

# ALL STUDENTS CAN SEE THE MATH! ROUTINES THAT WILL EMPOWER STUDENTS' DEPT OF THINKING A AND FLUENCY



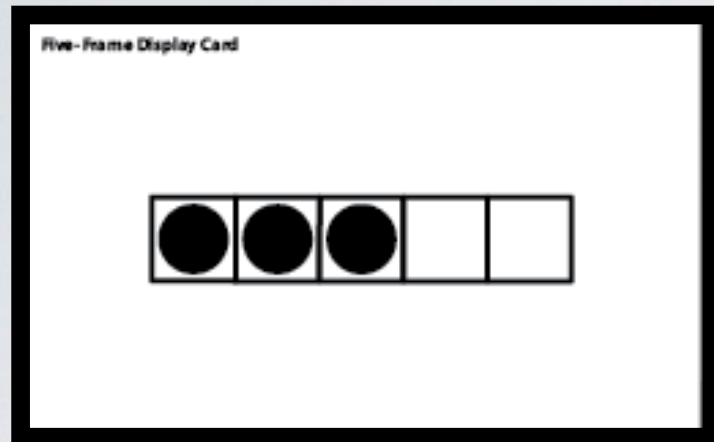
# RESEARCH ON VISUAL MODELS

- Start with 3-D\*
- Then 3-D side by side with 2-D
  - 2-D exact replica to 2-D “sketches”
- To mental images

