From Abundant to McNugget to Vampire: Using Fascinating Numbers to Build Competence and Enjoyment

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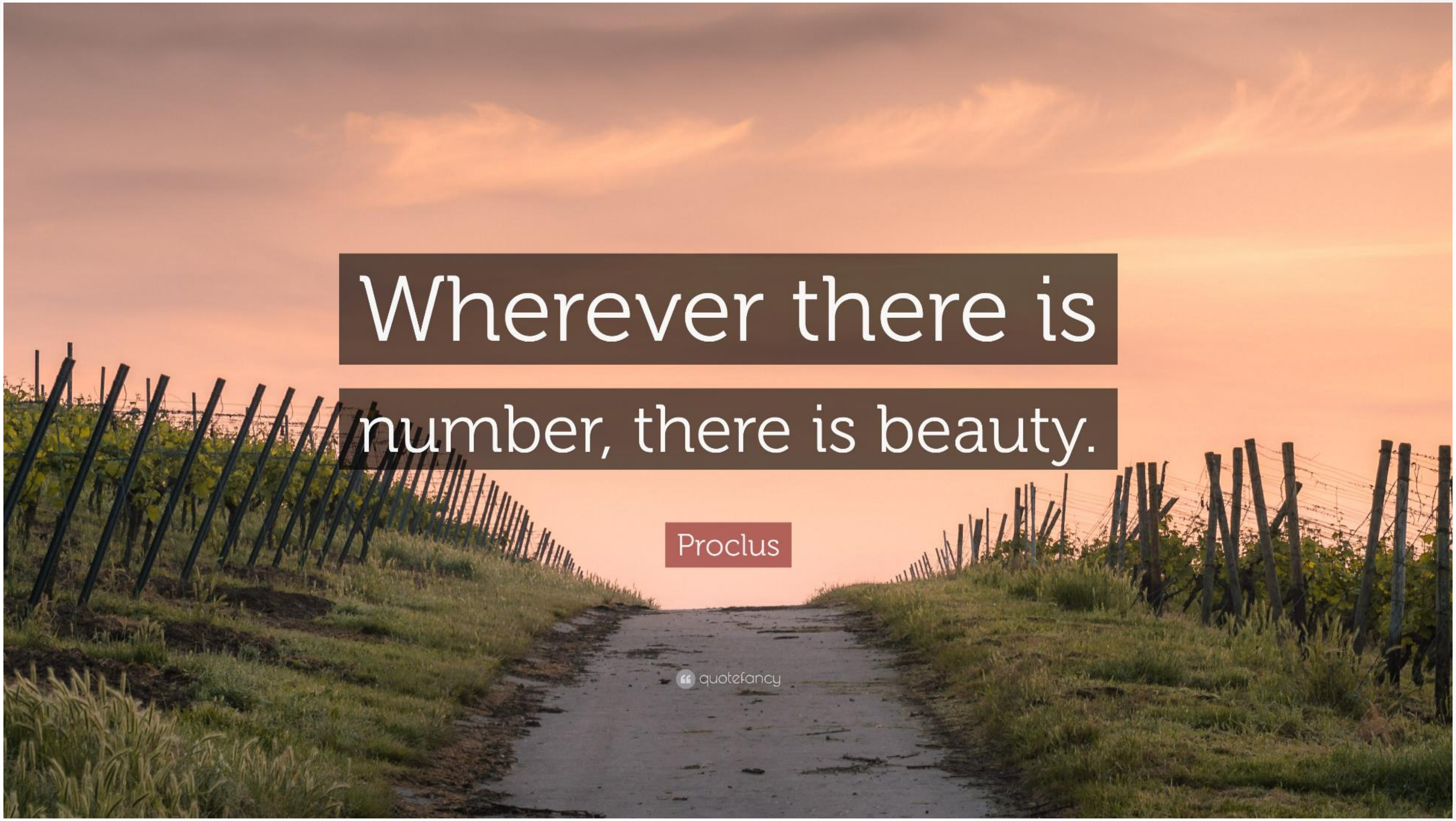

Wherever there is
number, there is beauty.
Proclus
quotefancy

Questions to Think About:

- What role does number sense play in mathematics learning?
- How might we nurture number sense for all learners?
- How might work with fascinating numbers lead to opportunities for productive struggle?

