

Growing Letters as Mathematical Models to Build Student Algebraic Understanding

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Principles to Actions (Theoretical Framework)

- Multiple representations
 - Bruner
 - Lesh
- Activity
- Student work
- Teaches:
 - Linear equations
 - Arithmetic sequences
 - Slope
 - Intercept

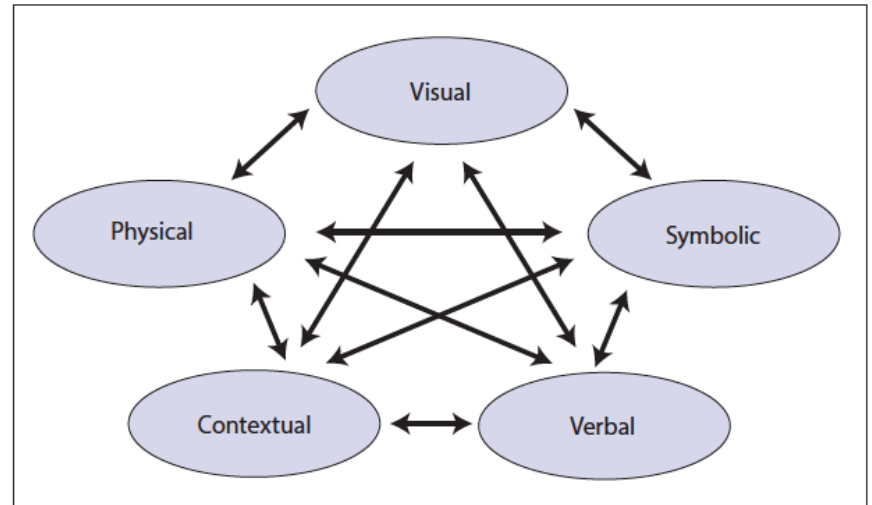
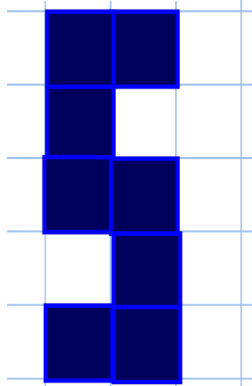


Fig. 9. Important connections among mathematical representations

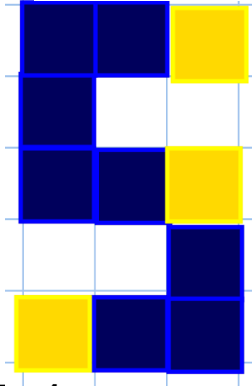
The Growing Letter S

- Concrete examples (from Abe)

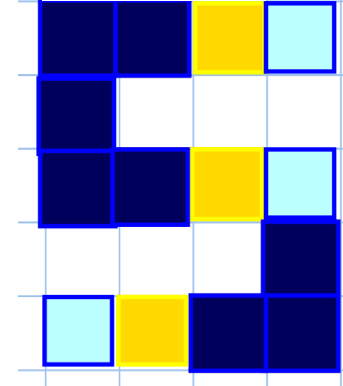
Stage 1



Stage 2



Stage 3



Stage	Value
0	
1	●
2	
3	

Stage	Value
0	
1	●
2	●
3	



Stage	Value
0	
1	●
2	●
3	●

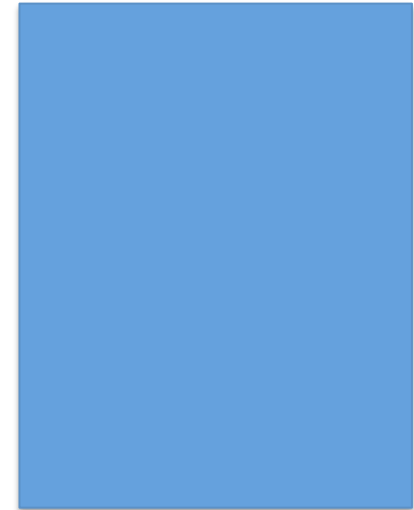
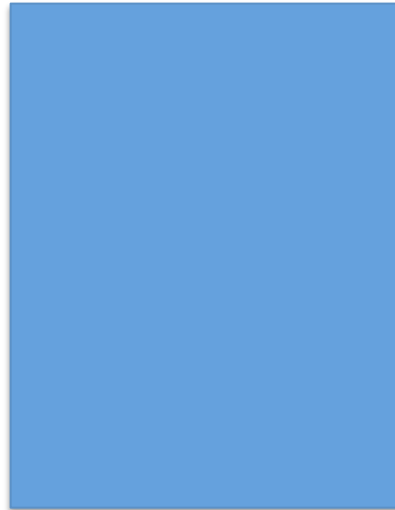
You grow **your** favorite letter

- Using the graph paper to grow your letter
- Record on a table the number of tiles used for each iteration (stage) as your letter grows
- Grow at least three iterations

Abe, Abe, how does your letter grow?

1. Abe
constructed a
table for first 3
stages

Stage	Value	Difference
0		
1	8	
2	11	
3	14	
4		

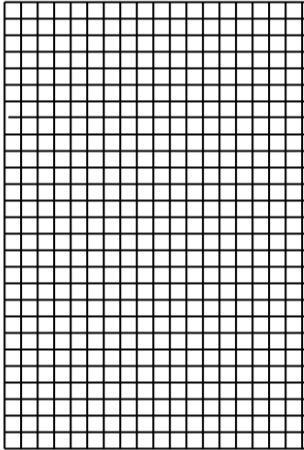


So now, how does your letter grow?

- You have the first three completed. Can you predict what the fourth number will be? Write down your guess on your table.
- Construct a recursive equation to model the growing pattern. Next =
- Build the fourth letter. Was your guess correct? Check with neighbor.