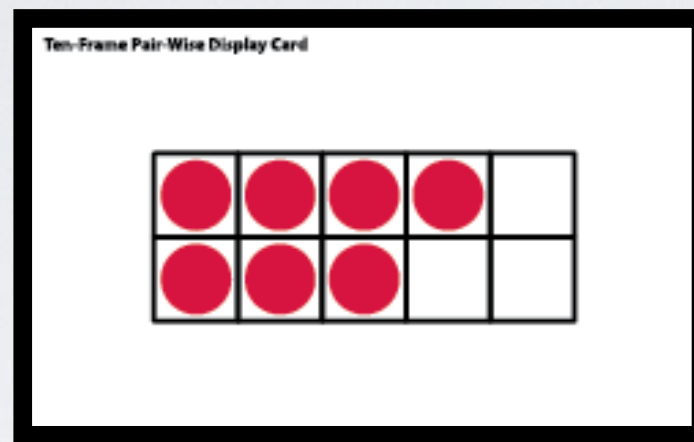


# NUMBER FRAMES

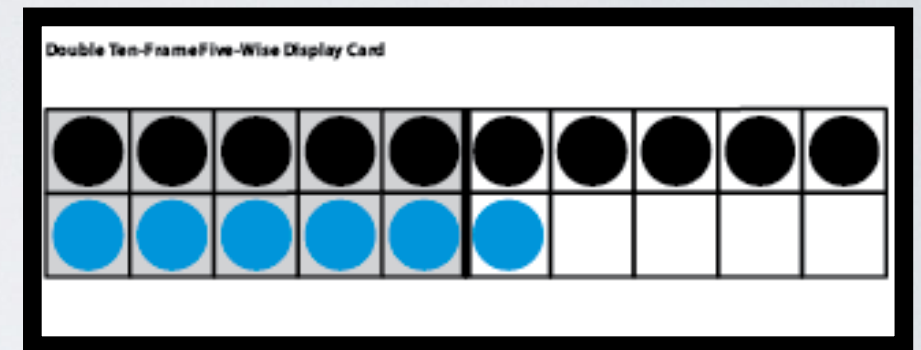
## BUNDLES & STICKS



Five-frame



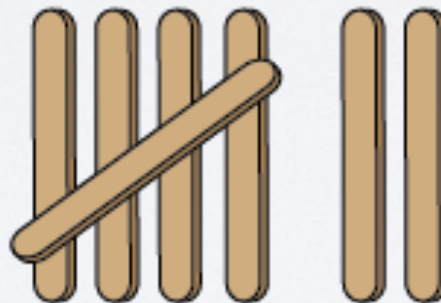
Ten-frame



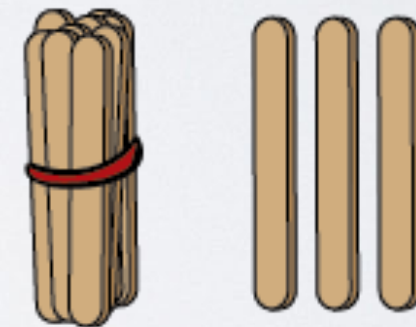
Double ten-frame



Tally marks showing 8



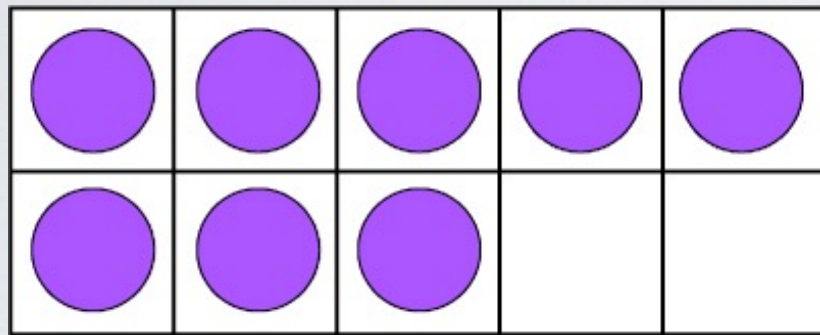
Craft sticks showing  
7 as tallies



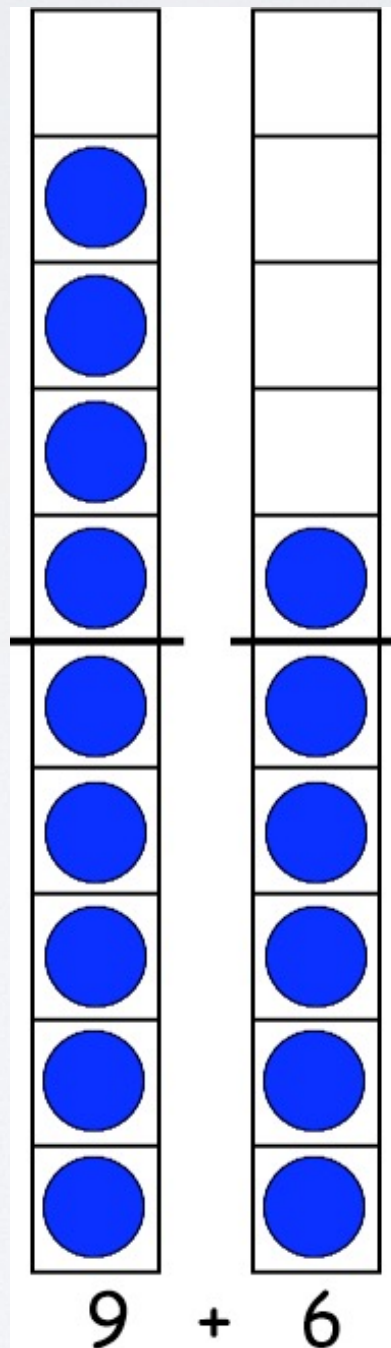
A bundle of 10 and  
3 more makes 13

# VISUAL MODELS

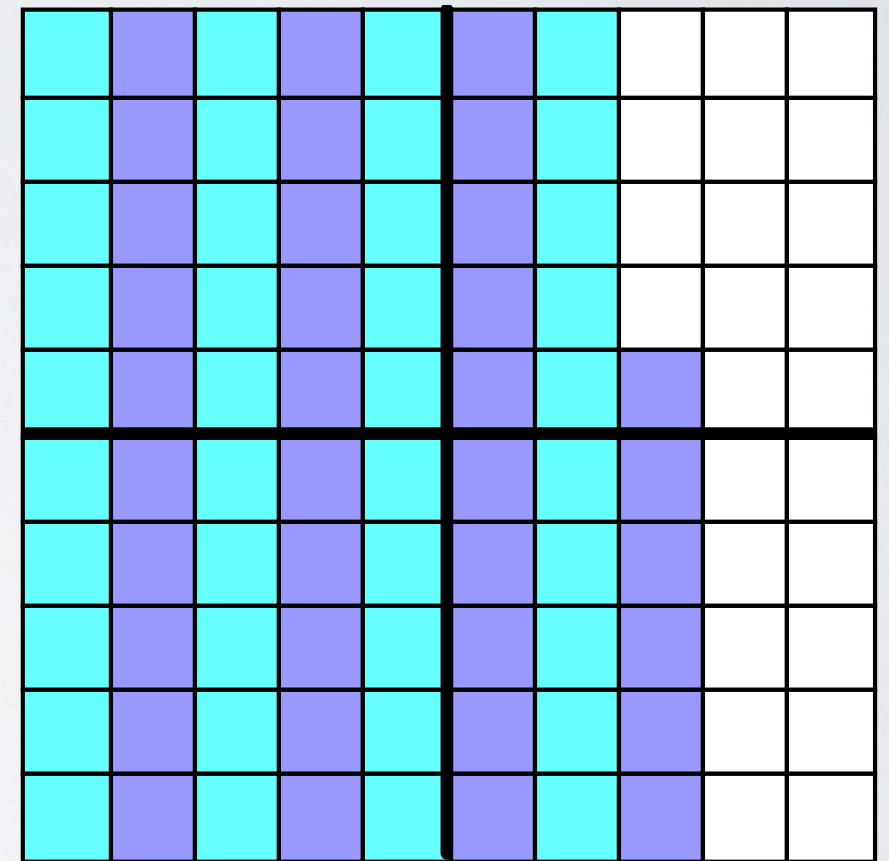
Ten Frames



Ten Strips



Hundreds Grid







The more we use...

nonlinguistic  
representations while  
learning, the better we  
can think about and  
recall our knowledge.

Robert Marzano, Classrooms That Work

# JO BOALER- [YOUNCUBED.ORG](http://YOUNCUBED.ORG)



## SEEING AS UNDERSTANDING: The Importance of Visual Mathematics for our Brain and Learning.



Jo Boaler, Professor of Mathematics Education  
with Lang Chen, Stanford Cognitive and Systems Neuroscience Lab  
Cathy Williams & Montserrat Cordero, youcubed.  
Stanford University

### *Introduction*

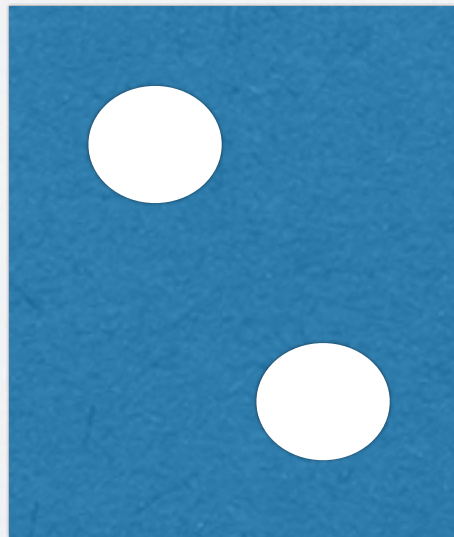
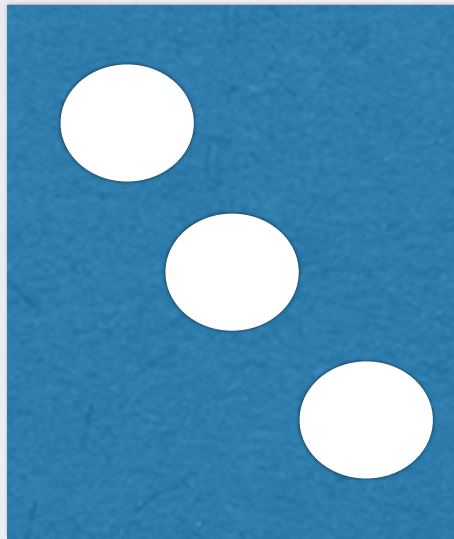
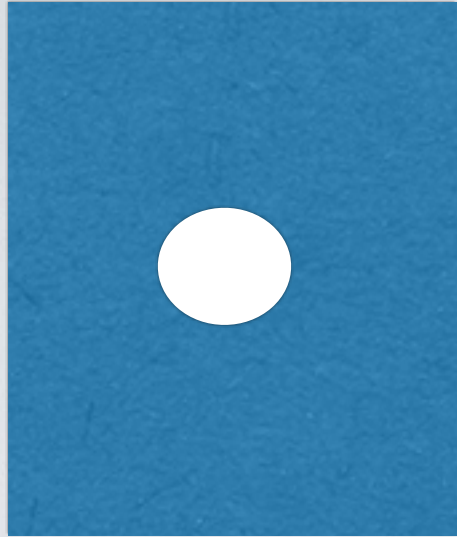
*A few weeks ago the silence of my Stanford office was interrupted by a phone call. A mother called to tell me that her 5-year old daughter had come home from school crying because her teacher had not allowed her to count on her fingers. A few weeks afterwards, when I told my undergraduate mathematics class that visual mathematics was really important, one of them asked: but it is only for low levels of math, isn't it?*



# WHAT IS SUBITIZING?

The term is derived from the Latin adjective *subitus* (meaning "sudden") and captures a feeling of immediately knowing how many items lie within the visual scene, when the number of items present falls within the subitizing range.