"Perfect numbers like perfect people are very rare."

--Rene' Descartes (1596-1650)

Perfect Numbers

- The proper factors of a number do not include the number itself.
- The proper factors of 12 are 1, 2, 3, 4, and 6.
- If a number is a perfect number, its proper factors sum to the number itself.
- Example: $6 = 1 + 2 + 3$, so 6 is classified as a perfect number.

Are both of these numbers perfect numbers? Explain your reasoning.

28

35

**“It is impossible
to be a mathematician
without being a poet in soul.”**

--Sofia Vasilyevna Kovalevskaya

(1850 – 1891, first major Russian female mathematician)

Abundant and Deficient Numbers

- If the proper factors of a number sum to less than the number itself, the number is classified as deficient.
- If the proper factors of a number sum to more than the number itself, the number is classified as abundant.
- Examples:

The number 12 is abundant, because 1 + 2 + 3 + 4 + 6 = 16, which is greater than 12.

The number 9 is deficient, because 1 + 3 = 4, which is less than 9.

"When you have mastered numbers, you will in fact no longer be reading numbers, any more than you read words when reading books. You will be reading meanings."

--W. E. B. Du Bois (1868-1963)

Semiperfect Numbers

- A semiperfect number is equal to the sum of SOME (or a subset of) of its proper factors.
- Example: 20 is semiperfect. Its proper factors are 1, 2, 4, 5, and 10.

  $$1 + 4 + 5 + 10 = 20$$

- Perfect numbers are also semiperfect. Why?

Classify the following numbers as abundant or deficient. Are any of these numbers also semiperfect?

25

30

49

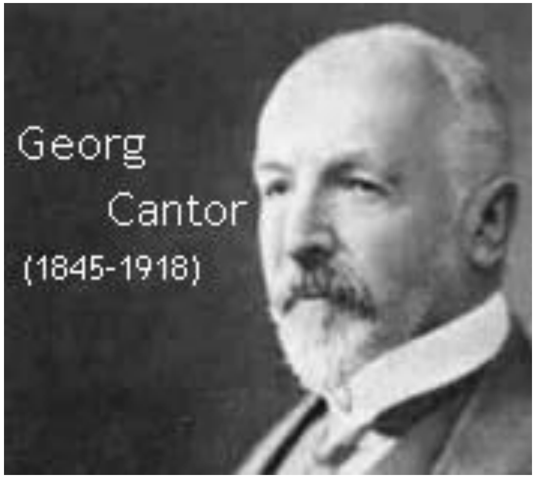
100

510

Would it be possible for a deficient number to be semiperfect?

- Explain your reasoning.

“In mathematics the art of proposing a question must be held of higher value than solving it.”



--Georg Cantor (1845-1918)

Weird Numbers

- A number is weird if it is abundant without being semiperfect.
- Example: 70 is a weird number. Its proper factors are 1, 2, 5, 7, 10, 14, and 35. These sum to 74 (so 70 is abundant), but no subset of the proper factors sum to 70 (so 70 is not semiperfect).

Which of these numbers are weird?

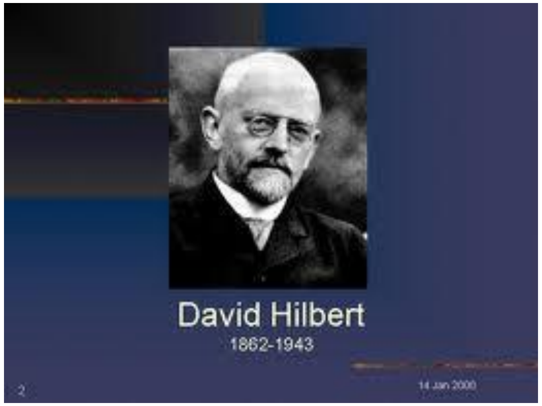
836

912

4030

“Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.”