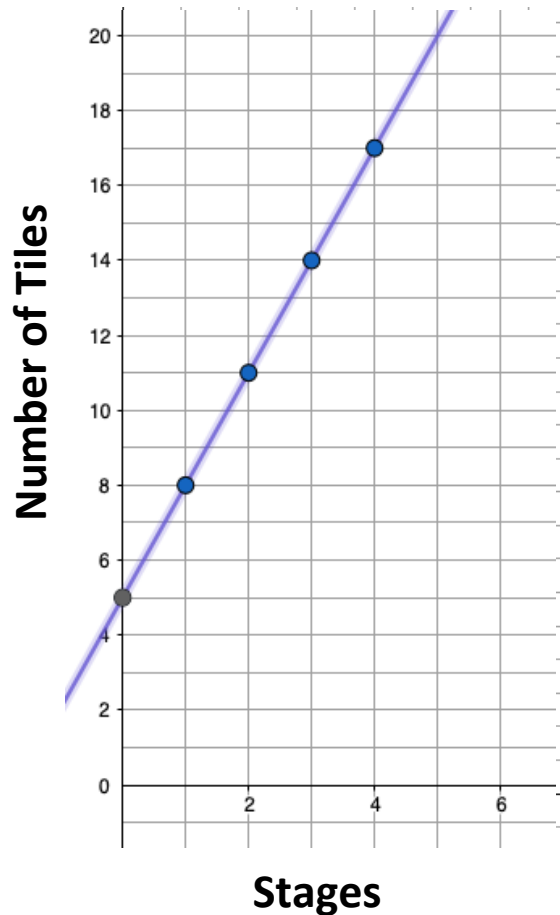
Algebra Four Square

- Table
- Written description and Picture
- Graph
- Equation

Table		Written Description (Picture)														
<table border="1"> <thead> <tr> <th>Stage</th> <th>Value</th> </tr> </thead> <tbody> <tr><td></td><td></td></tr> <tr><td>1</td><td></td></tr> <tr><td>2</td><td></td></tr> <tr><td>3</td><td></td></tr> <tr><td>4</td><td></td></tr> <tr><td>n</td><td></td></tr> </tbody> </table>	Stage	Value			1		2		3		4		n			
Stage	Value															
1																
2																
3																
4																
n																
Graph		Equation														
																

Make a graph from your table

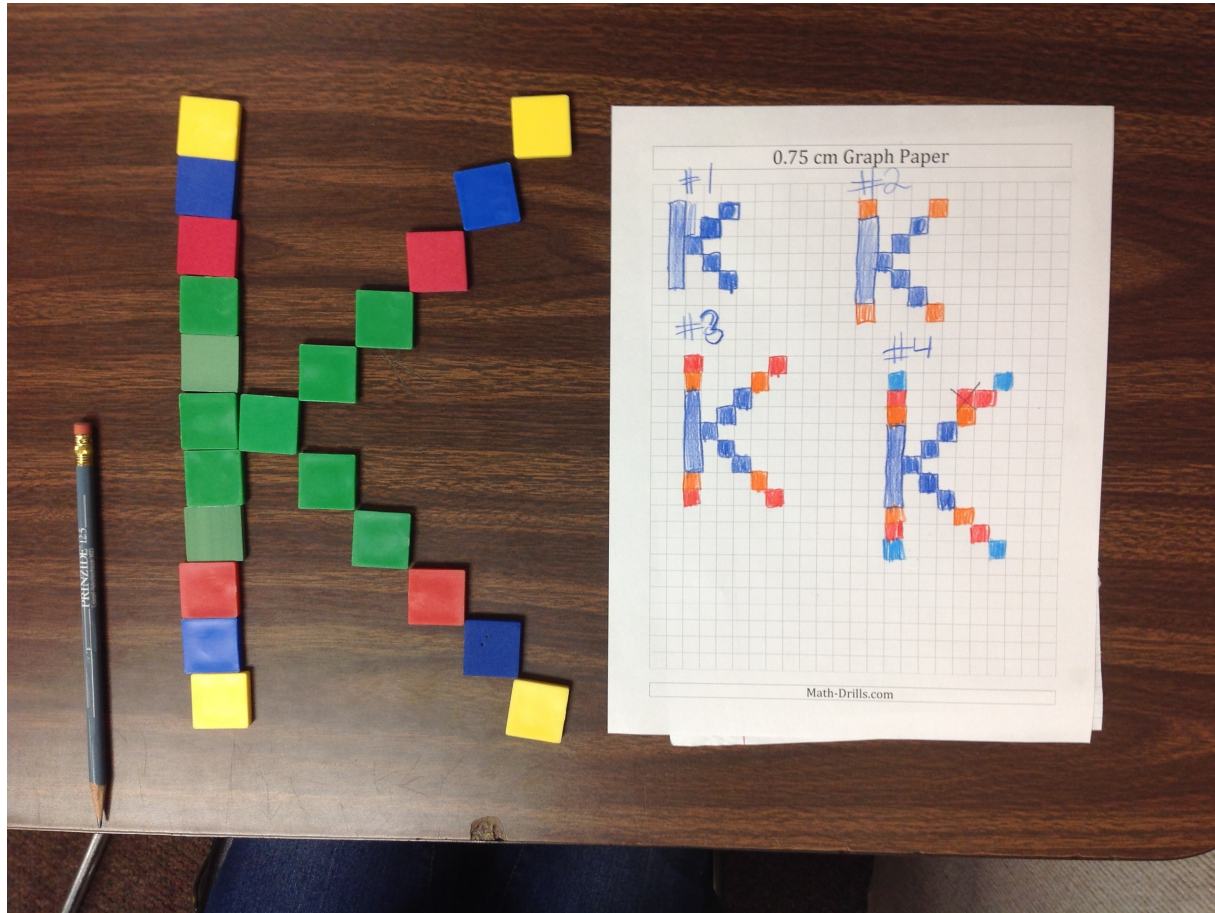
- Abe's Graph



Compare the room

- Arithmetic sequence is a sequence of numbers such that the difference between two consecutive terms is constant (Wikipedia).
- How many have something that models an arithmetic sequence?
- How many have a graph that is a straight line?
- How many have a graph that is not a straight line?