# SUBITIZING RESEARCH





# RATIONAL- LAYS THE GROUNDWORK FOR OPERATIONS

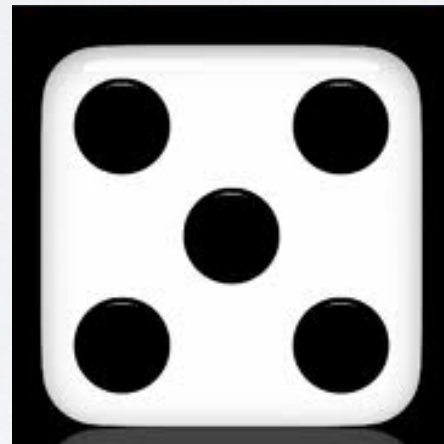
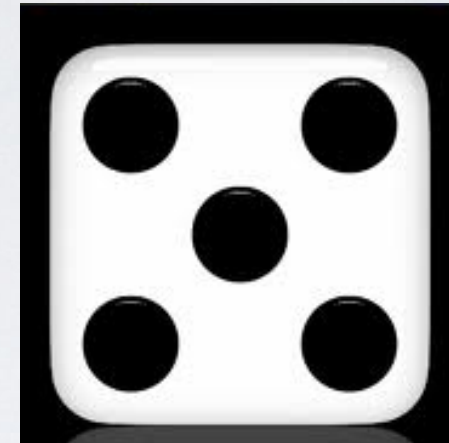
- Quick images give students experience counting and seeing (*subitizing*) numbers.
- Students can make connections between how they see the dots and the possible symbolic equations.
- 2 Phases of Subitizing:
  - **Perceptual subitizing**- ability to see a group and know how many without counting
  - **Conceptual subitizing**- ability to see more than one group and add them together to find the total.

# STRUCTURE WITHIN STRUCTURE





# STRUCTURE WITHIN STRUCTURE



# STRUCTURE WITHIN STRUCTURE

How many?



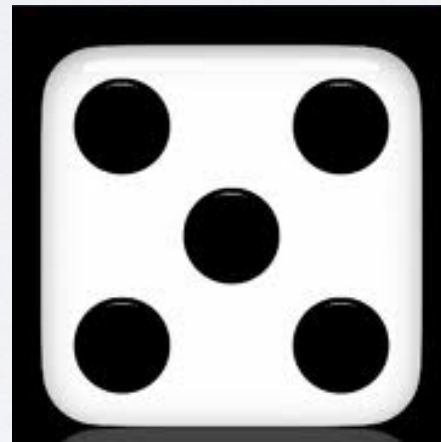
# VISUAL MODELS THAT ARE FLEXIBLE



Repeated  
Addition



Fractions



Multiplication



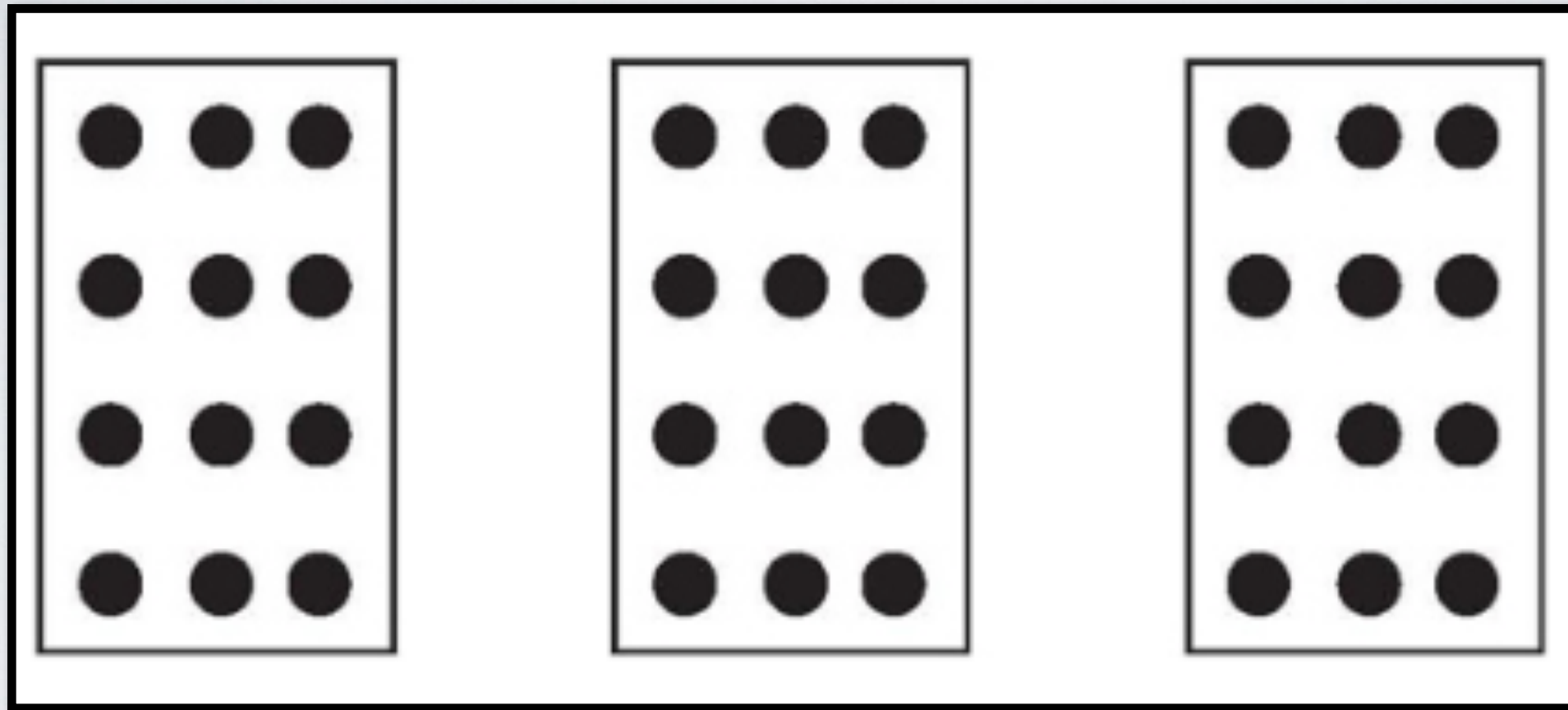
# SEEING NUMBERS AS GROUPS

## SUPPORTS:

- Counting on strategies
- Learning basic facts
- Decomposing numbers
- Development of addition strategies
- **Development of multiplication strategies**



