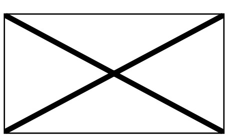
Tetralope

What do you get when you cross a jack rabbit with an antelope? Why, a jackalope, of course! What do you get when you cross a tetrahedron with an envelope? You guessed it—a **tetralope**!

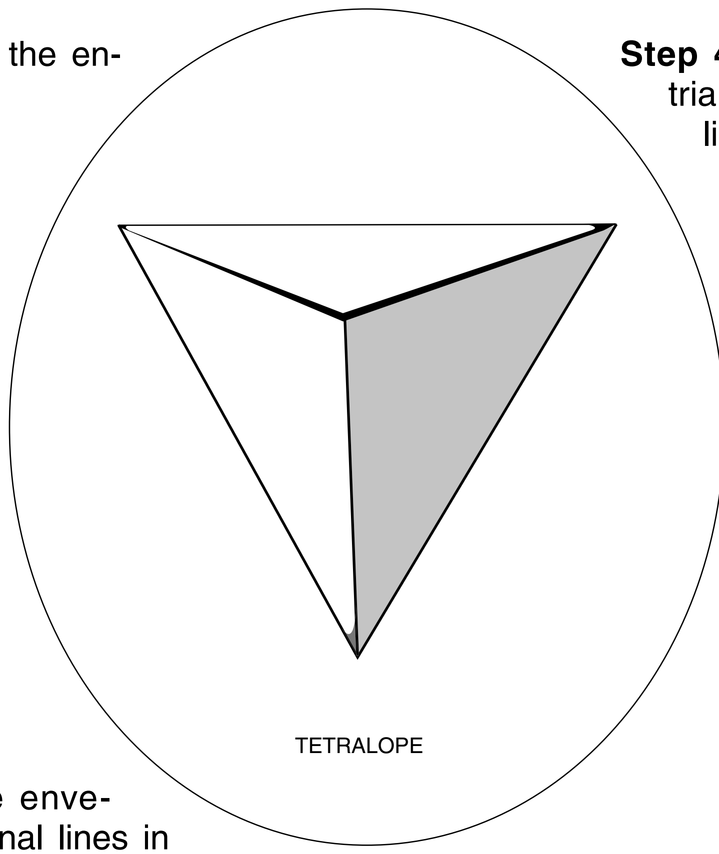
If you follow the directions below carefully, you will be able to cross a tetrahedron with an envelope. You will need two letter-sized (approximately 6.5” × 3.5”) envelopes, a straightedge or ruler, a pair of scissors, and a pen.

Step 1: Seal one of the envelopes.

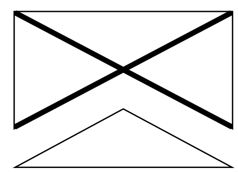
Step 2: Draw diagonals on one side of the envelope with your straightedge and pen. Make sure your marks are heavy and dark.



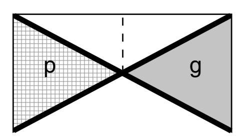
Step 3: Crease the envelope along the diagonal lines in both directions, forward and backward. If your marks were heavy, you should have no problem.



Step 4: Cut out one obtuse triangle along the diagonal lines. Save this piece.



Step 5: Take the concave pentagonal piece and make one more fold (forward and backward) down the middle.



The editors wish to thank David Masunaga, Iolani School, Honolulu, HI 96826, for the ideas used in this issue of NCTM *Student Math Notes.*

Step 6: Place your thumbs inside the envelope on the middle fold you just made and stretch the envelope open. A “Kermit”-like puppet should result. (See fig. 1.)