Examples of student work



Examples of student work

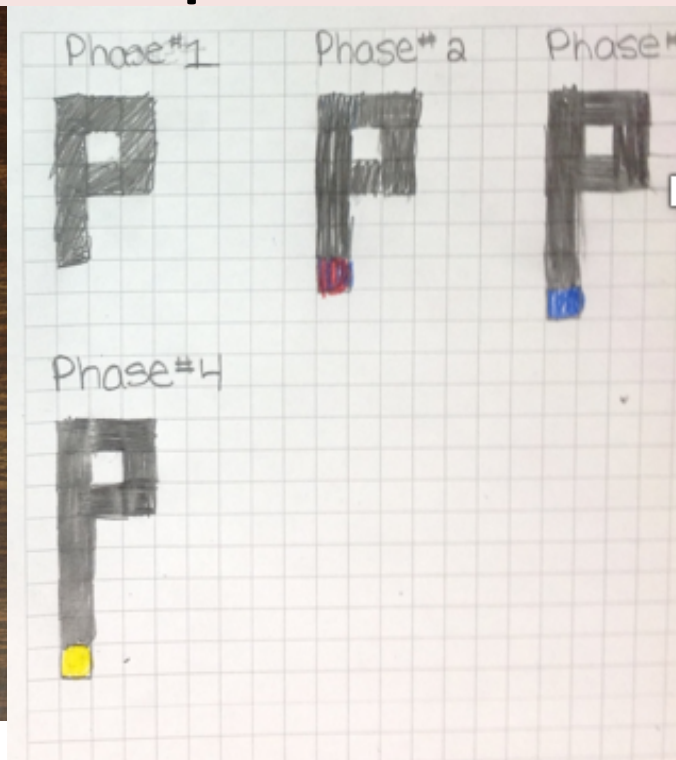
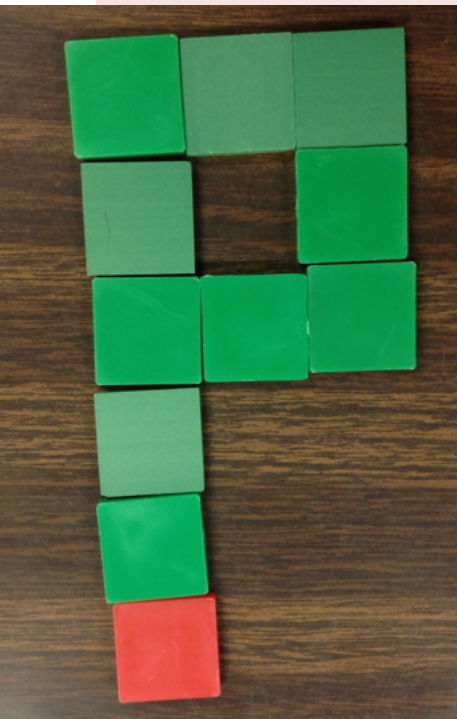

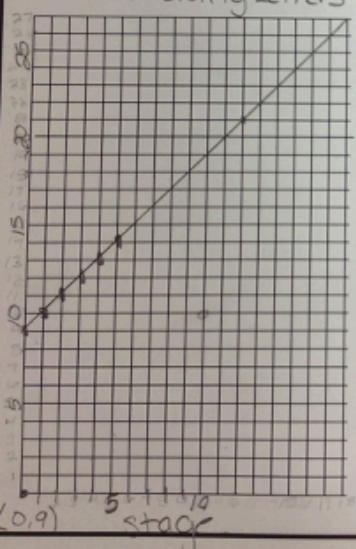
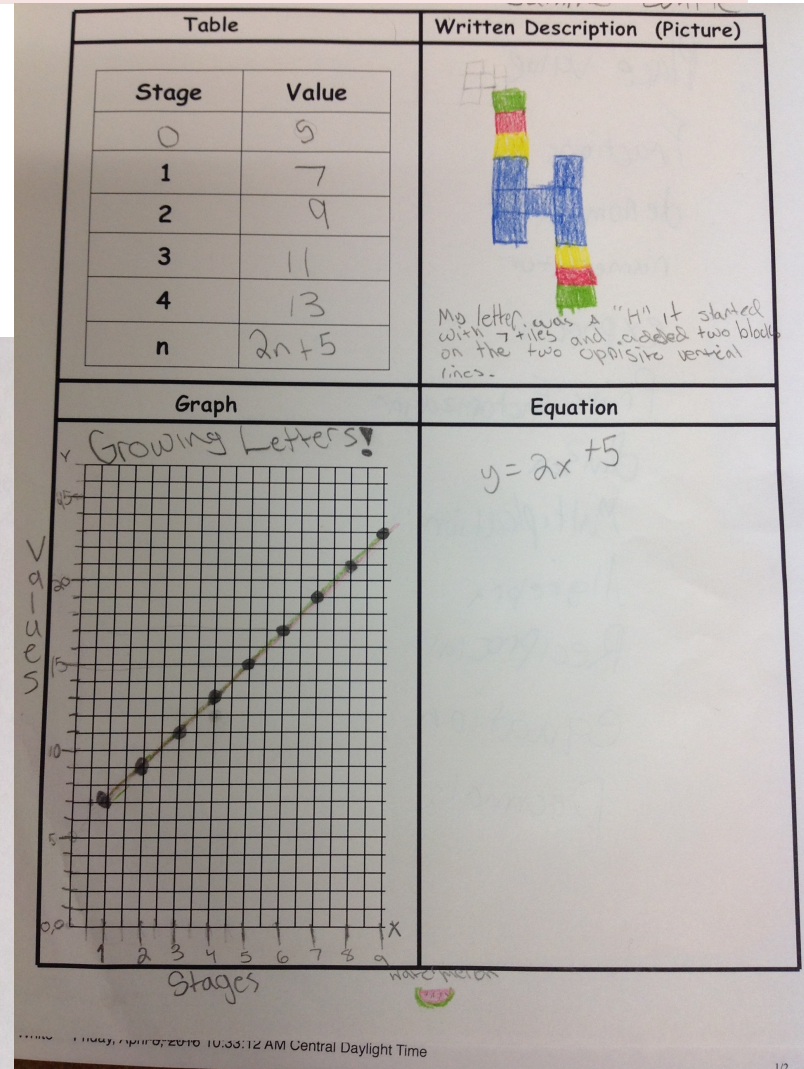
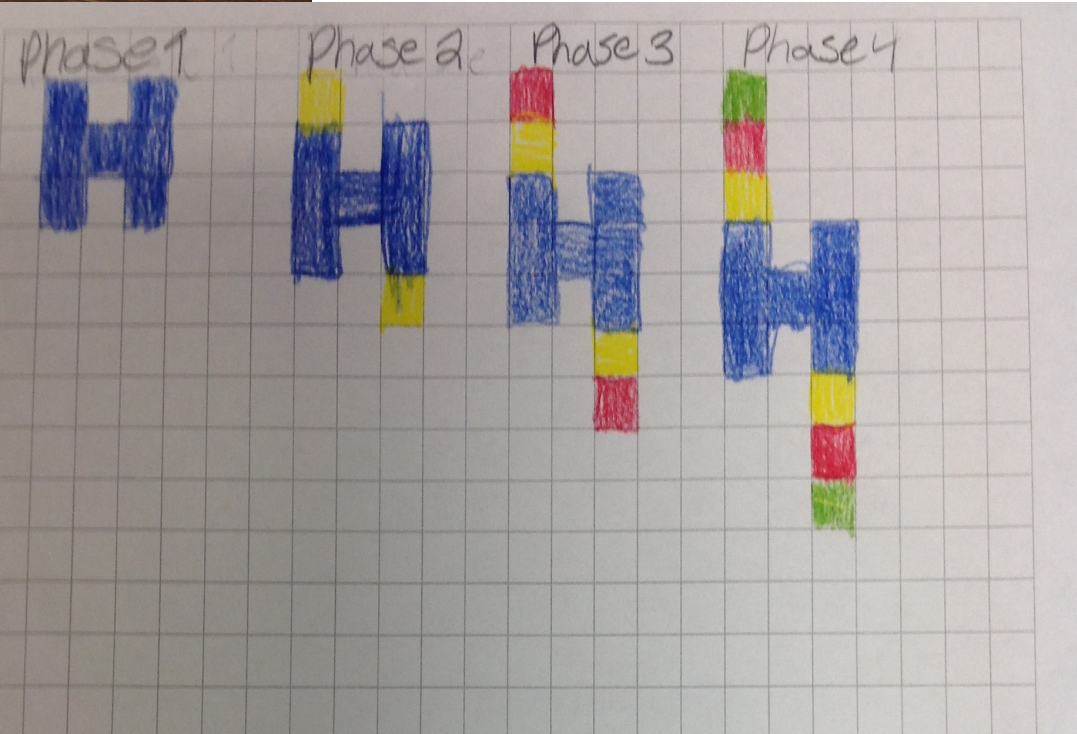
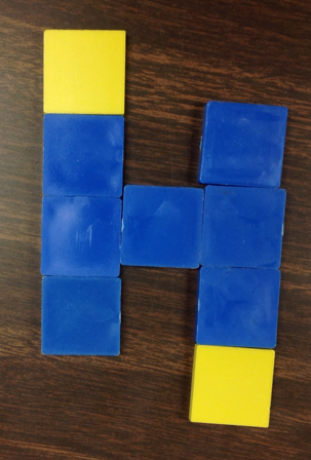
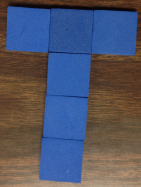