# SUBITIZING AT A 3 - 5 LEVEL



**Could you see them?**

$$4 \times 3$$

$$6 \times 2$$

$$3 \times 4$$

$$3 (2 \times 2)$$

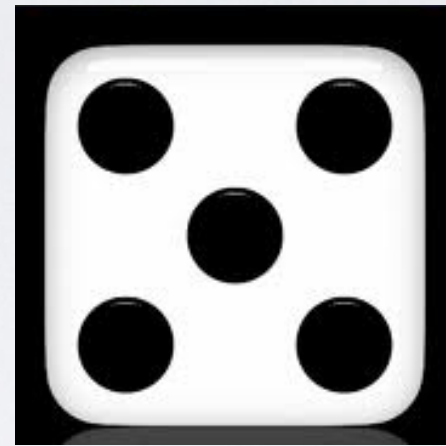
$$3 (4 \times 2)$$

$$4 (3 \times 2)$$

$$2 \times 6$$

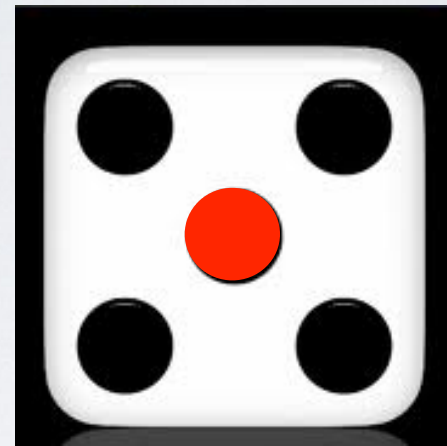
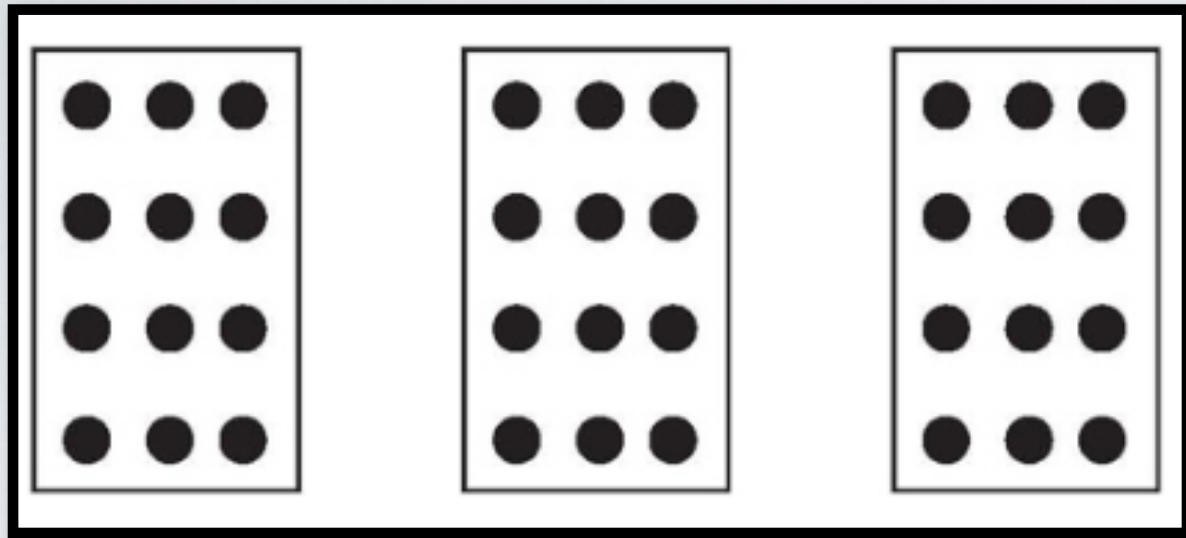
$$2 (3 \times 2)$$