--David Hilbert (1862-1943)



Untouchable Numbers

- An untouchable number is a positive integer that is not the sum of all proper factors of any number.
- The proper factors of 26 are 1, 2, and 13. These sum to 16, so 16 is *not untouchable*.
- The proper factors of 40 are 1, 2, 4, 5, 8, 10, and 20. These sum to 50, so 50 is *not untouchable*.

Untouchable Numbers

Examples:

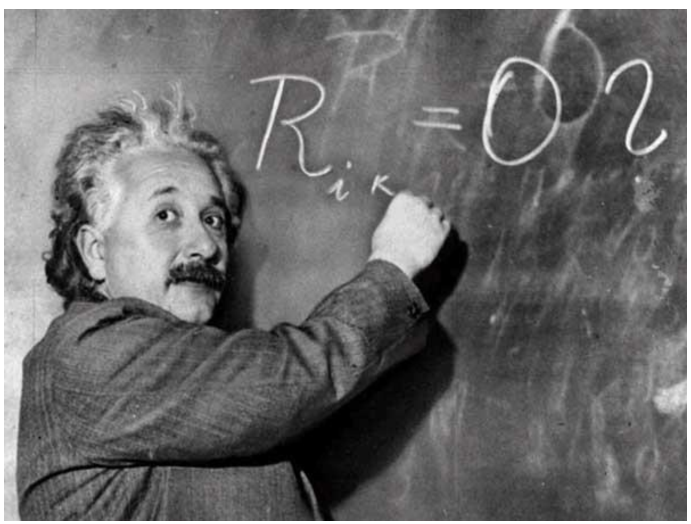
- 5 is untouchable because there is not a number whose proper factors sum to 5.
- 52 is untouchable because there is not a number whose proper factors sum to 52.

Try These:

- 26 is *not untouchable*. Verify this by finding a number whose proper factors sum to 26.
- 40 is *not untouchable*. Verify this by finding a number whose proper factors sum to 40.
- Which of the following numbers is *untouchable*: 87 or 88? Explain your reasoning.

“Pure mathematics is, in its way, the poetry of logical ideas.”

-- Albert Einstein (1879-1955)



Friendly Numbers

- Friendly Numbers have a special relationship. They share the same abundancy.
- Abundancy is the sum of **all** factors of a number (not only the proper factors) divided by the number itself.
- The abundancy of 30 is 12/5, because (1 + 2 + 3 + 5 + 6 + 10 + 15 + 30) divided by 30 is 72/30 or 12/5.
- The abundancy of 140 is 12/5, because (1 + 2 + 4 + 5 + 7 + 10 + 14 + 20 + 28 + 35 + 70 + 140) divided by 140 is 336/140 or 12/5.

Friendly Numbers

- Thus, 30 and 140 have the same abundancy. Therefore, they are classified as *friendly numbers.*
- Pairs of friendly numbers are known as *friendly pairs*. There are also friendly triples, friendly quadruples, and friendly quintuples.
- Numbers that share the same abundancy are called friends.
- Numbers that do not share abundancy with any other numbers are called *solitary numbers*.

Which of the following are friends? Explain your reasoning.

