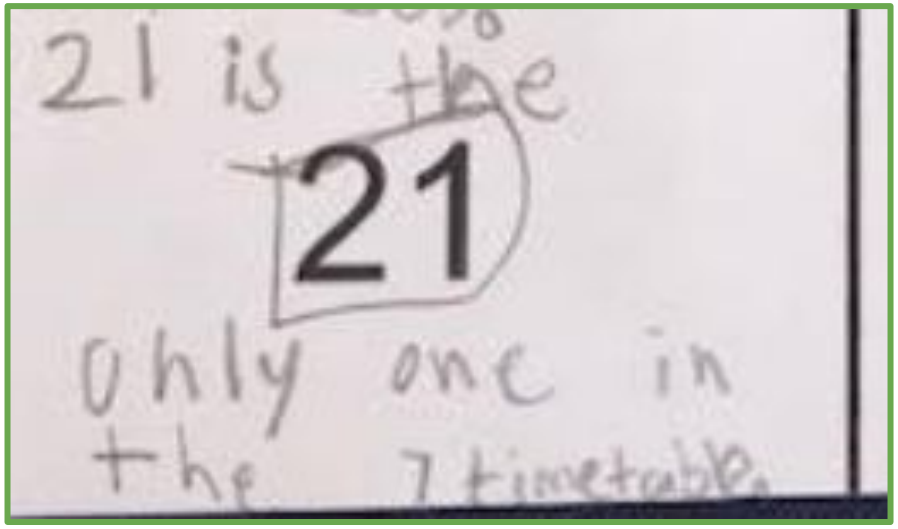
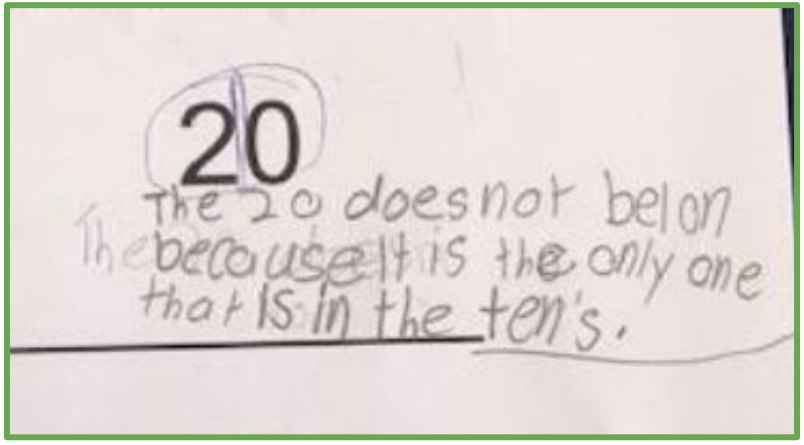


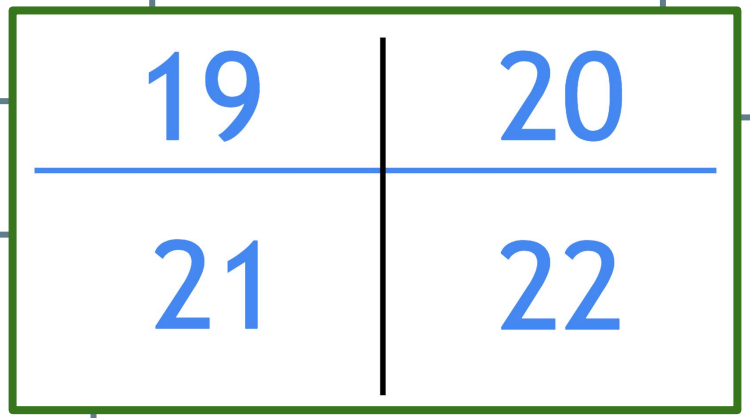




A beautiful and authentic discovery of the meaning of “prime.”

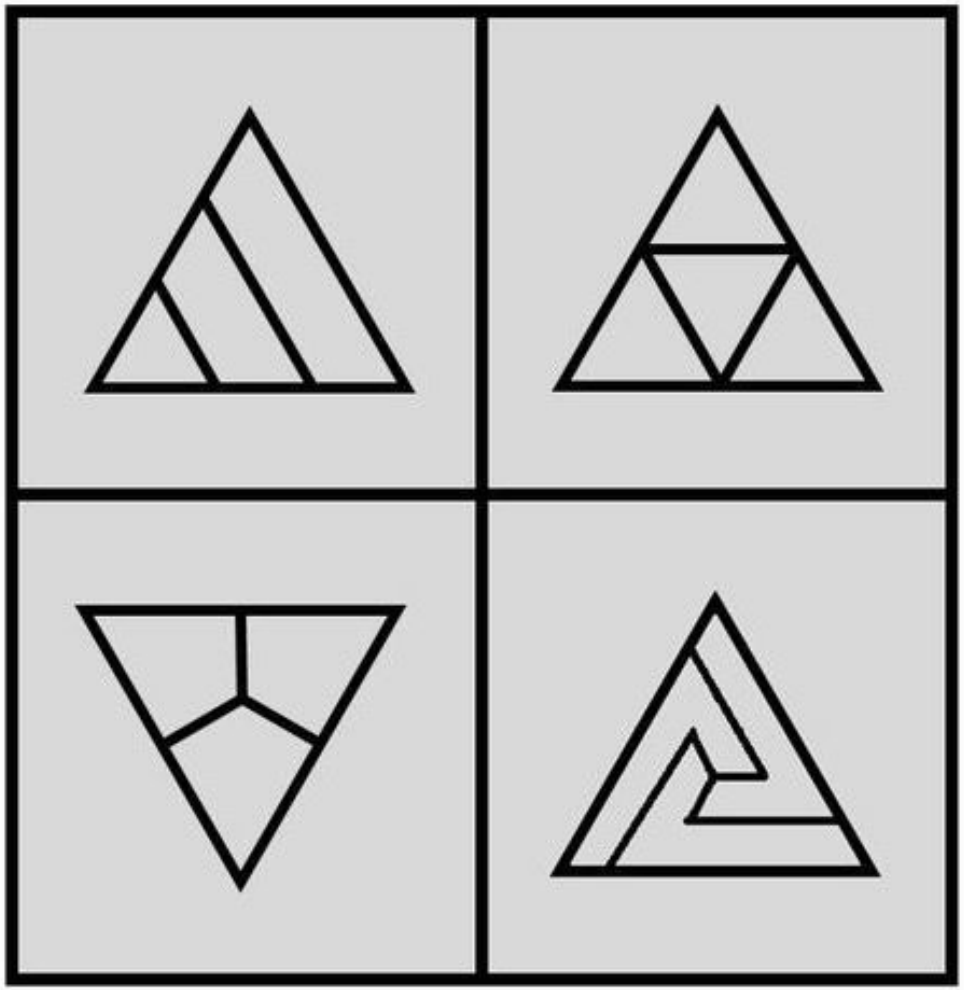
“The 19 doesn’t belong because it doesn’t have a 2 on it.”

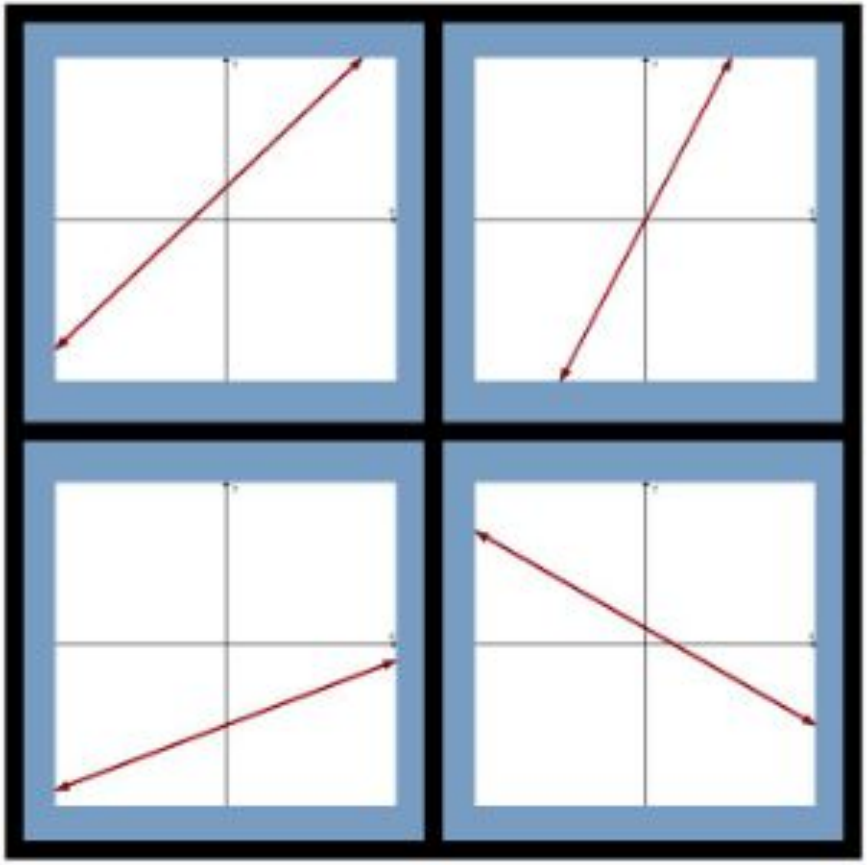
“The 20 doesn’t belong because it is the only one that has a zero.”