# SUBITIZING AT A 3-5 LEVEL





# SUBITIZING AT A 3 - 5 LEVEL



**Can you see them?**

$$4 \times 9$$

$$3 (4 \times 3)$$

$$6 \times 6$$

$$4 (3 \times 3)$$

$$3 \times 12$$

$$2 \times 18$$

$$3 (2 \times 6)$$

$$2 (3 \times 6)$$

$$5 + 5 + 5 + 5$$

$$4 \times 5$$

$$2 (2 \times 5)$$

$$2 \times 10$$

$$(4 \times 4) + (1 \times 4)$$

$$4 \times 5 = 5 \times 4$$



3<sup>rd</sup> Grade

10/17/18

Subitizing





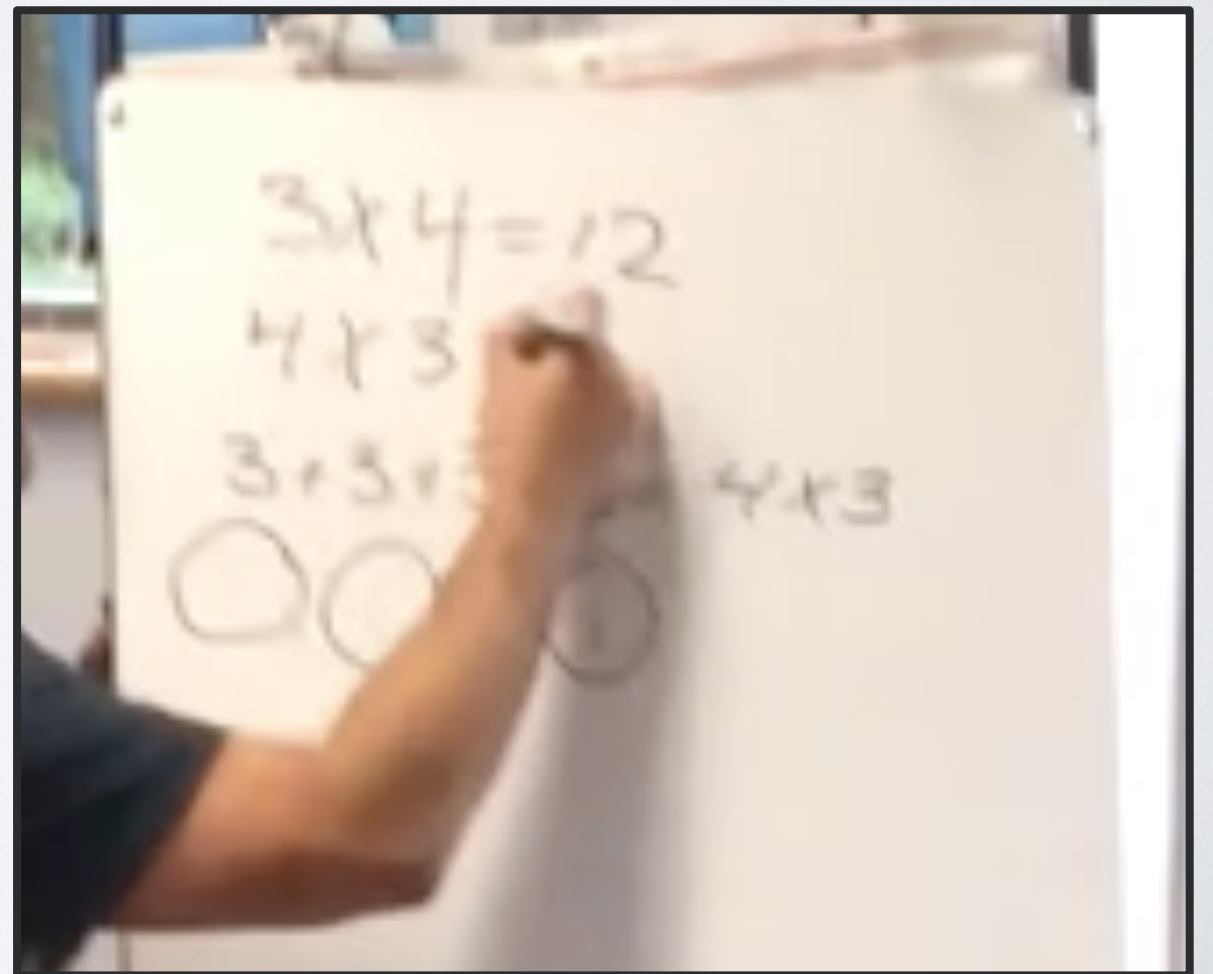
W. Knoultan III



# Images

- What do you notice?
- What do you wonder?