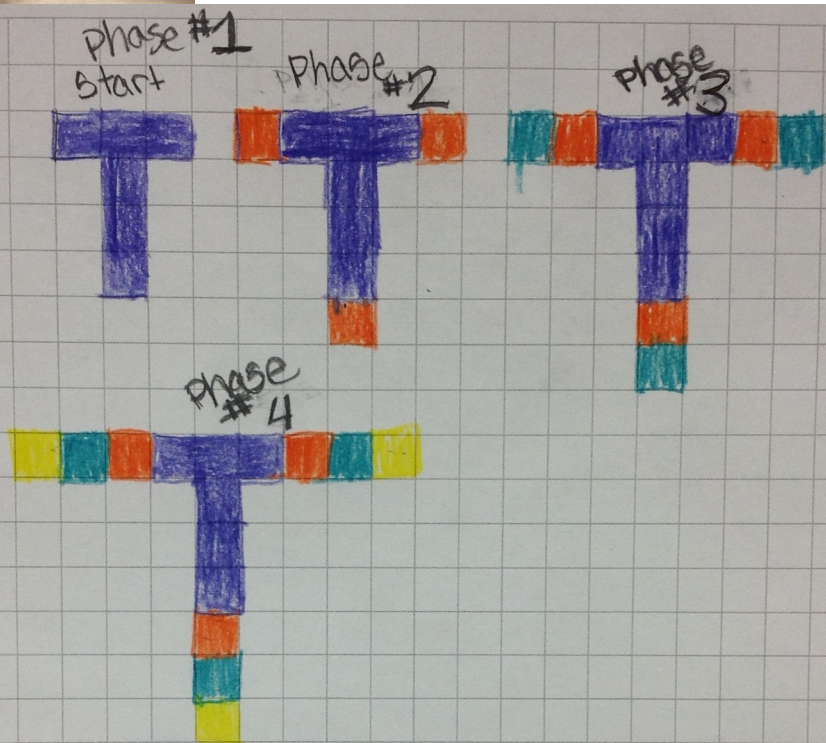
Table		Written Description (Picture)
0	9	my letter is a Capital P I used 10 blocks for the first stage  Then I added 1 block to the end of the capital P which made 11 blocks I just kept adding a new block every time to the end of the capital P
Stage	Value	
1	10	
2	11	
3	12	
4	13	
5	14	
n		
Graph		Equation
Growing Letters 		$x + 9$

Examples of student work

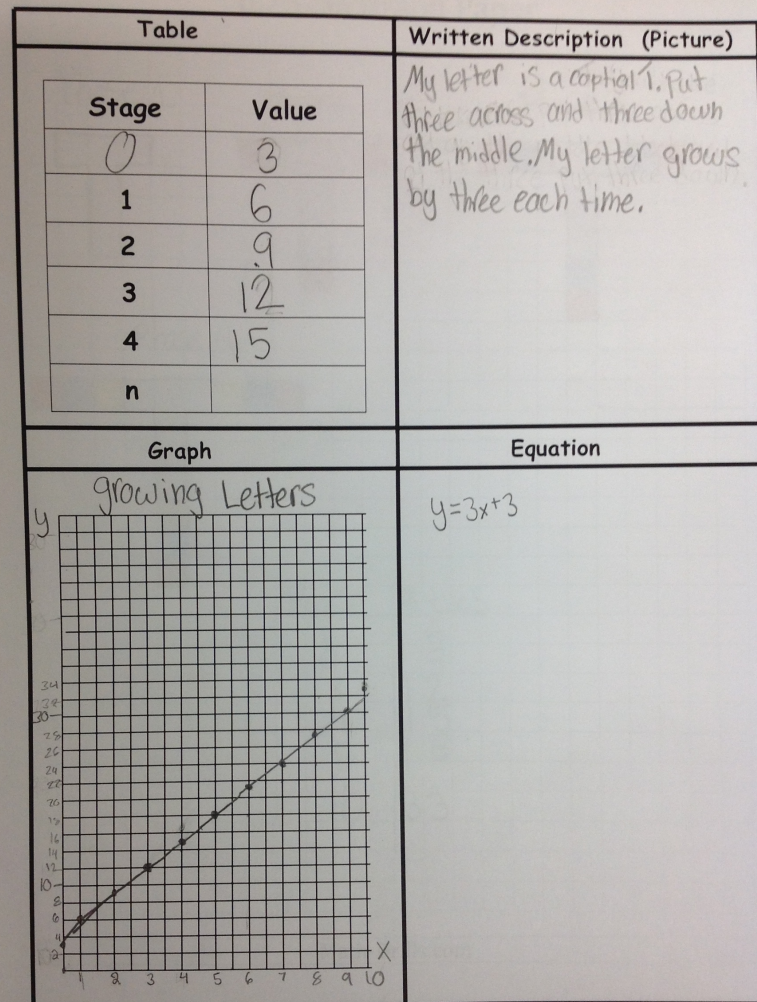




Examples of student work



Phase	# of tiles
1	6
2	9
3	12
4	15



Principles to Actions (Theoretical Framework)