“The 21 doesn’t belong because it the only one that if you switch the digits it is a 12.”

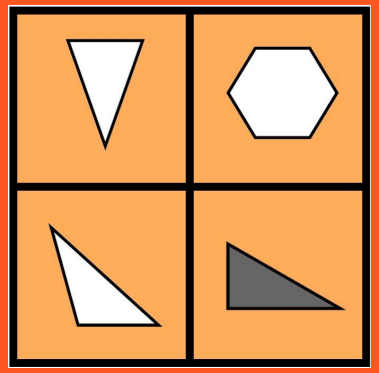
What kind of feedback could we give here to prioritize the mathematics?

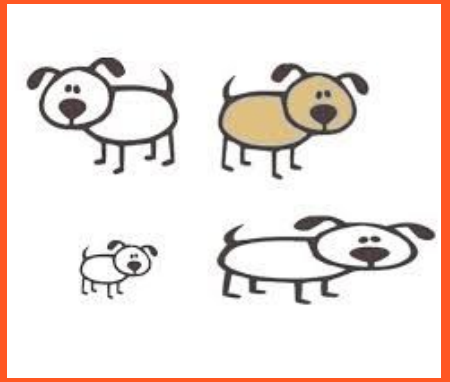




The WODB Instructional Routine:

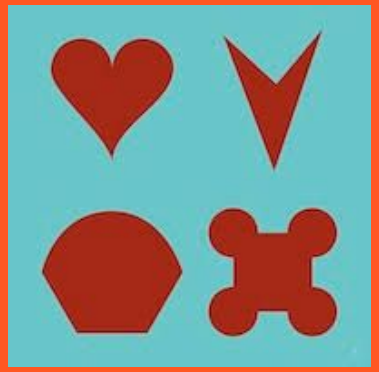
- Can, but does not have to be connected to content
- Provides access for a variety of learners
- Includes quiet think time and talk time
- Multiple correct answers
- A focus on explanation
- An invitation to see things in new ways
- Opportunities to talk about each other's thinking, respectfully challenge claims and revise them





Want more?

wodb.ca



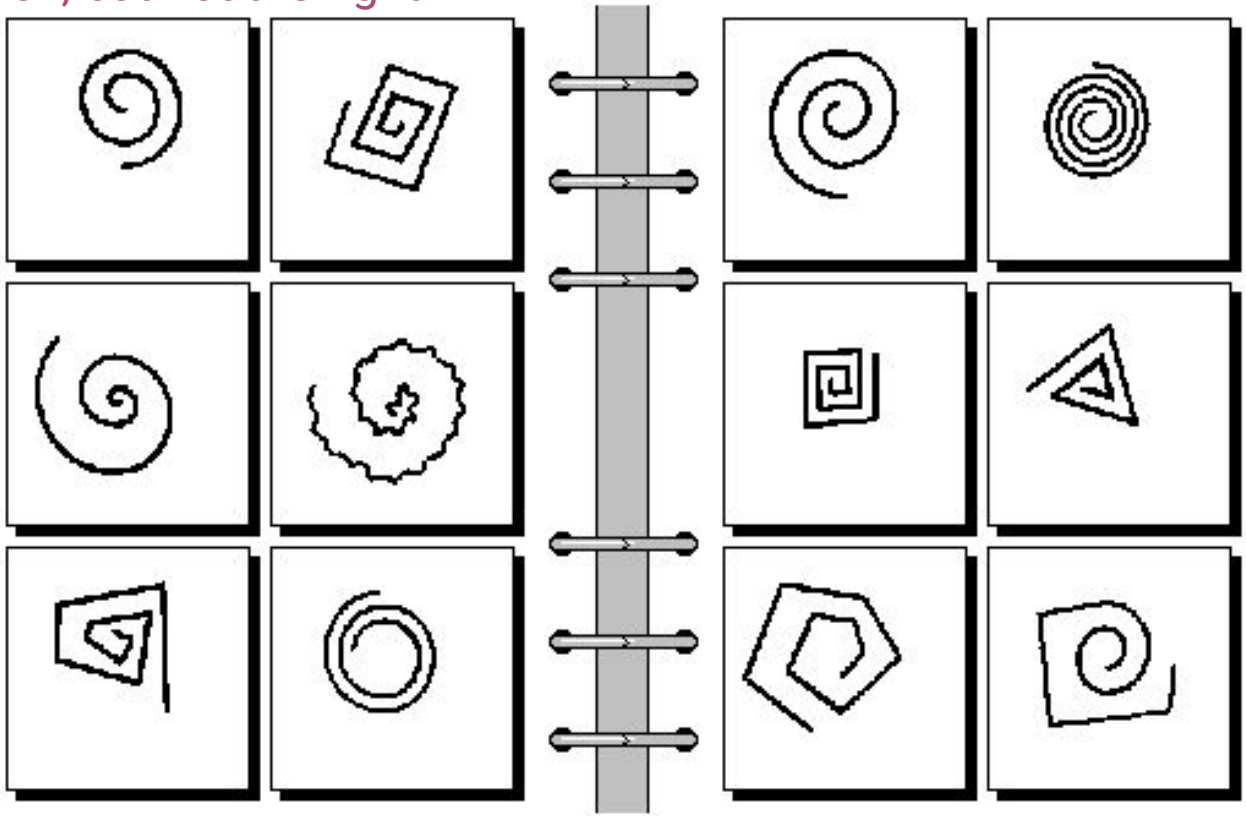
Bongard Problems

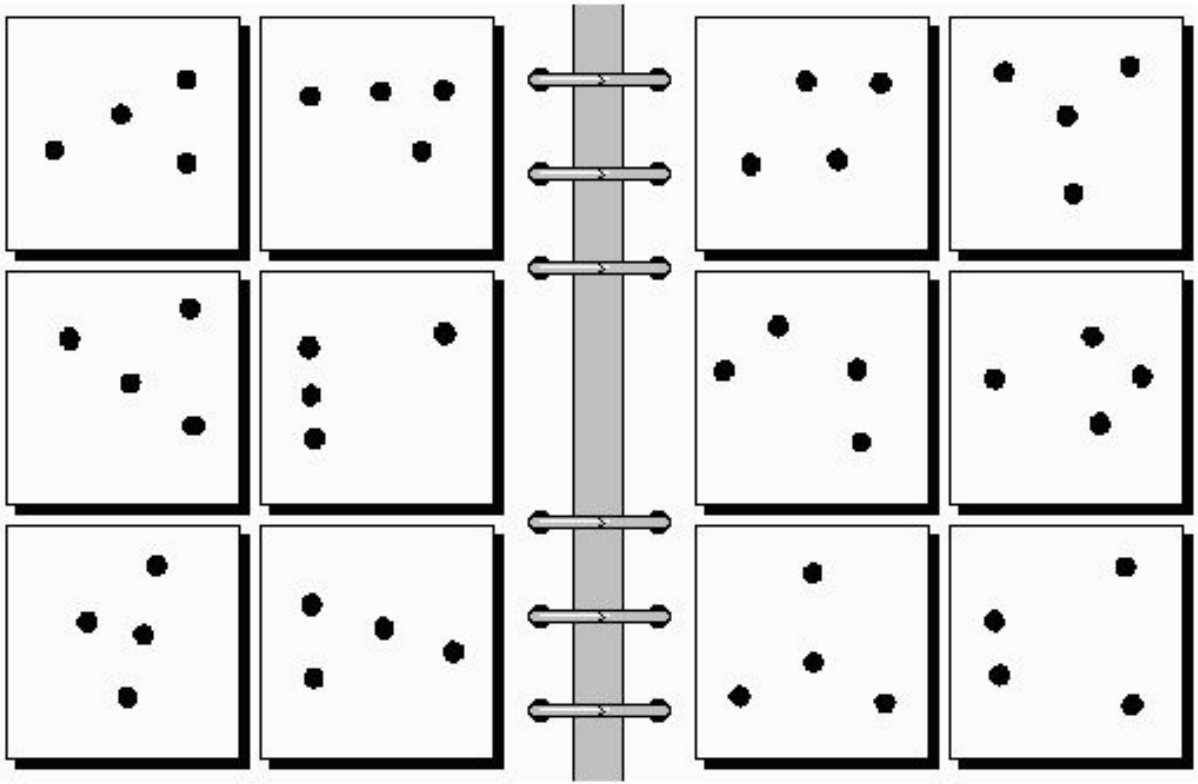
Bongard Problems:

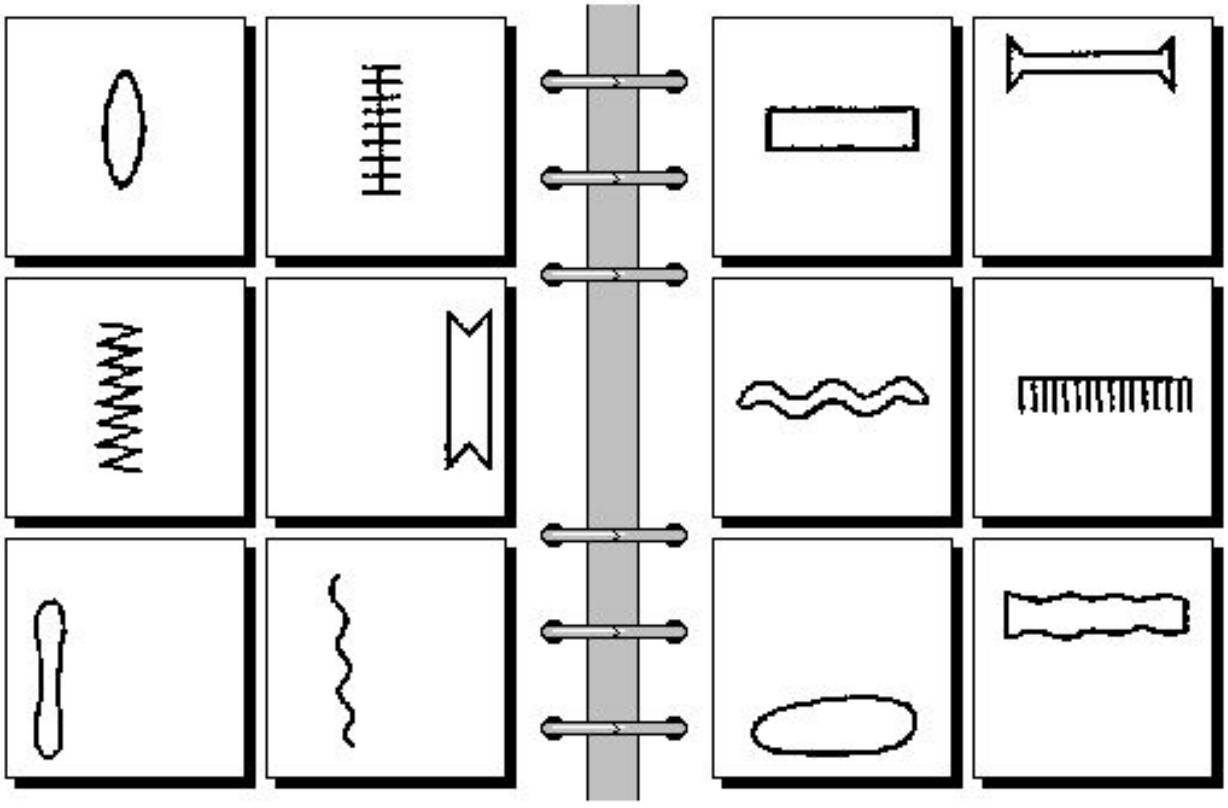
Come up with a rule that holds true of all of the images on the left side of the notebook, but not the right.

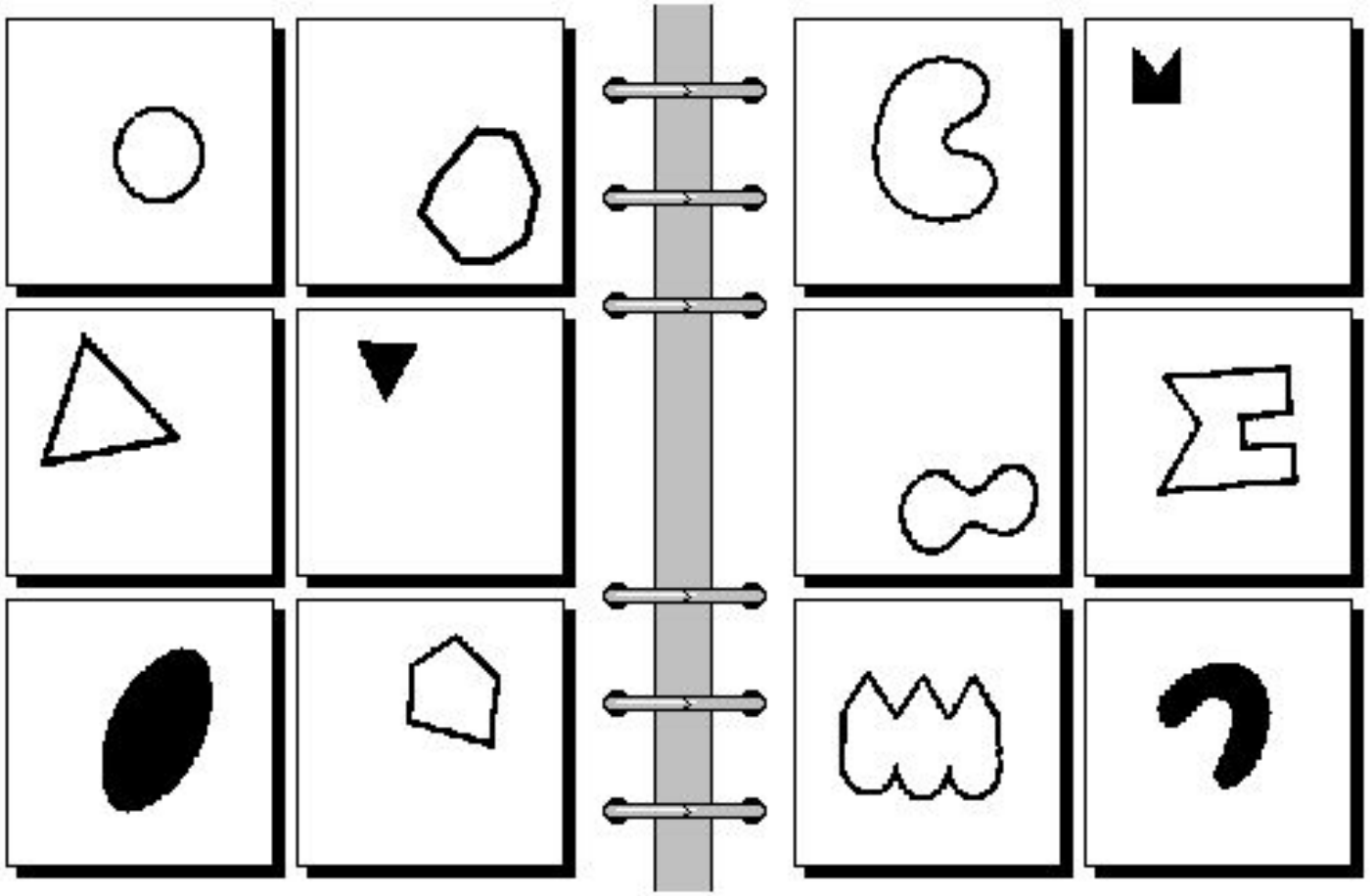
Bongard Problems:

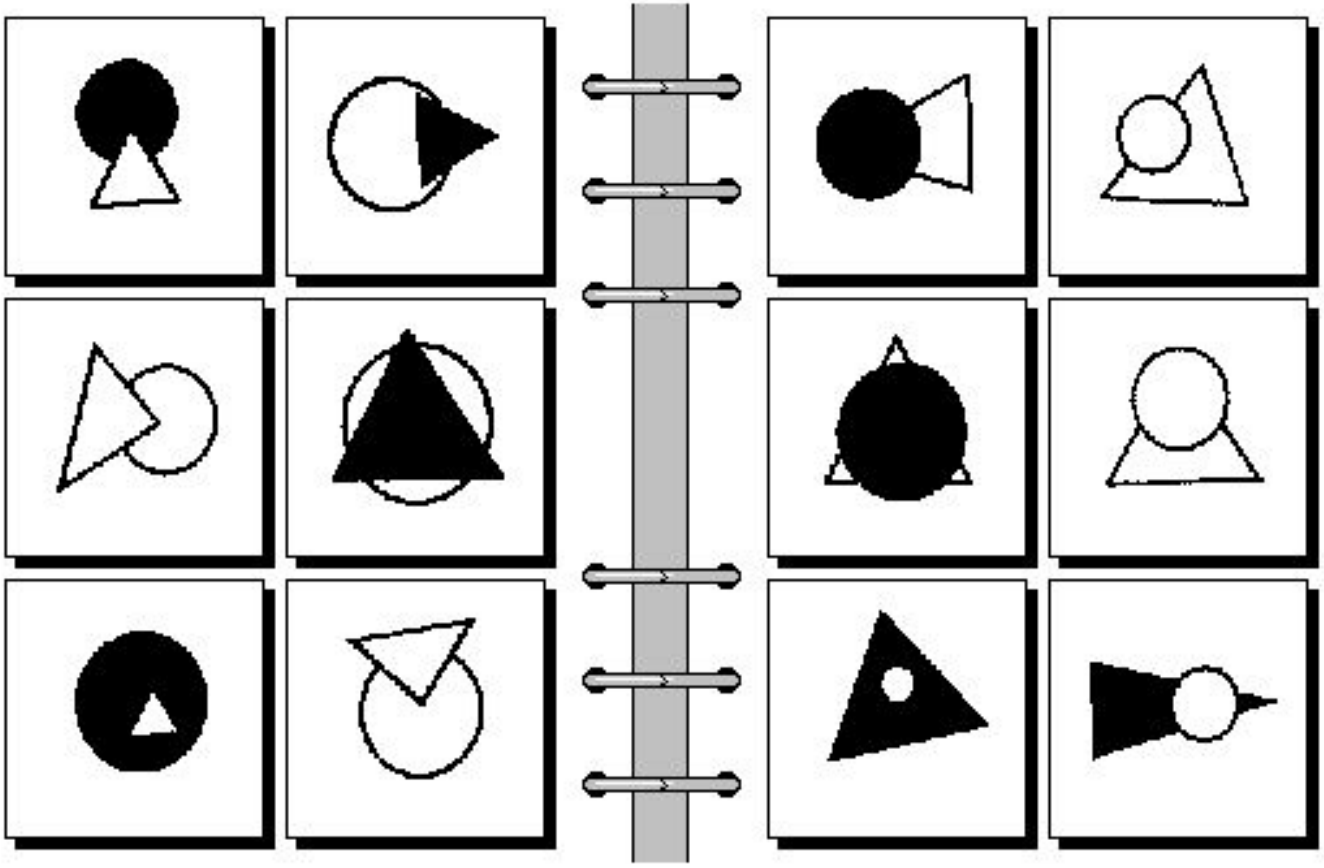
Come up with a rule that holds true of all of the images on the left side of the notebook, but not the right.

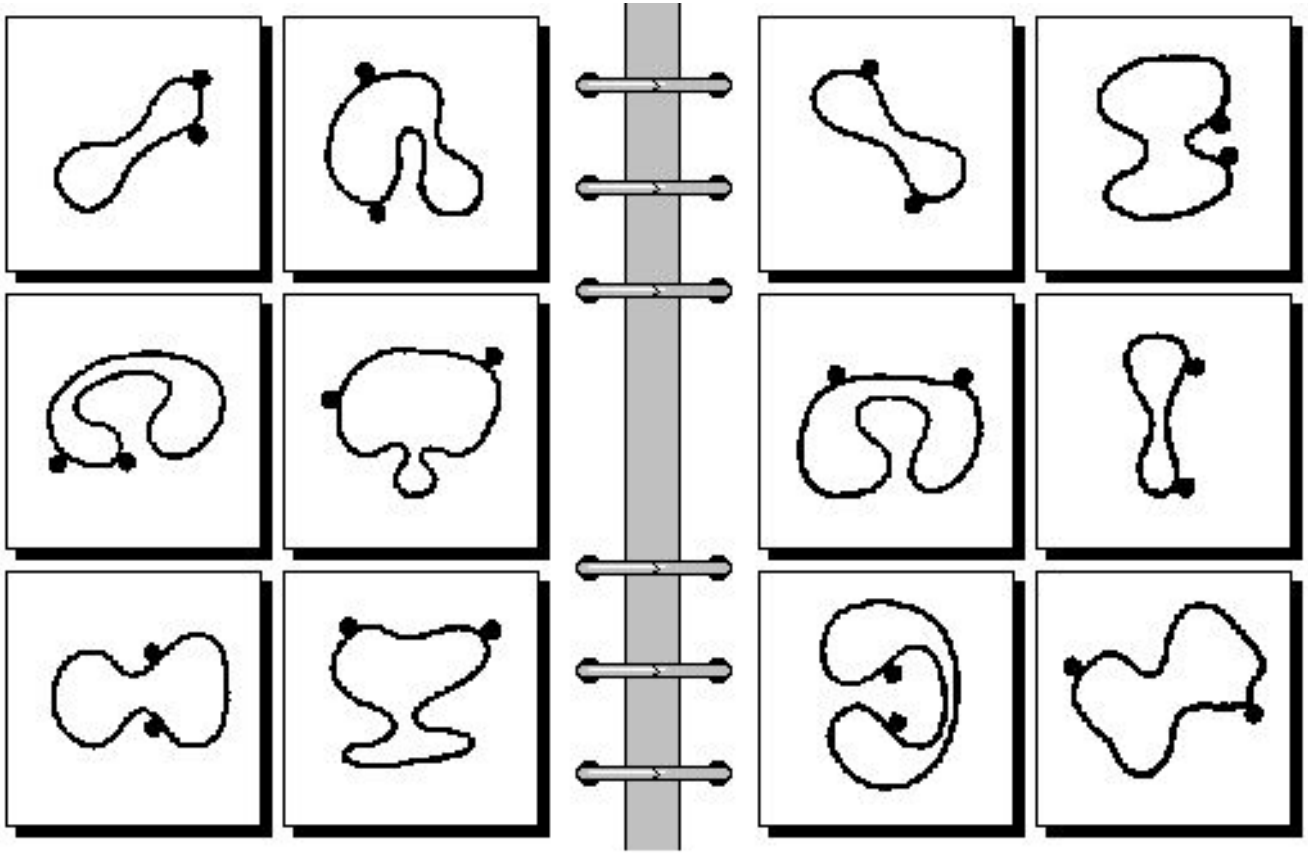


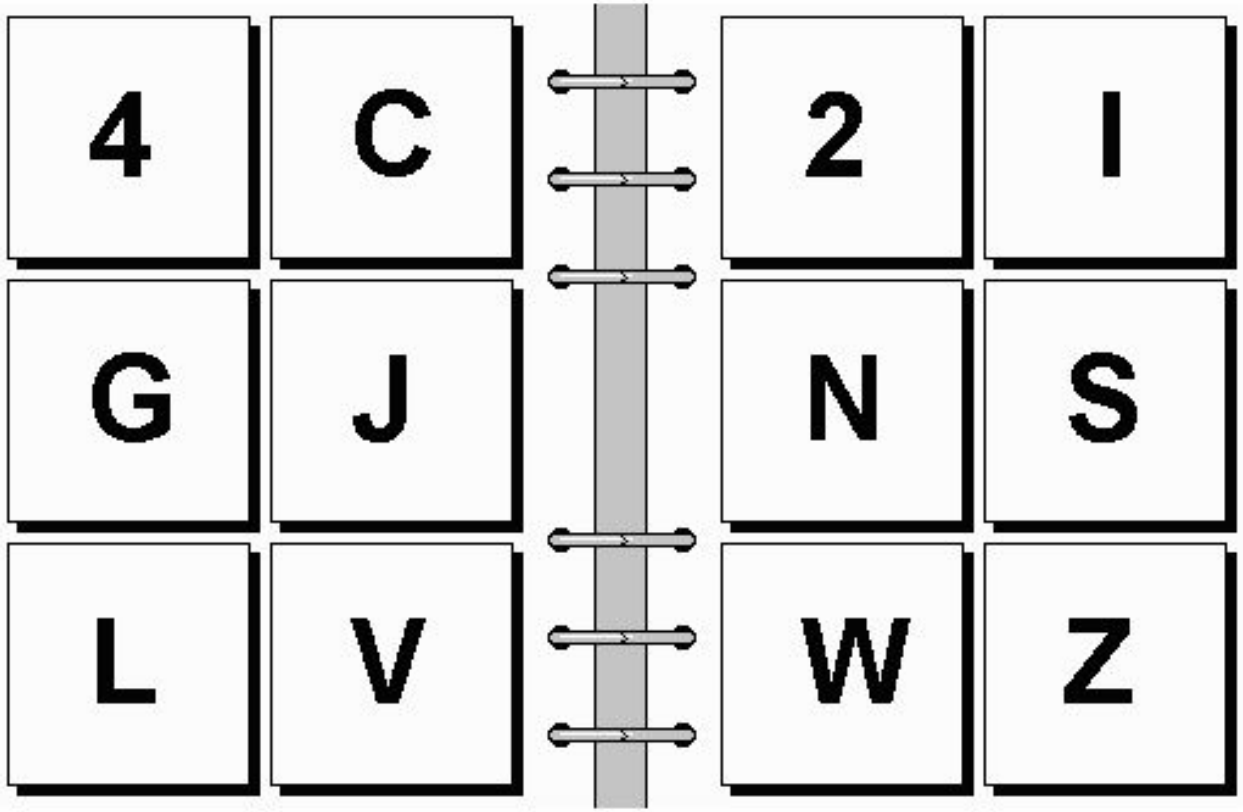








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Estimation

180

ESTIMATION 180

Building number sense one day at a time.