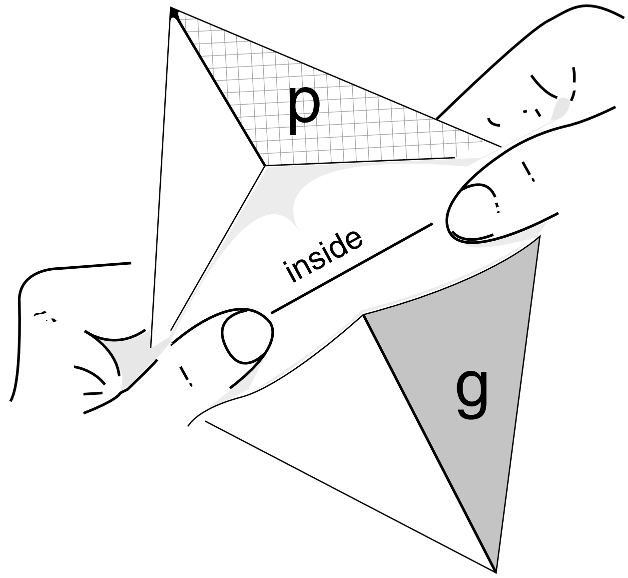


Fig. 1

Step 7: Stick Kermit’s upper jaw completely into his lower jaw, and with a little bit of adjustment, you should get a tetralope! (See fig. 2.)