# CONNECTING REPEATED ADDITION AND MULTIPLICATION

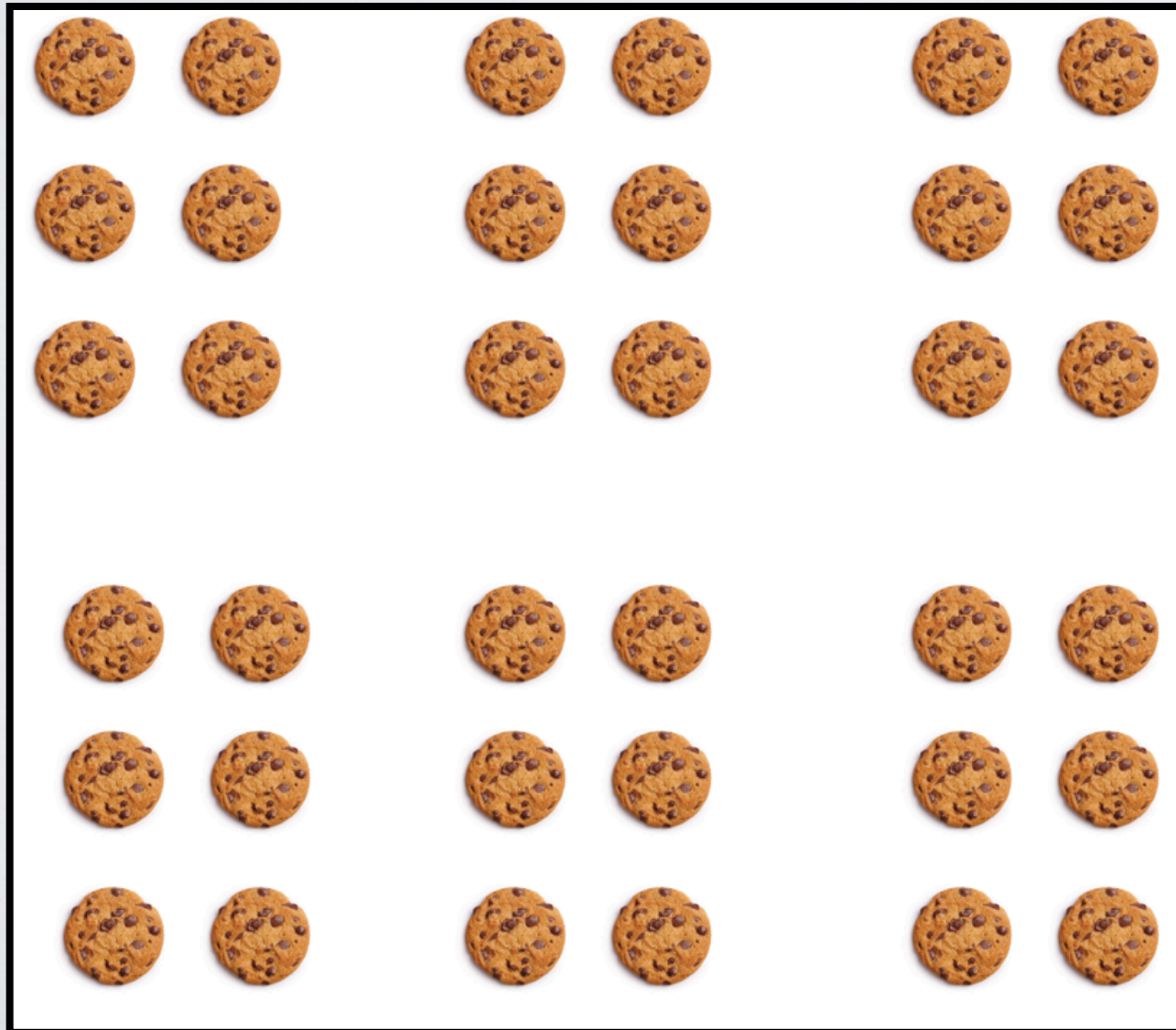




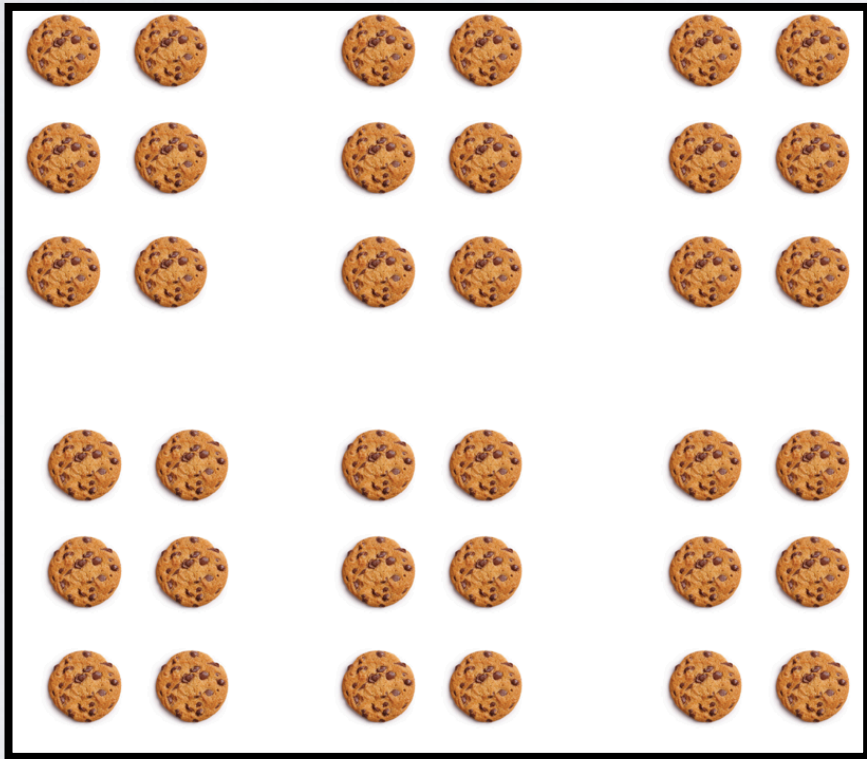




# GROUPS WITHIN GROUPS



# BEYOND 3RD GRADE



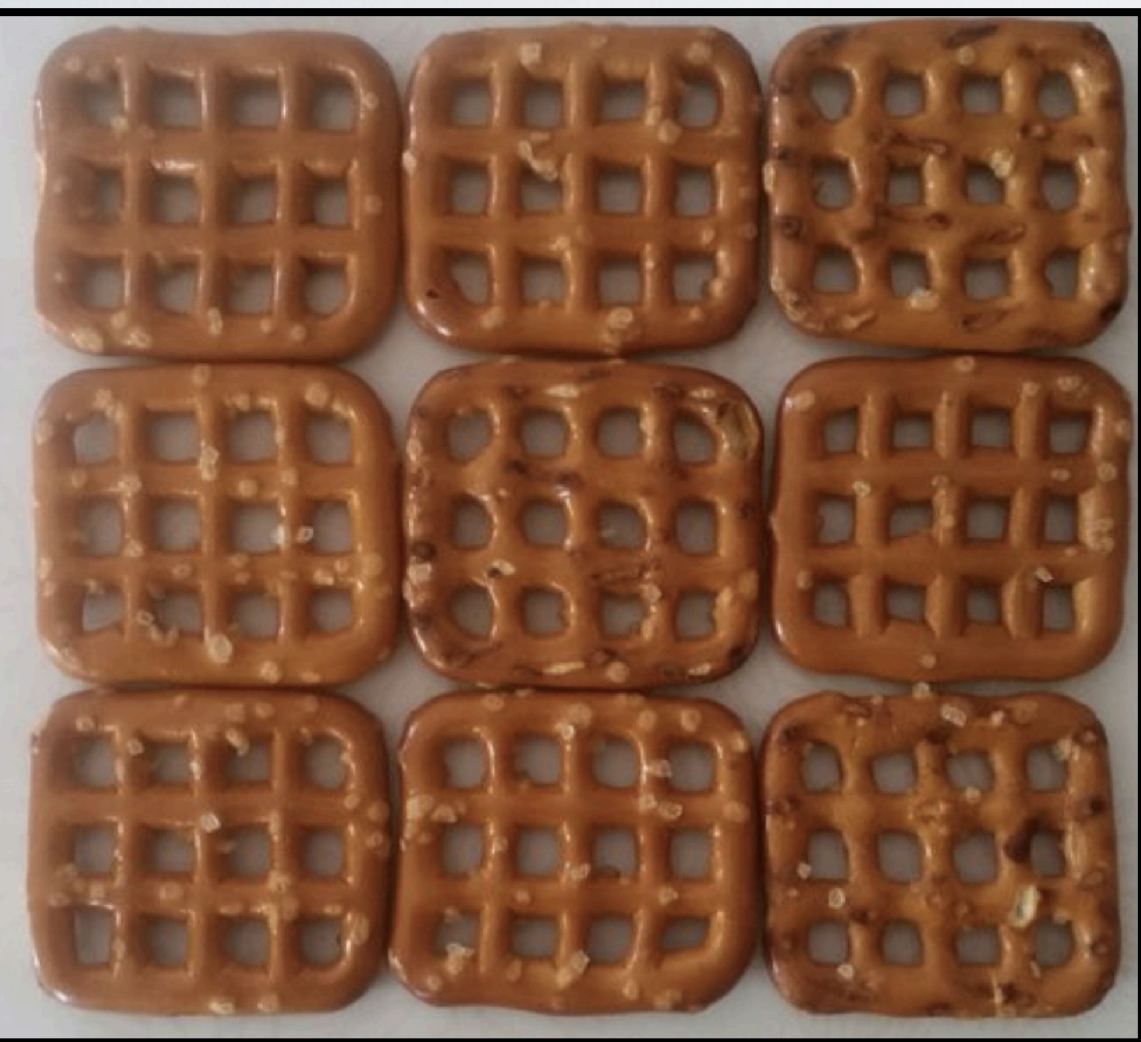


# GRAPEFRUIT FRACTIONS





# PRETZEL HOLES



$$12 + 12 + 12 + 12 + 12 + 12 + 12 + 12 + 12$$

$$[(2 \times 12) + (2 \times 12)] + [(2 \times 12) + (2 \times 12)] + 12$$

$$9 \times 12$$

$$48 + 48 + 12$$

$$(4 \times 12) + (4 \times 12) + 12 \text{ or}$$

$$2(4 \times 12) + (1 \times 12)$$

Others?

$3 \times 3 = 9$   
 $3 + 3 + 3 = 9$

$(2 + 12 + 12 + 12) + (2 + 12 + 12 + 12) + 12$   
 $\rightarrow 48 + 48 + 12$

$12 \times 9 = 108$   
 $\rightarrow 9 \times 12 =$

$\begin{array}{r} 48 \\ + 48 \\ \hline 96 \end{array}$   
 $96 + 12 = 108$



# CUPCAKES





# SEE MATH EVERYWHERE!





# SEE MATH EVERYWHERE!





# RESOURCES

- <http://ntimages.weebly.com/>

**NUMBER TALK IMAGES**

ACCUEIL | HOME

POINTS | DOTS

PHOTOS

SUITES | STRINGS

ÉLÈVES | STUDENTS

MORE...

## Number Talk Images

Un projet collaboratif qui a pour but de recueillir des images intéressantes qui peuvent servir comme point de départ pour des jasettes mathématiques au sujet des nombres.

A collaborative project dedicated to gathering interesting images to be used as a launching point for Number Talks.

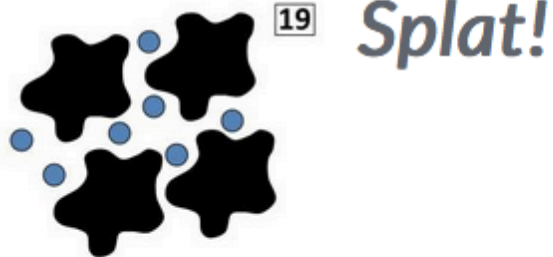
CONTRIBUER | CONTRIBUTE



# SOME RESOURCES TO CONSIDER

- <https://www.stevewyborney.com/?p=893>

## Steve Wyborney's Blog: I'm on a Learning Mission.



✉ By [stevewyborney@gmail.com](mailto:stevewyborney@gmail.com),  
Feb 9, 2017

Welcome to Splat! You are only **moments away** from a VERY POWERFUL, highly interactive number sense strategy that can be used **at any grade level!**

This post includes **50 (fifty!)** free, downloadable PowerPoint math lessons!