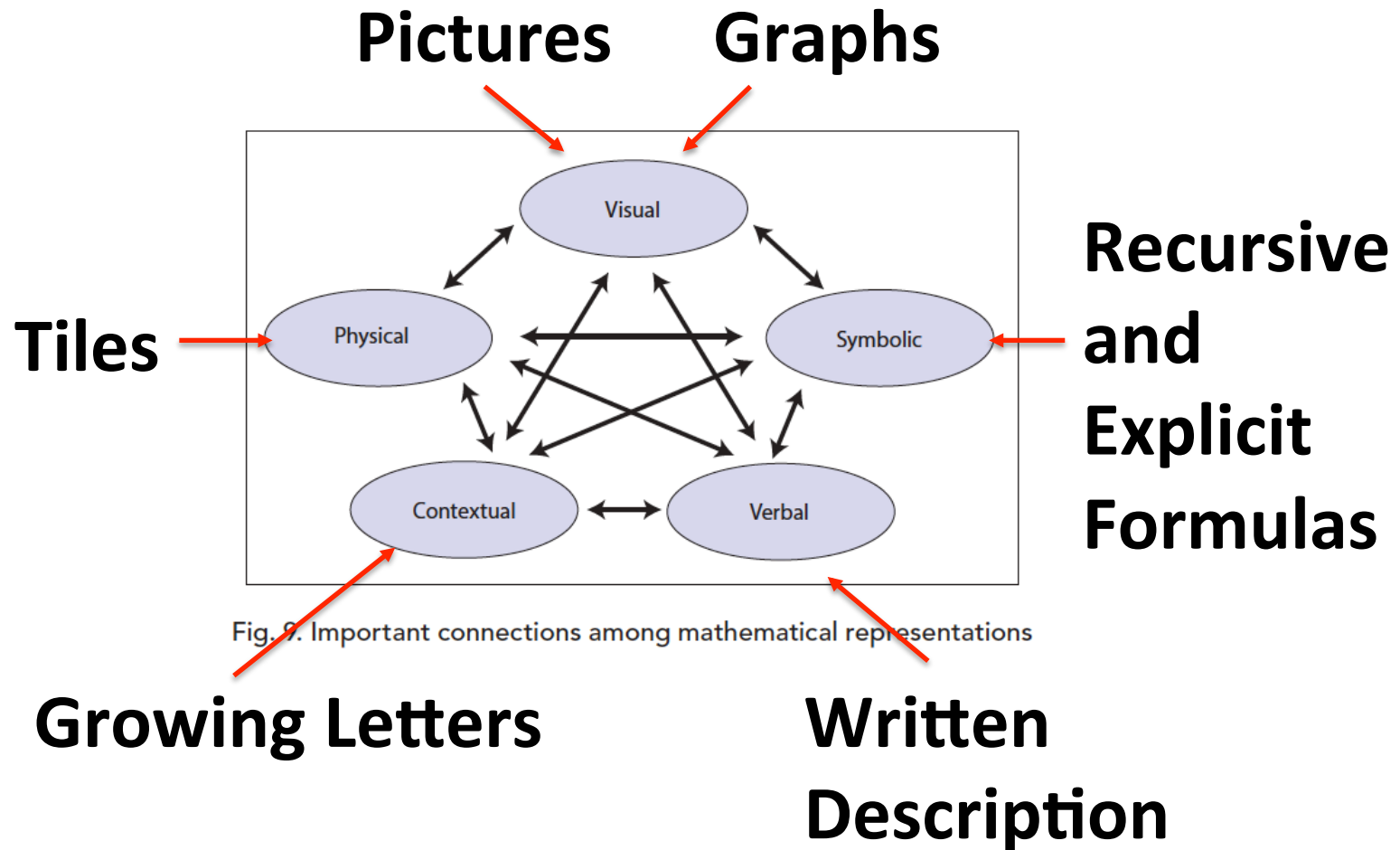
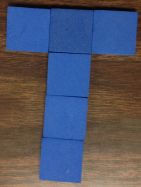
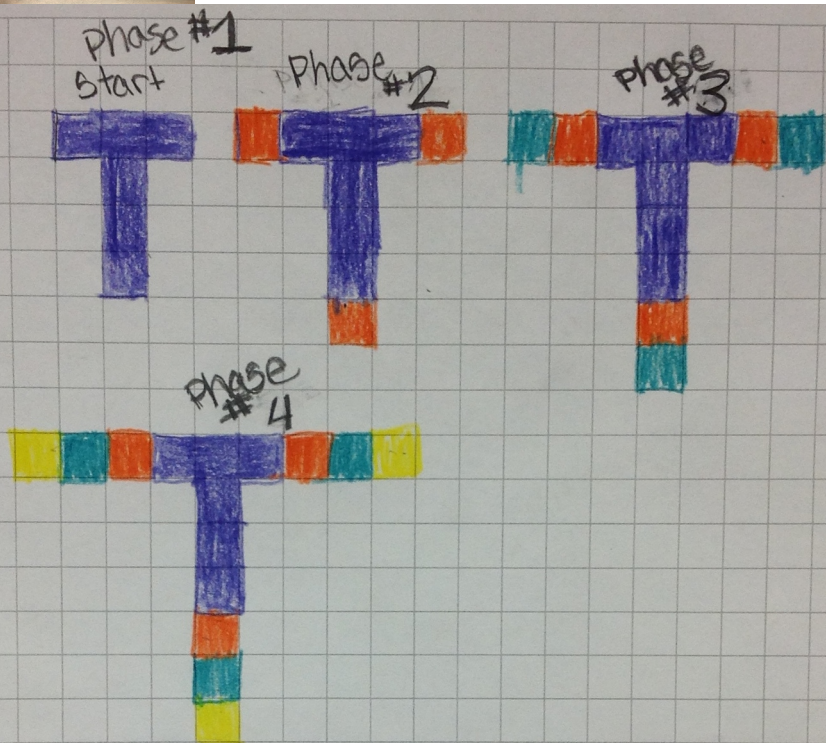