What is Mr. Stadel's height?

Day 1

What is Mrs. Stadel's height?

Day 2

What is my son's height?

Day 3

What is the height of the lamppost?

Day 4

What is Mr. Stadel's height?

Make an estim

* Required

What's too LOW? *

What's too HIGH? *

Your estimate. *

Your reasoning. *
Do better than "I guessed."

Your name. *

How many pieces of red licorice are in my hand?

Answer

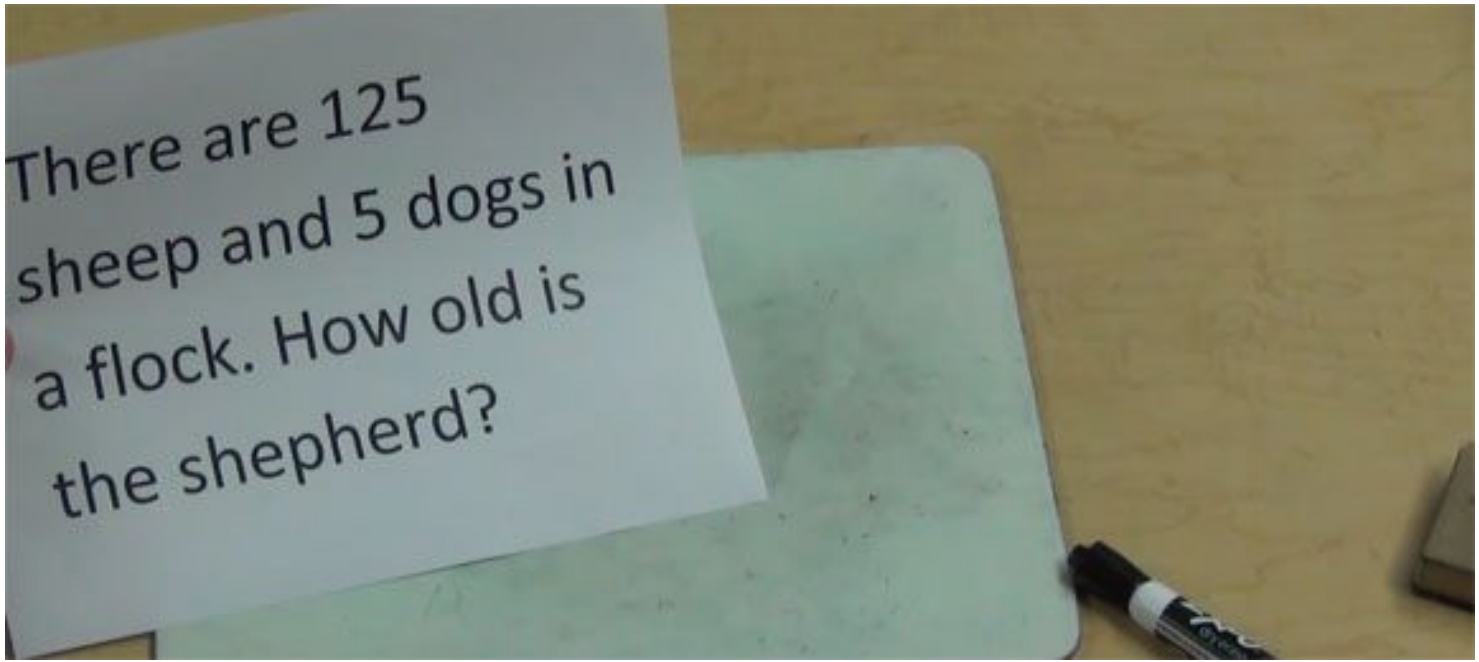
Video

3 Act Tasks

Goal of a 3 Act Task

To authentically engage students in making sense of a situation, asking questions, requesting relevant information, modeling with mathematics, solving interesting, real life problems and explaining their solutions.

What do we mean by “make sense”? SMP 1



RobertKaplinsky.com

3 Act Task - The Breakdown

Act 1 - An engaging and perplexing context, a video or photo hook to get students interested.

Act 2 - Students notice, wonder, pose questions, estimate and request information to solve the problem.

Act 3 - Solution discussion and THE REVEAL!

3 Act Task - Humpty Dumpty from Gfletchy.com

Cape Town, South Africa - Grades 1 and 2

ACT ONE - Notice and Wonder

Your only job now is to watch the video.

(no talking yet - just 

)

Be prepared to share something that you noticed.

VIDEO

What did you notice?

What did you see?

“I noticed.....”

OR

“One thing I saw was...

What do you wonder about?

What are some mathematical questions we can ask?

The Class Question:

How many eggs are broken?

ACT TWO - Request information

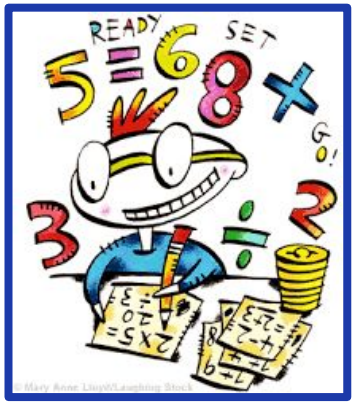
To solve the problem, if would be helpful to know _____.

How many eggs were in the carton to begin with?

There were 9 eggs in the carton to begin.

How many eggs did NOT break?

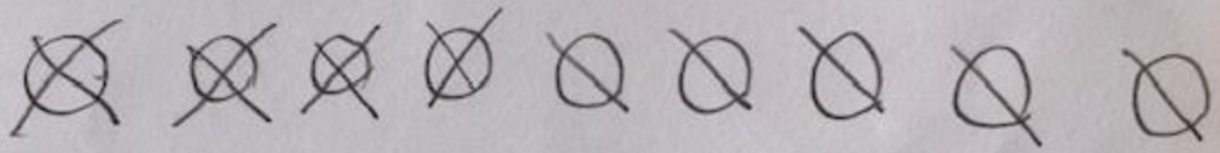
ACT THREE - Solve and Share



How many eggs are broken?

Explain how you figured it out.

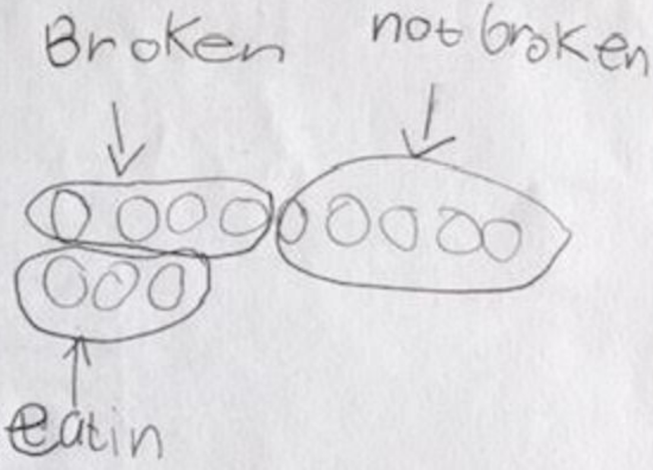
There wor 9 egg
in the carten.