Watch the video, download some (or all) of the lessons, and experience what happens when you unleash this resource in your classroom!

I've been excited to click the Publish button on this post for several weeks! It's time!

UPDATE: [The Fraction Splat! Series](#) (with 20 more downloadable lessons) is now

### SUBSCRIBE HERE!

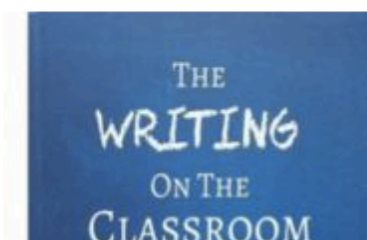
You will receive the latest updates as soon as they are posted!

Subscribe

We won't send you spam. Unsubscribe at any time.

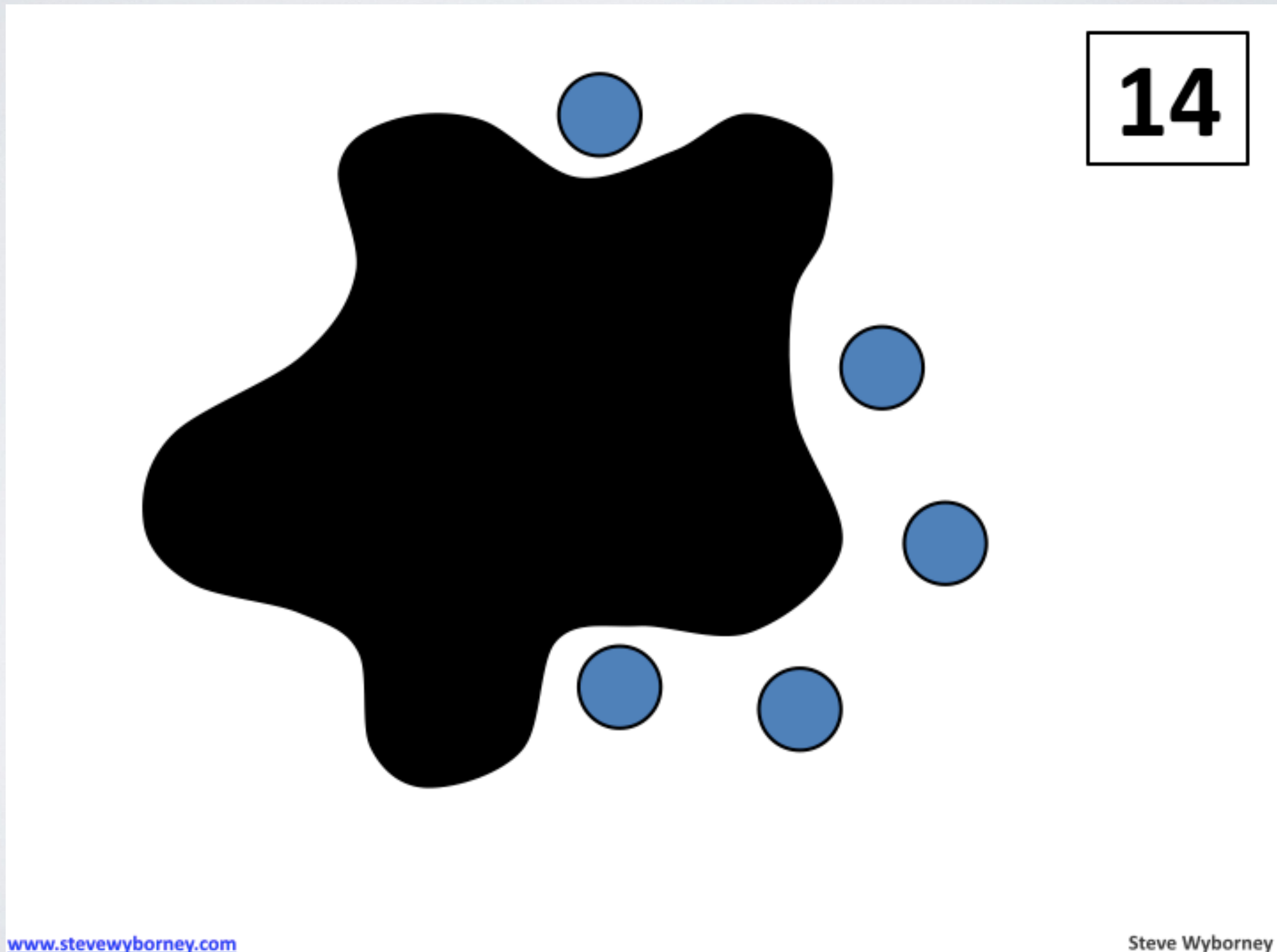
Powered by ConvertKit

**WHAT WERE THE 24 MESSAGES THAT STEVE WYBORNEY POSTED ON HIS CLASSROOM WALL?**



# SOME RESOURCES TO CONSIDER

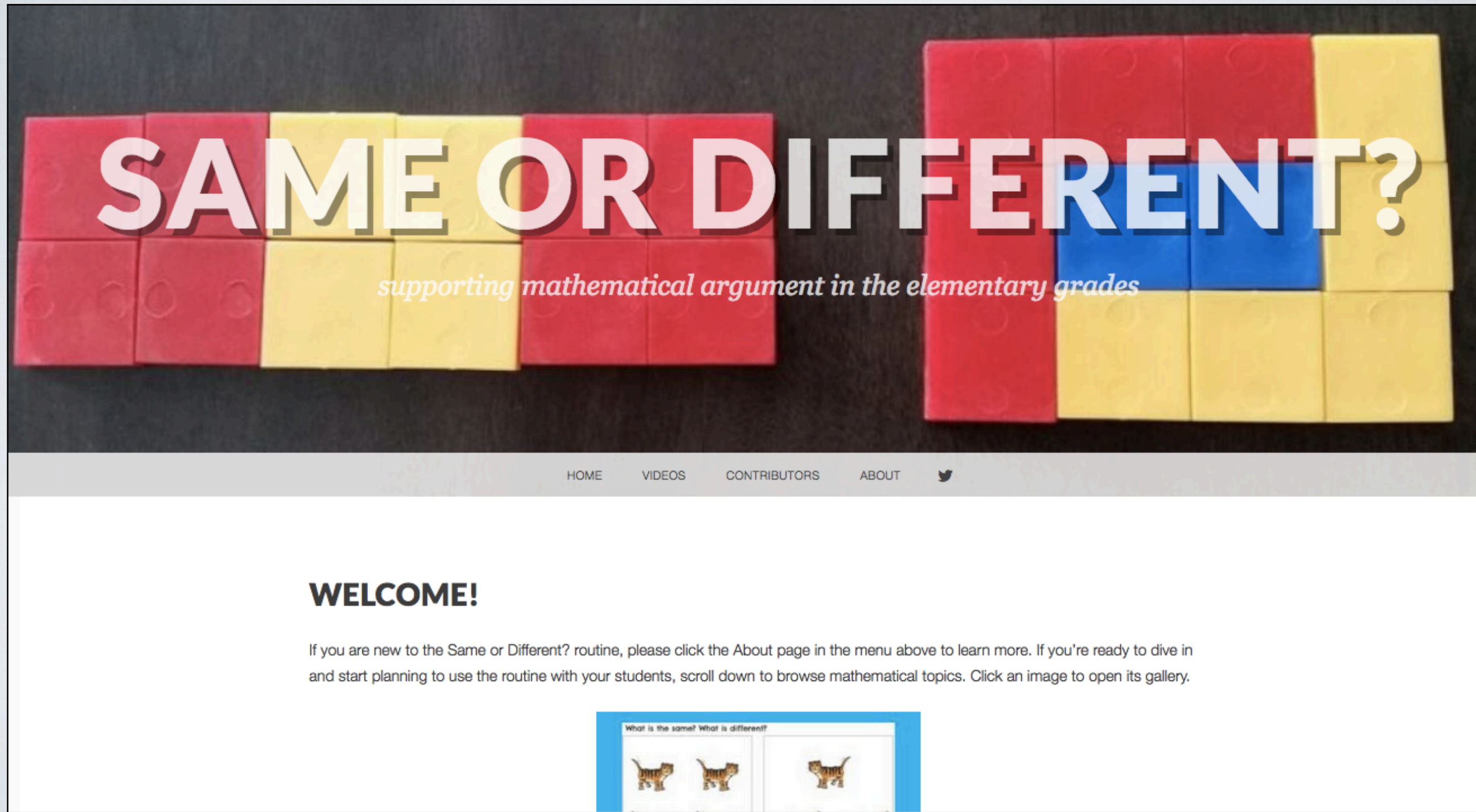
- <https://www.stevewyborne.com/?p=893>