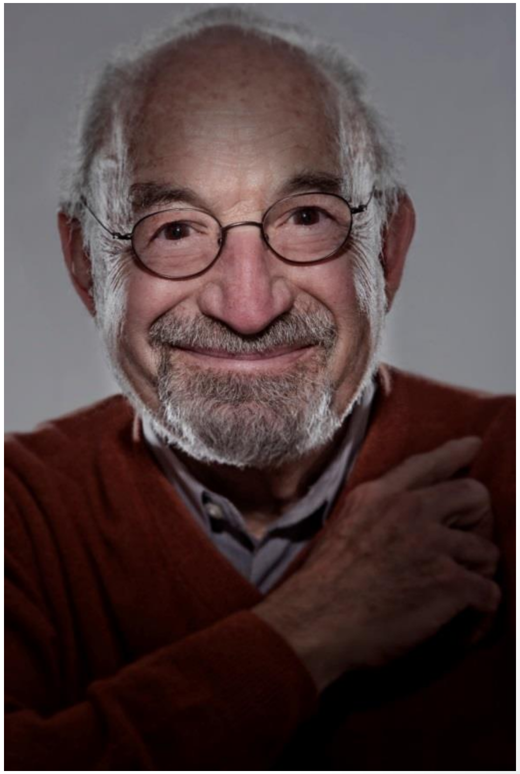
80 and 200

50 and 112

40 and 224

"The essence of mathematics is not to make simple things complicated, but to make complicated things simple."

--Stanley Gudder (born 1937)



Happy Numbers

- To determine if a number is a *happy number*, we begin by squaring each of the number's digits and adding these squares together.
- Example: For 28, we would add 2^2 and 8^2. These are 4 and 64, which sum to 68. We repeat the process.
- $6^2 + 8^2 = 36 + 64 = 100$. We repeat the process.
- $1^2 + 0^2 + 0^2 = 1 + 0 + 0 = 1$.
- Since we arrived at 1, 28 is classified as a happy number.
- If we are working with a number and never arrive at 1, the number is not a happy number.

Happy Numbers

- As another example, consider the number 7.
- Squaring digits and adding the results lead to the sequence 7, 49, 97, 130, 10, 1.
- So, 7 is a happy number.

Which of the following are happy numbers?

19

31

50

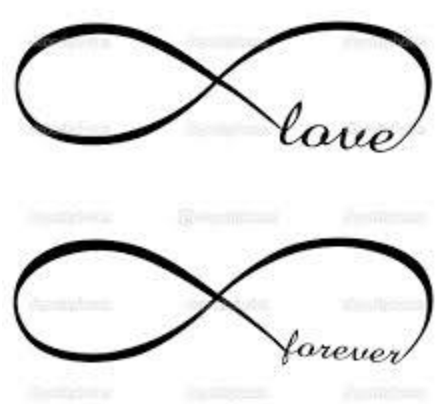
70

82

- Can you find another happy number (or numbers)?

“There was a young man from Trinity,
Who solved the square root of infinity.
While counting the digits,
He was seized by the fidgets,
Dropped science, and took up divinity.”

--Author Unknown



Vampire Numbers

- A four-digit vampire number has digits that can be rearranged to form two two-digit numbers that multiply to give us the four-digit number.
- Example: 1395 is a vampire number because 15 x 93 = 1395. 15 and 93 are called *fangs*.
- Example: 1530 is a vampire number because 51 x 30 = 1530. 51 and 30 are the *fangs*.
- It is not permissible for **both** of the fangs to have trailing zeroes (zeroes in the ones place).
- There are vampire numbers that are larger than four digits.

Try these:

- 1260 is a vampire number. Verify this.
- 1435 is a vampire number. Verify this.
- Which of the following numbers is a vampire number: 1827 or 1829? Verify your answer.

"If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is."

--John Von Neumann (1903-1957)

Lazy Caterer Numbers

- Lazy Caterer numbers represent the maximum number of pieces of a pancake that can be made with a certain number of straight cuts.
- With 1 cut, a pancake may be cut into a maximum of 2 pieces.
- With 2 cuts, a pancake may be cut into a maximum of 4 pieces.
- How about 3 cuts? What would be the maximum number of pieces?

Lazy Caterer Numbers

Try These:

- What is the maximum number of pieces (of a pancake) that could be made with 4 straight cuts?
- What is the maximum number of pieces (of a pancake) that could be made with 5 straight cuts?
- Do you notice any type of pattern?

“I'll tell you once, and I'll tell you again. There's always a prime between n and 2n.”

--Paul Erdos (1913-1996)

Narcissistic Numbers

- An n-digit number that is the sum of the nth powers of its digits is called a *narcissistic number.*
- Example: 370 is *narcissistic* because

$3^3 + 7^3 + 0^3 = 27 + 343 + 0 = 370.$

- Example: 1634 is *narcissistic* because

$1^4 + 6^4 + 3^4 + 4^4 = 1 + 1296 + 81 + 256 = 1634.$

Which of the following are narcissistic numbers?