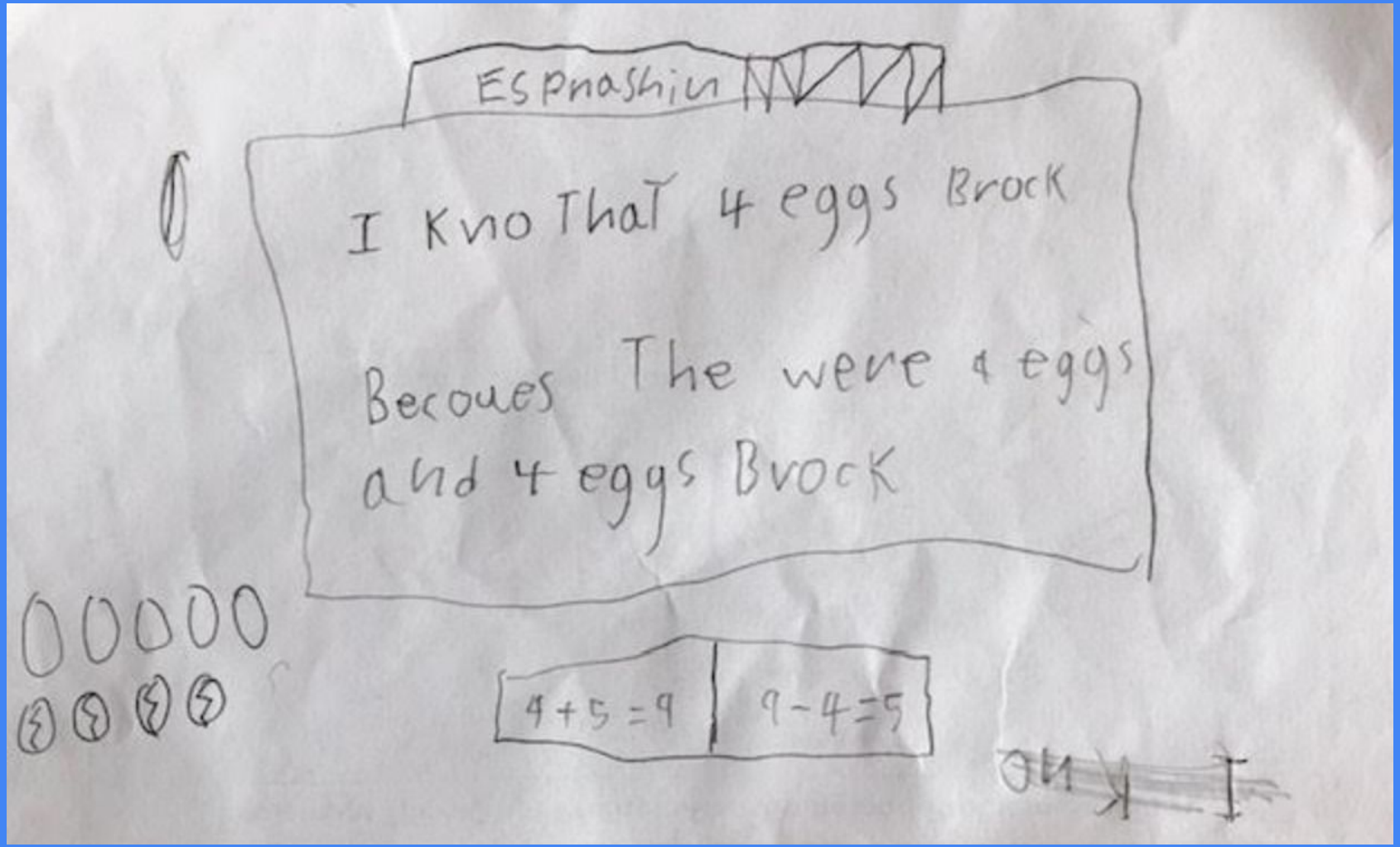


9-4=5

5

I Know beacase 5[illegible]4 = 9 and So 4 did brack and five didnotbrack




Espnashin
I Kno That 4 eggs Brock
Becoues The were 4 eggs
and 4 eggs Brock
4+5=9
9-4=5

ise There were 9 eggs in the carton to begin.

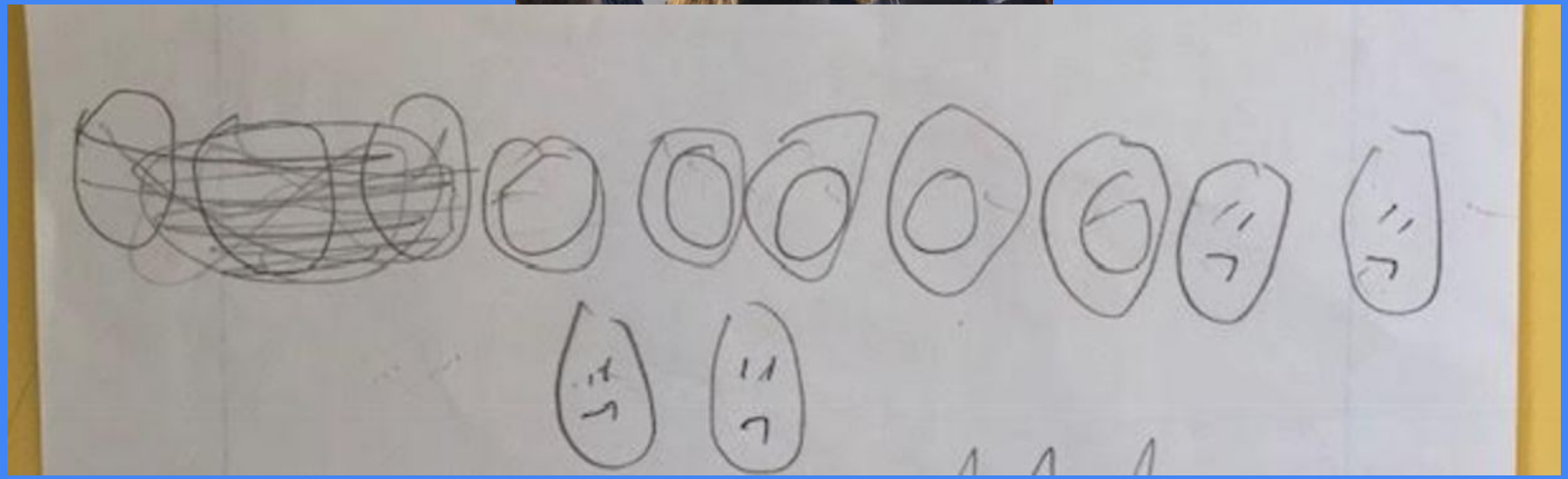
000005 000000000P

SmaST
0000
4

P=4-5

I We started with nine and five did not break so 5+4=9 so 4 left.


There were 9 eggs in
the carton to begin.

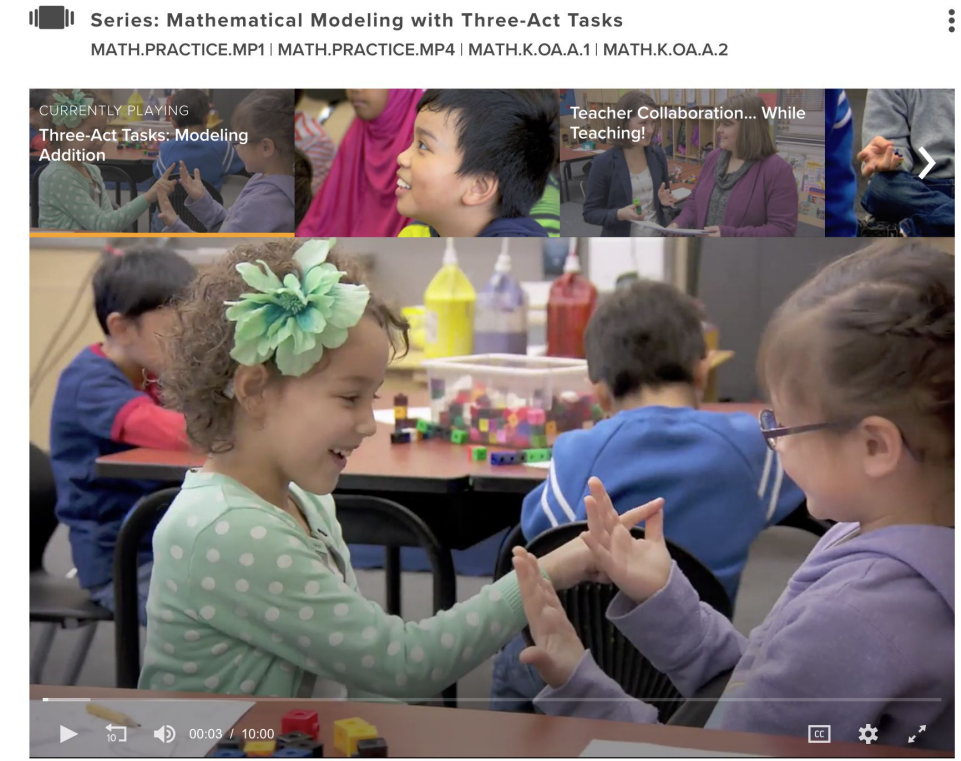



NUTRITION FACTS
Not a significant source of Dietary Fiber or Sugars. Percent Daily Values (DV) are based on a 2,000 calorie diet.

Features of a 3 Act Task

- Whole group math activity
- Low floor, access for everyone
- Engaging, interesting and authentic context
- Student agency - Students ask mathematical questions
- Students identify and request relevant information, develop a strategy to solve the problem
- Students share solutions, ask for clarification on others' explanations.