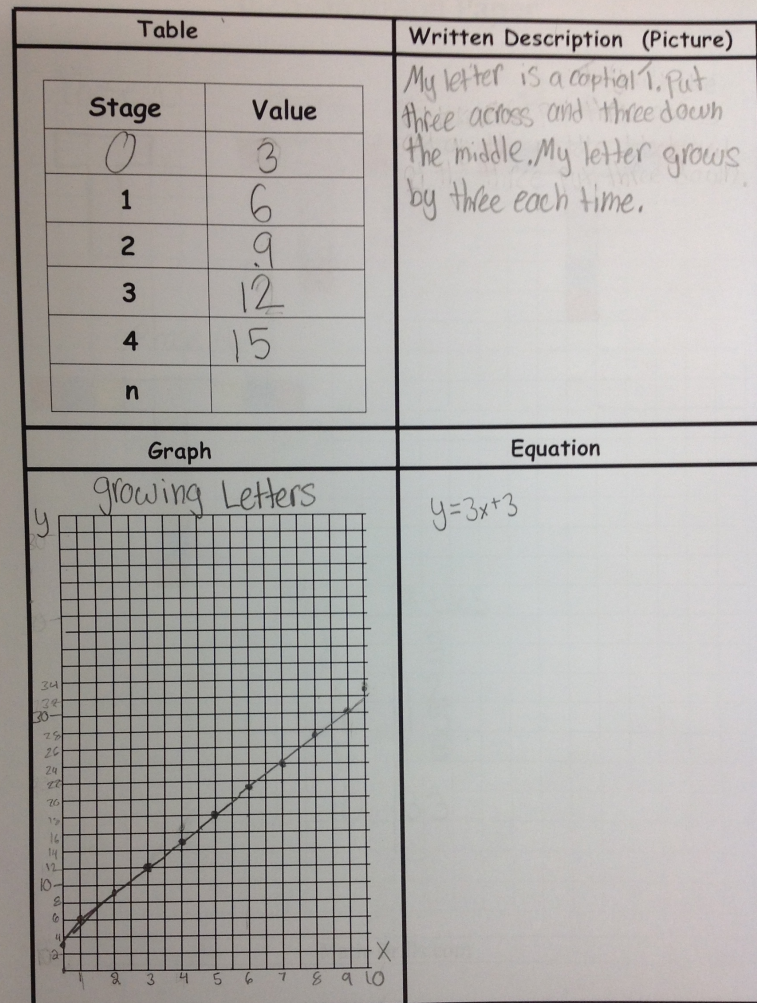
Fig. 9. Important connections among mathematical representations