153

371

407

451

8208

“Mathematics directs the flow of the universe, lurks behind its shapes and curves, holds the reins of everything from tiny atoms to the biggest stars .”

--Edward Frenkel (born 1968)

McNugget Numbers

A McNugget number is a positive integer that can be obtained by adding together orders of McDonald's® Chicken McNuggets™ (prior to consuming any), which originally came in boxes of 6, 9, and 20.

Examples:

- 75 is a McNugget number, because

  20 + 20 + 20 + 9 + 6 = 75.
- 32 is a McNugget number, because 20 + 6 + 6 = 32.
- 34 is not a McNugget number, because it would not be possible to order exactly 34 McNuggets.

Which of the following are McNugget numbers?

22

23

24

35

40

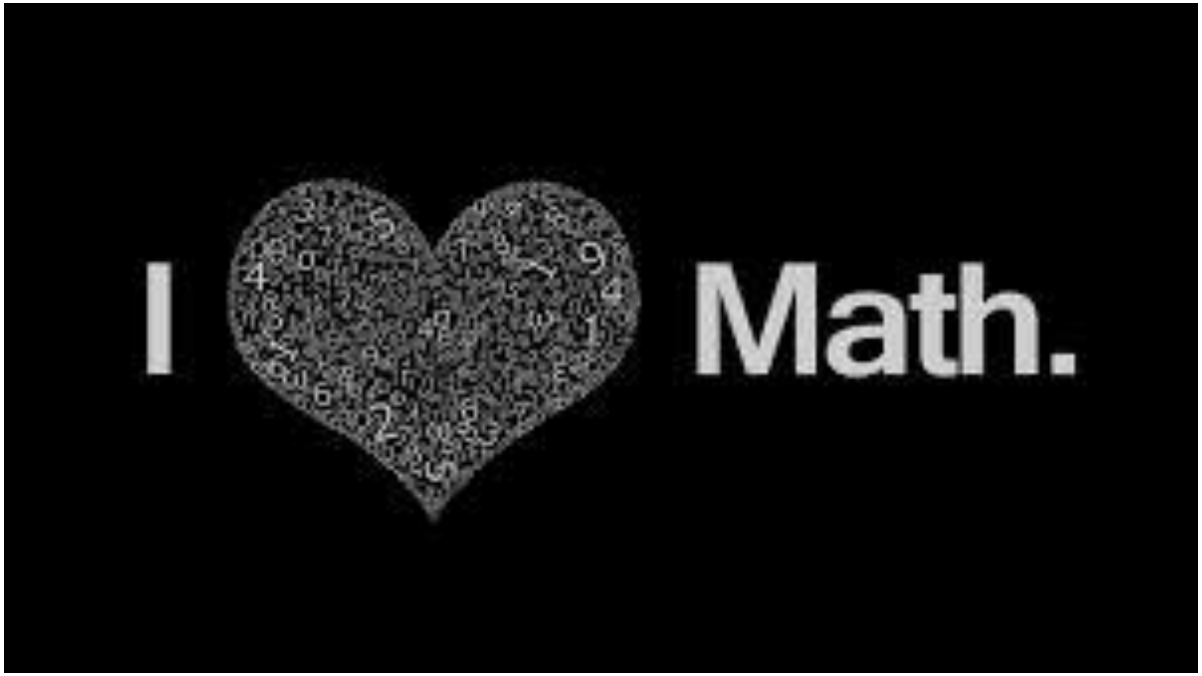
45

51

101

McNugget Numbers

- There is a number that is the largest *non-McNugget* number.
- Can you discover what this number is?


I
Math.

Special Sets of Numbers

Learning about special sets of numbers….

- Enhances enjoyment of numbers
- Strengthens mental mathematics
- Sharpens problem solving abilities
- Builds confidence
- What else?

How might you use one or more of these special sets of numbers in a mathematics teaching/learning environment?

Thank you so much for attending!