# SOME RESOURCES TO CONSIDER

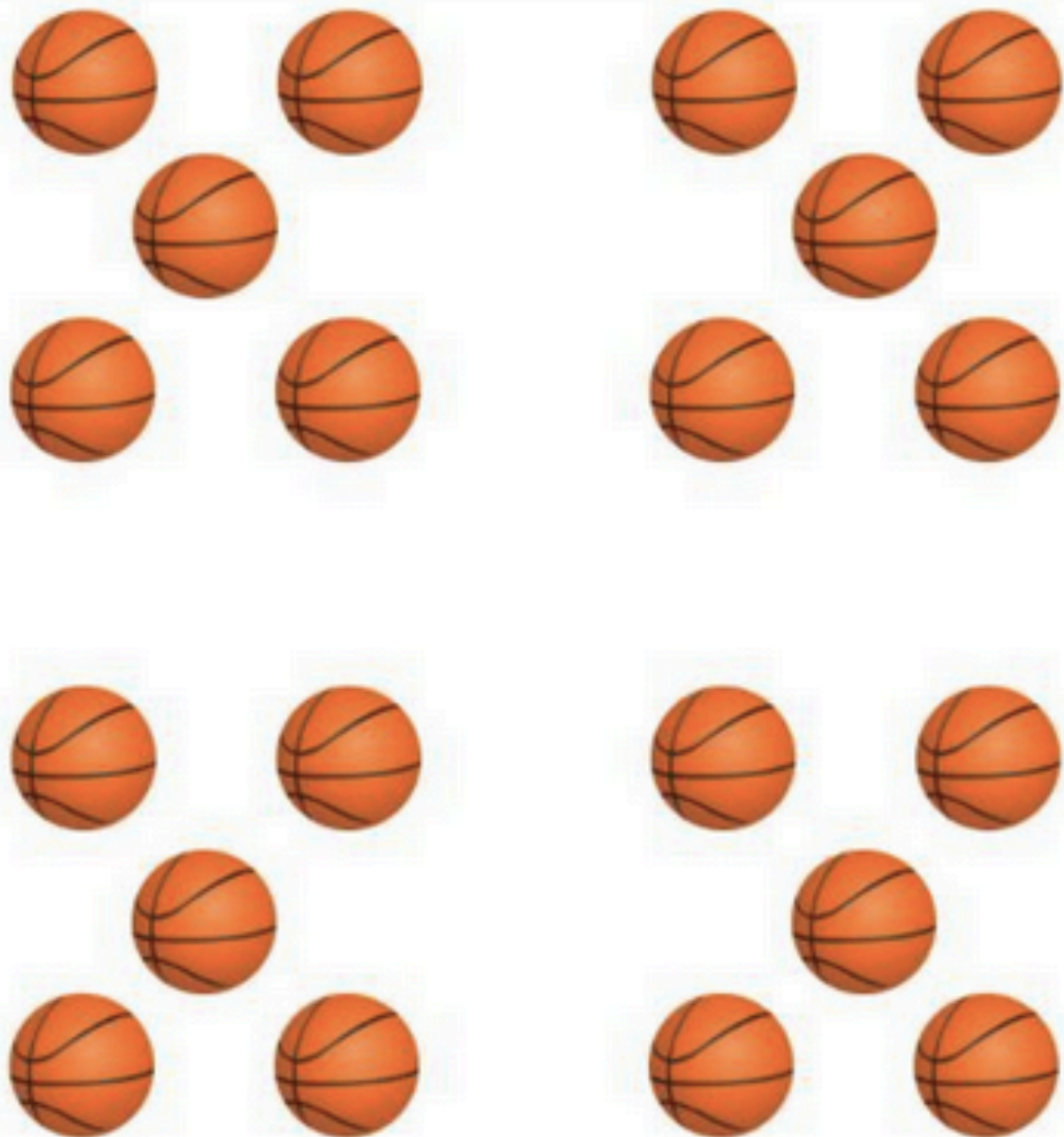
- <https://samedifferentimages.wordpress.com/>



# SOME RESOURCES TO CONSIDER

- <https://samedifferentimages.wordpress.com/>

What is the same? What is different?





# PLAY!

- Check out some of the websites
  - <http://ntimages.weebly.com/>
  - <https://www.stevewyborne.com/>
  - <https://samedifferentimages.wordpress.com/>
- Create your OWN visual image using icons
- Create your OWN visual representation using images (from online or your own!)

# CALL TO ACTION

- How can you take this information back to your school or district and use it with students?



# *Empowering the Mathematics Community*



Thanks for coming!  
Enjoy the rest of the  
conference.

**@teedjvt**

**#NCTMSDI9**

**@bbblankephd**

FOLLOW AND TWEET US!