Model 3 Act Lesson - Kindergarten - theteachingchannel.org



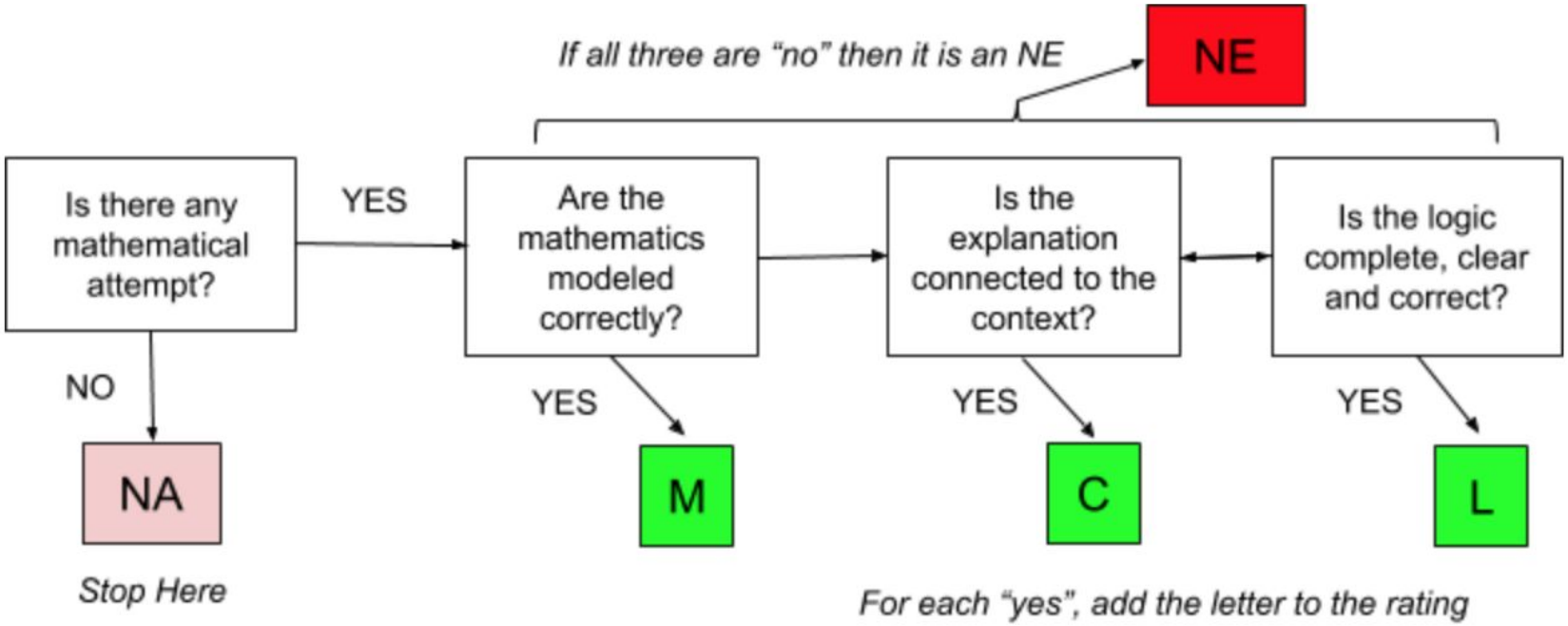
Primary Performance Tasks

How do we know if our students are getting better at this?

What are we looking for in a beautiful explanation?

- Make a claim
- Provide context (units, labels)
- Include math to support your claim
- Write (or say) a logically complete thought or statement

Analysis Flowchart:



Features of our PTs

- Low stakes, formative
- Short, 1 or 2 questions
- Securely held content
- Aimed to inform instruction
- Students have opportunities to get feedback and revise

1st Grade

Each garden box is 10 units long.

1 unit	1 unit	1 unit	1 unit	1 unit	1 unit	1 unit	1 unit	1 unit	1 unit

There are 2 different kinds of vegetables the students can c[illegible]

Carrots  need 1 unit of space to grow.

Broccoli  needs 2 units of space to grow.

Randy's Garden

									
1 unit	1 unit	1 unit	1 unit	1 unit	1 unit	1 unit	1 unit	1 unit	1 unit

Dina's Garden

									
1 unit	1 unit	1 unit	1 unit	1 unit	1 unit	1 unit	1 unit	1 unit	1 unit

Co-Developed by:

Antelope Valley 1st Grade Team,

Los Angeles County Office of Education,

Stanford Center for Assessment, Learning and Equity (SCALE)

and

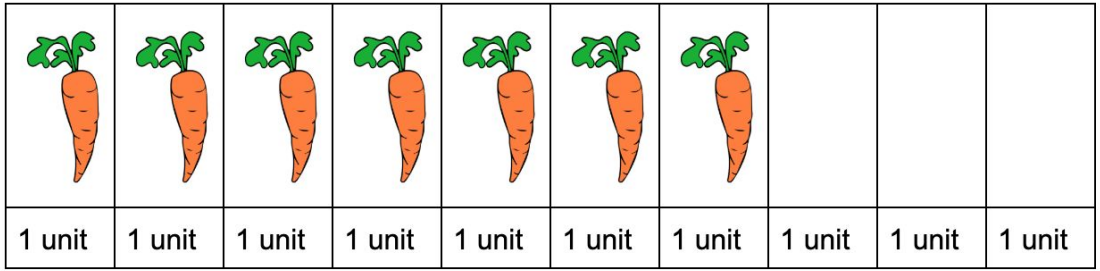
Envision Learning Partners (ELP)

Randy claims he can plant 2 broccoli plants in the rest of his garden.

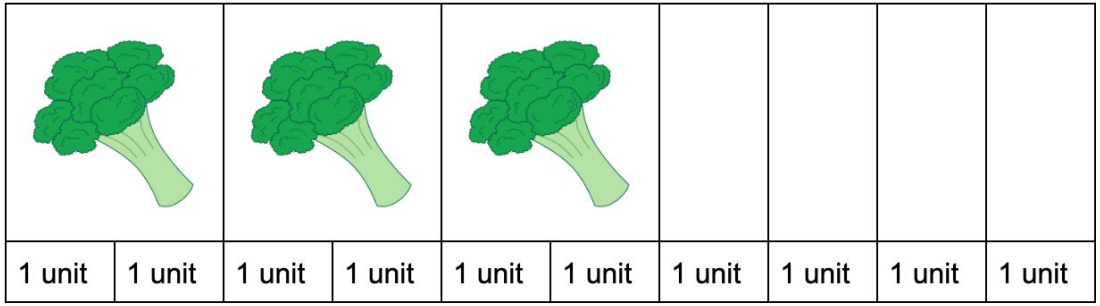
Do you agree with his claim?

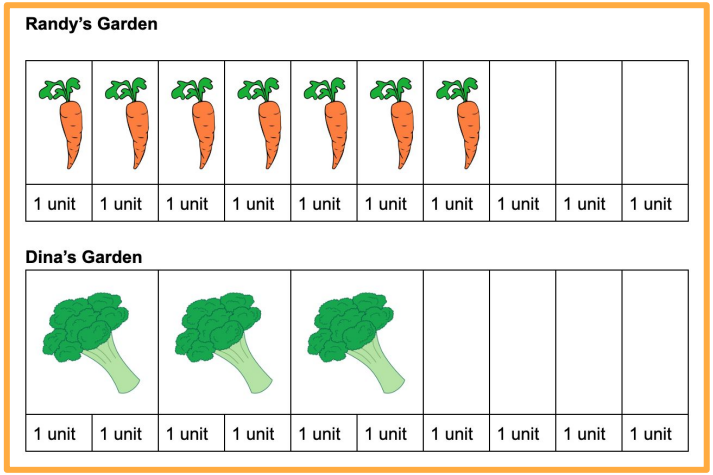
Explain to Randy why you agree or disagree.

Randy's Garden



Dina's Garden





"No because you need 4 and there are only 3."

"No because 2+2=4 and 4>3."

"No because 4 is more than 3."

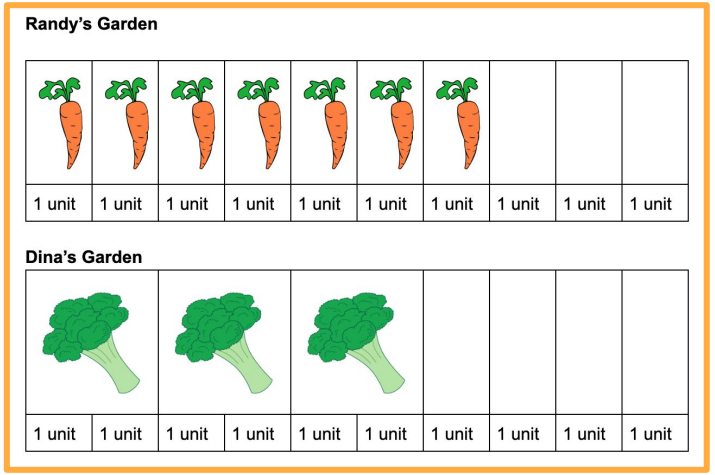
Claim?