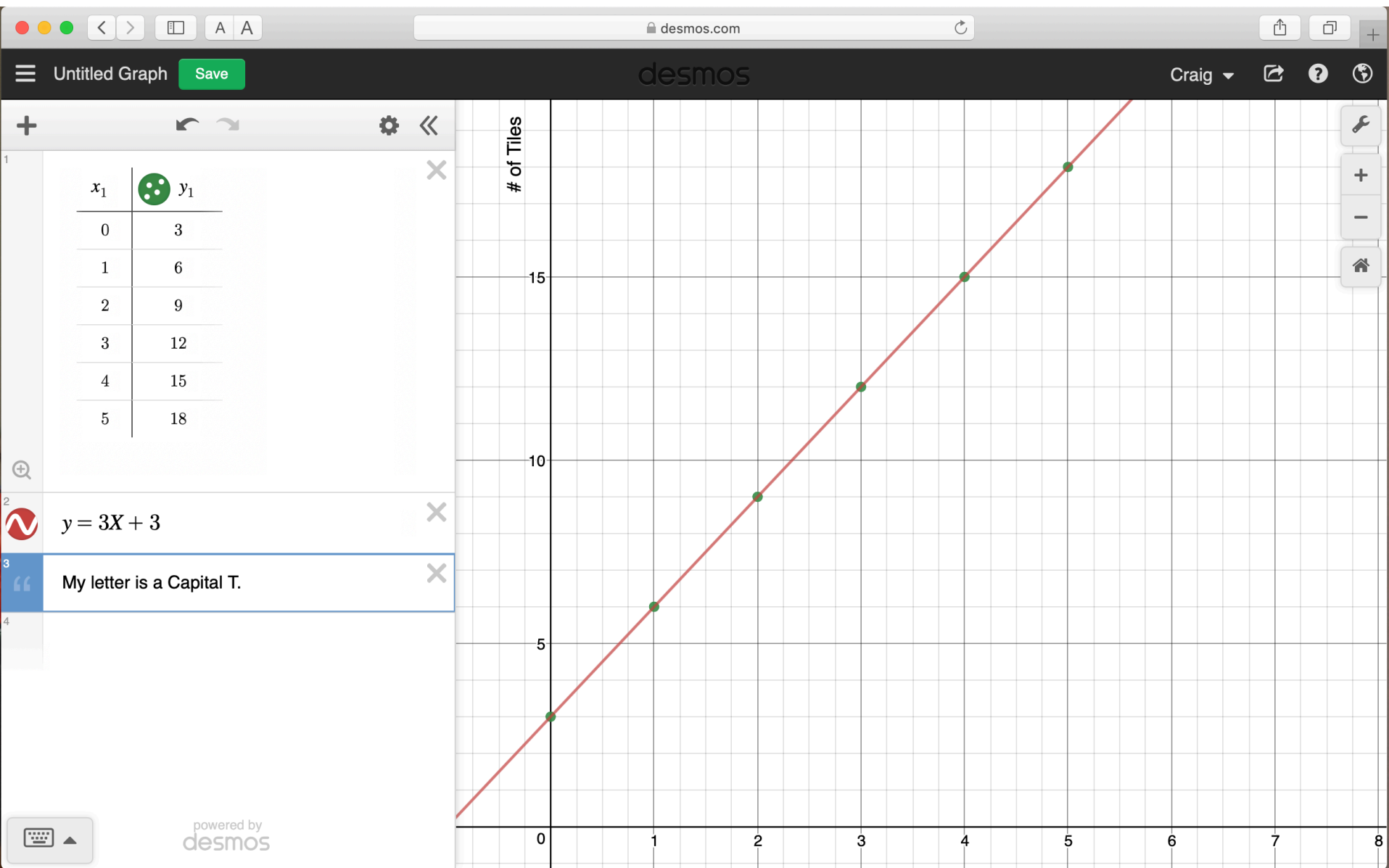
Examples of student work



Phase	# of tiles
1	6
2	9
3	12
4	15



CAPITAL LETTER T



Examples of student work

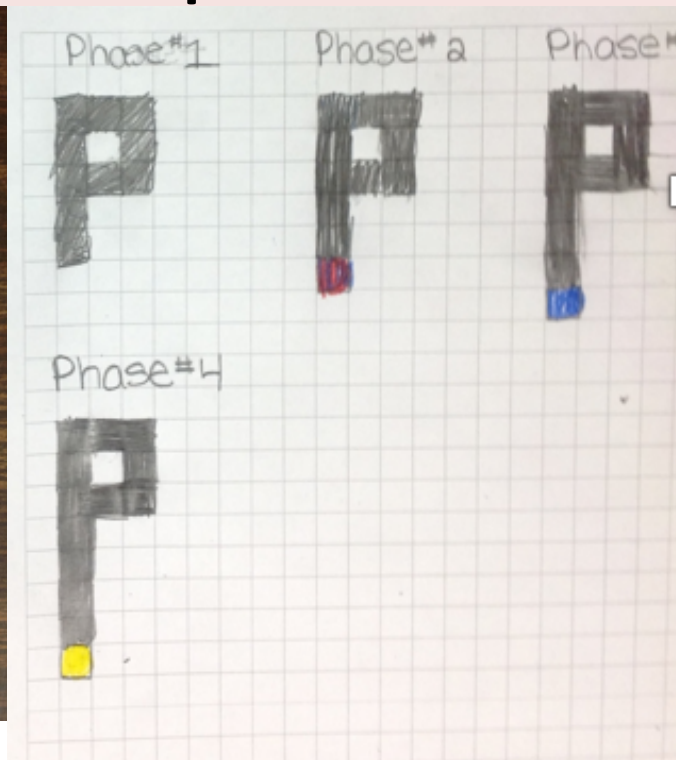
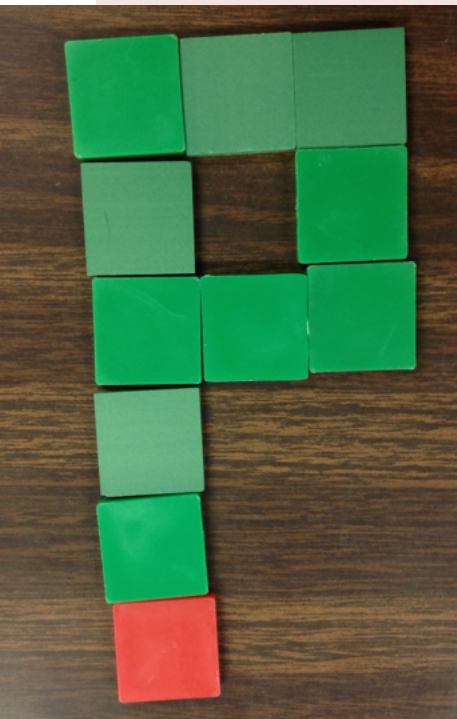

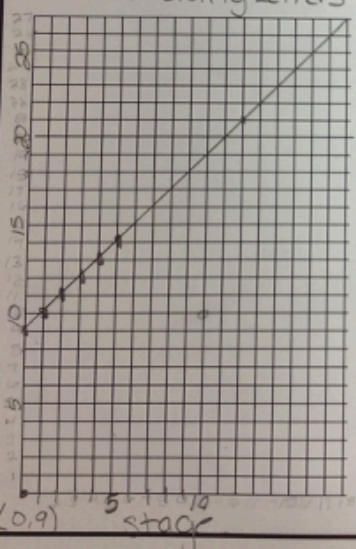
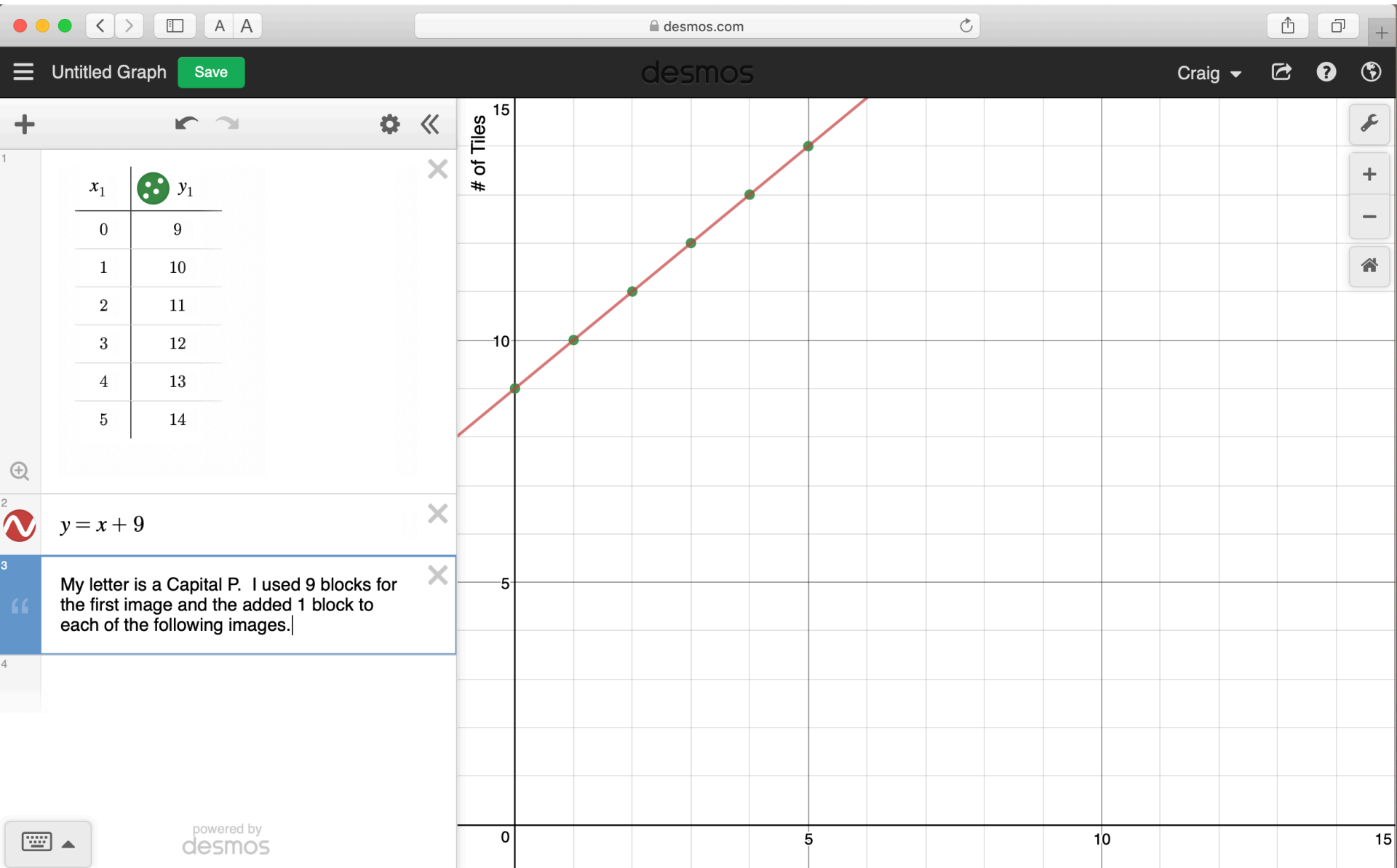
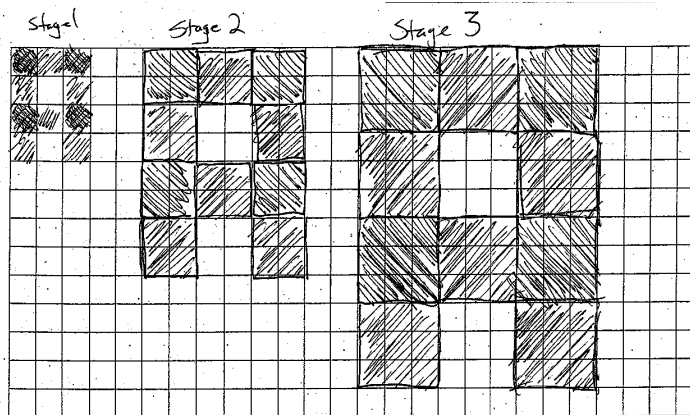