Context?

Math to support?

Logical conclusion?

How can we help here?



Claim?

Context?

Math to support?

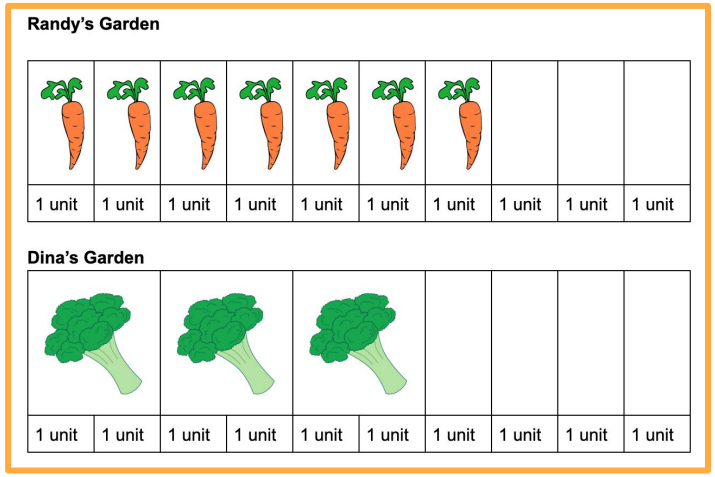
Logical conclusion?

"No because those are too big and they would take up 4 spaces."

"No. Because broccoli = 2 and 2+2=4. Broccoli needs 4 places."

"No because he had 10 spots and filled 7 with carrots so there are only 3 more."

How can we help here?



"No since broccoli are really big and they take up two spots. He could put one broc and 1 carrot but if he wanted to put two broccoli he would have to take out one of those other carrots that he out. Then they can fit."

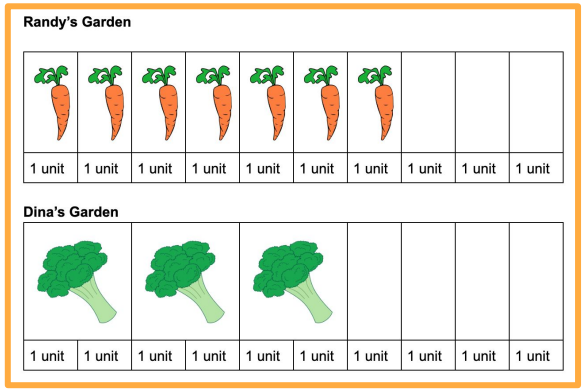
Claim?

Context?

Math to support?

Logical conclusion?

"Right now, no, but maybe he could dig one extra spot on the outside. The broccoli each take up 2 spots so they need 4 spots but he only has 3 open. If he wants 2 broccoli he has to dig another hole."



“Randy, I’m sorry but no. You only have 3 little rectangles left so here are your choices:
You could do 3 more carrots. 1+1+1 = 3
You could do a broccoli and a carrot
2+1=3
But two broccolis?! That is 2+2=4 and I’m sorry Randy but no.”

Claim?
Context?
Math to support?
Logical conclusion?

Sample Kinder PT

Teacher Version

Pancake Party!

These items will give your students the chance to show their ability to count and compare numbers and explain their reasoning.

Item	Claim	Content
1	2	K.CC.C.6 - Identify whether the number of objects is greater/less/equal to
2	3	K.CC.C.7 - Compare two numbers within 10

Before the Performance Task - Whole Group Frontloading:
The goal for this component of the PT is to do a little frontloading with students to ensure they all understand the context of the items that will follow.

At some point before the performance task, please choose from the books or videos below to share with your students as a whole group. You can work this into your day/week whenever it seems reasonable; read aloud time, play time, science etc. Use this time to have discussions about baking, cooking, helping in the kitchen. By no means are you expected to use all of the resources below, but if it works incorporating them into your day, great! Also, feel free to supplement with any other related resources that could build background knowledge and understanding of the context.

This frontloading piece is very flexible but should focus on students understanding that we can cook and bake things that we can share with friends and family.

Possible vocabulary to discuss:

bake - oven - stove -ingredients - cook - kitchen - pancakes - share - breakfast

Possible discussion prompts:

Have you ever helped out in the kitchen? Or pretended to cook in a play kitchen?
What are your favorite things to eat or cook for breakfast? What do you like on your pancakes? Fruit? Syrup?
Have you ever visited a bakery where a real baker

Related Literature:

- The Runaway Pancake - Mairi Mackinnon
- Pancakes, Pancakes - Eric Carle
- Pancakes for Breakfast - Tomie De Paola
- Perfect Pancakes If You Please - William Wi
- If You Give a Mouse a Cookie - Laura Nume

Possible video Links:

- Peppa Pig Making Pancakes
- Runaway Pancake Read Aloud
- Mr.Wolf's Pancakes
- Pancake Party (fun to dance to?)
- Sing! Early Years Song - Pancake Day

Play:

- Pancake Activity Ideas

Administering the Performance Task

Pass out the 2017 PT 1 - Student Version to each student and have them write their name on it. Each student (hopefully in a small group) could also have manipulatives (counters, beans, cubes) to sort and count themselves if you think that is appropriate.

Decide how you would like to administer the two item PT. Small groups are recommended where the teacher can help with spelling or can scribe answers. Remember the goal of the PT is not to rate or rank students, rather to gather evidence of their mathematical thinking. However you think is best collect this evidence is up to you.

Explain that this is a picture of some of the pancakes you have made. Have a small group or whole group discussion about the picture. Possible discussion topics.

- What do you notice about these pictures?
- Which plate would you rather have? Why?

Students should be on front page of the PT. Direct them to Item #1.

Say: Which plate has the most pancakes on it? Circle or mark the plate with the most pancakes.

Have students mark their own papers.

Say: How did you know that plate has more? Teachers scribe student responses.

5 friends are coming to eat pancakes.

Are there enough pancakes on this plate for each friend to have a whole pancake?

_____

Tell me how you figured that out.

Kinder Responses

“I think there will be enough because my stomach told me.”

5 friends are coming to eat pancakes.

Are there enough pancakes on this plate for each friend to have a whole pancake?

_____

Tell me how you figured that out.

"No. I figured it out because I counted 3 and 5 is greater than 3"

Claim?
Context?
Math?
Logically complete?

How can we help here?

5 friends are coming to eat pancakes.

Are there enough pancakes on this plate for each friend to have a whole pancake?

_____

Tell me how you figured that out.

"No because there are only 3 cakes."

OR

"No because there are 5 friends that want a pancake."

Claim?
Context?
Math?
Logically complete?

How can we help here?

“I think if 2 friends didn't come then there would be enough. There is only 3 cakes and I have 5 friends. So we are missing 2 pancakes.”

“No. You would need 2 more pancakes. I know because there are 3 there and 3 + 2 = 5. So if we had 2 more, then all 5 people can eat. Wait, what about mine?”

“Maybe if you used this other stack from the front because look, there are 1, 2, 3, 4, 5, 6 so all my friends could have one and one for me! I hope there is syrup and butter!”

A middle school example

I got the answer 35 because i divided 1/2 to 3 1/2 and got 7. Then I multiplied it to 5 to know how many minutes it will take me, which is 35.

September 2018

She left at 8:25 and stopped at 8:30. So, that means that she'd been riding her bike for 5 minutes. So, I divided 0.75 by 5, to find out how many miles did she ride her bike in a minute. After, I got the answer 0.15. That means she rode her bike for 0.15 mile in a minute. After that, I divided 3.42 by 0.15, in order to find out how many minutes she's going to ride her bike. Then, I got the answer 22.8. That means Emily could arrive at school in approximately 23 minutes. Since she left at her house in 8:25, she'll arrive at her school in 8:48. In conclusion, she'll arrive at her school on time.

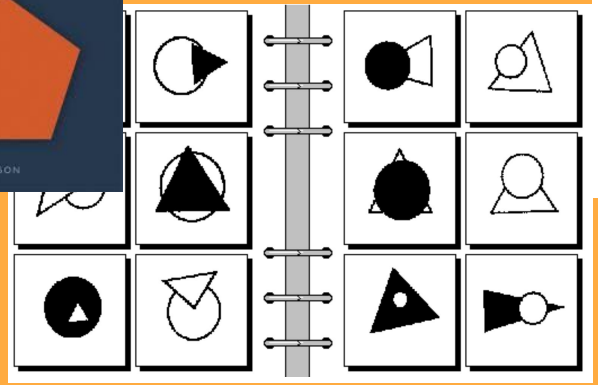
December 2018

If we want students to get better at explaining their mathematical thinking we should:

1. **Ask questions worth talking about**
2. **Provide opportunities for students to practice**
3. **Give feedback and opportunities for students to revise and strengthen their explanations**

Call to Action!

TRY ONE NEXT WEEK!



THANKS!

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