Table		Written Description (Picture)
0	9	my letter is a Capital P I used 10 blocks for the first stage  Then I added 1 block to the end of the capital P which made 11 blocks I just kept adding a new block every time to the end of the capital P
Stage	Value	
1	10	
2	11	
3	12	
4	13	
5	14	
n		
Graph		Equation
Growing Letters 		$x + 9$

CAPITAL LETTER P



Extensions



Try a growing A.

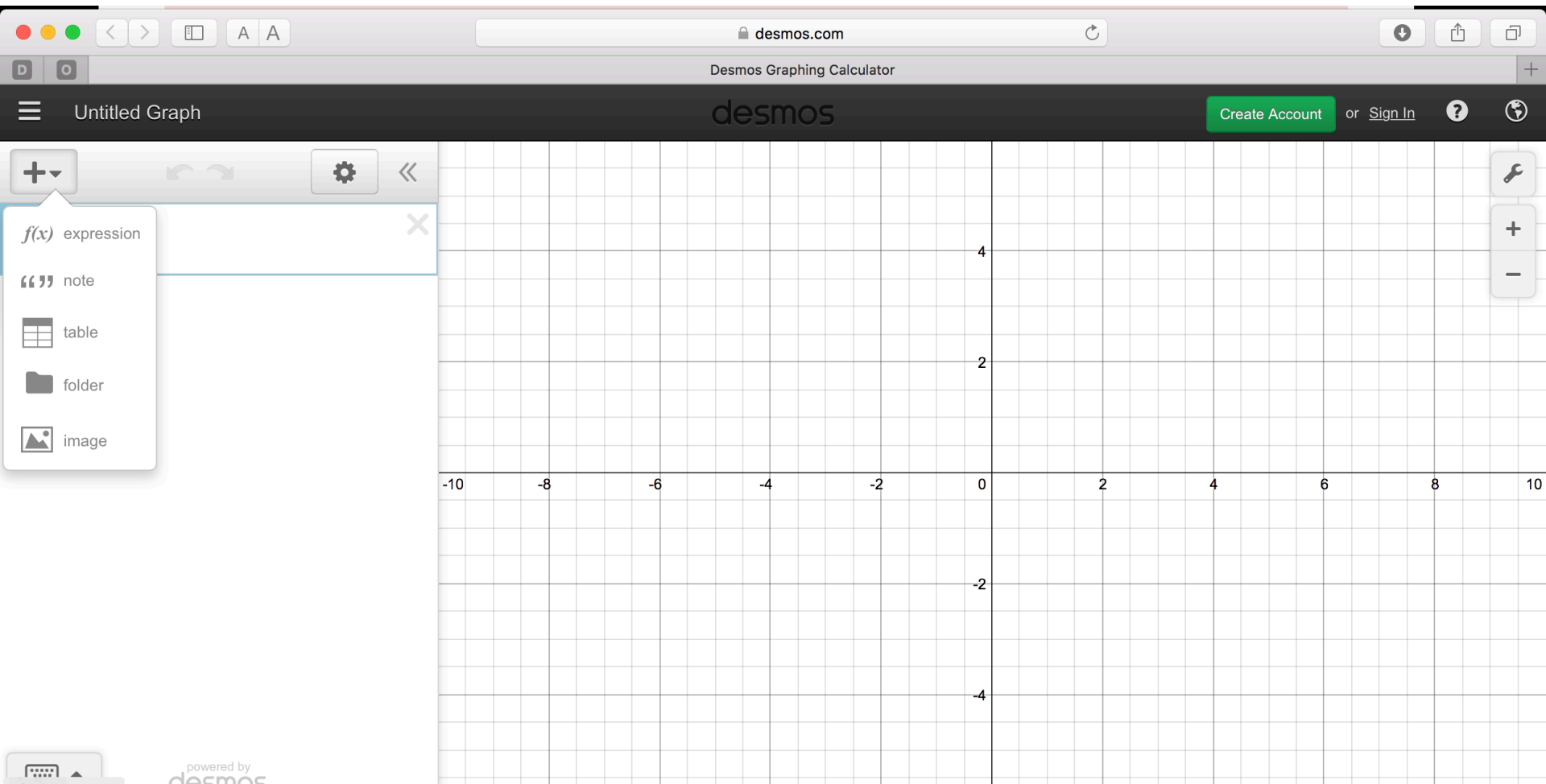
Is this what exponential growth looks like?

(HINT: No this is not exponential growth!)

Table		Written Description (Picture)												
<table><tr><th>Stage</th><th>Value</th></tr><tr><td>1</td><td>10</td></tr><tr><td>2</td><td>40</td></tr><tr><td>3</td><td>90</td></tr><tr><td>4</td><td></td></tr><tr><td>n</td><td></td></tr></table>		Stage	Value	1	10	2	40	3	90	4		n		<p>Ten growing squares in the shape of an A.</p>
Stage	Value													
1	10													
2	40													
3	90													
4														
n														
Graph		Equation												
		$\text{Next} = \text{Now} + (\text{Now} - \text{Prev}) + 20$ $\text{Next} = 2\text{Now} - \text{Prev} + 20$ $B = 10A^2 + 0$												

Resources

Desmos



Resources

Geogebra

≡ GeoGebra Graphing Calculator

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