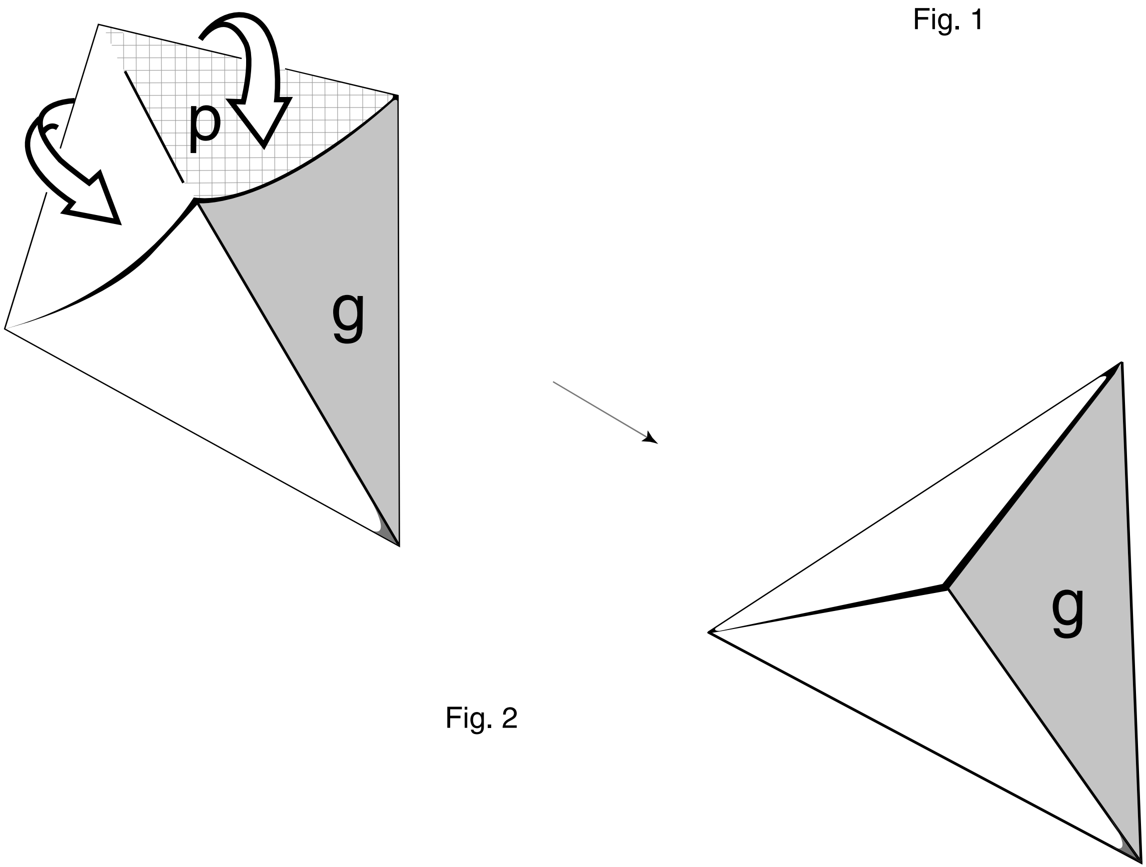


Fig. 2

Tetralope—*Continued*

1. *a)* How many faces does your tetralope have? ______________________

   b) edges? ______________________

   c) vertices? ______________________

2. Measure each edge of the tetralope.

3. Describe each face.

4. What is the area of each face? You will need to measure the height of the face to find its area.

5. What is the total surface area of the tetralope?

Take a pen or colored marker and shade all the faces of the tetralope so that when the envelope is opened and flattened, you can tell which parts of the envelope were the faces of the tetralope.

6. Open and flatten the envelope and reattach the obtuse triangle you cut out. Look at both sides of the envelope. What do you notice?

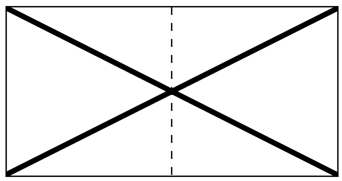
7. Use your ruler to take measurements, and find the area of one side of the envelope.

8. How does it compare to the surface area of the tetralope?

With the second envelope, try the variation on the next page for making a tetrahedron. A *tetrahedron* is the mathematical name for a tetralope.

Repeat steps 1, 2, and 3 from the first page.

Step 4: Now, fold the envelope in half and crease well, as shown below.



Step 5: Make any cut that is point-symmetric with respect to the intersection point of the diagonals. See figure 3 for some possibilities.

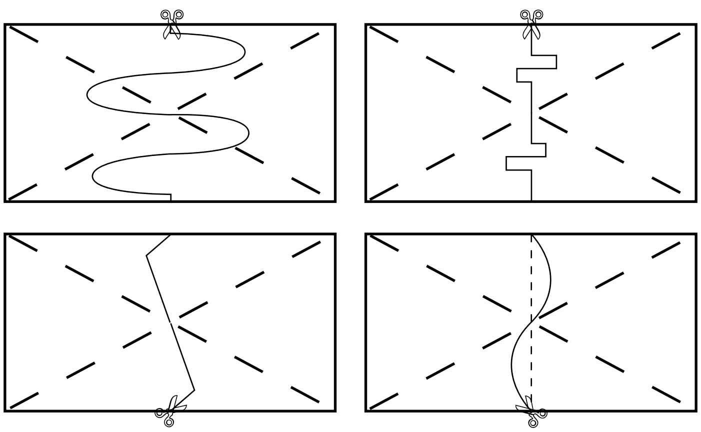


Fig. 3

9. Fold both halves along the straight segments creased earlier and you will get two separate tetrahedra. Explain why, when you fold the two half envelopes, the cut edges of the tetrahedra fit exactly.

Can you …

- examine what happens by repeating the original directions with envelopes of different proportions, such as legal sized, greeting-card sized, and so on?

Did you know …

- a regular tetrahedron can be created by folding two one-dollar bills?

Mathematical content

- Measurement, area of rectangle, area of triangle, surface area of tetrahedron

Bibliography/Resources

Edmonson, Amy. *A Fuller Explanation: The Synergetic Geometry of R. Buckminster Fuller.* Boston: Birkhauser, 1987.

Pedersen, Jean. “Letter to the Editor.” *Mathematics Magazine* 61 (1988): 270.

Steen, Lynn Arthur, ed. *On the Shoulders of Giants: New Approaches to Numeracy.* Washington, D.C.: National Academy Press, 1990.

Answers

1. *a)* 4
   b) 6
   c) 4
2. Answers will vary, but all edges will be about the same length.
3. All about the same size, equilateral triangles
4. Answers will vary.
5. Answers will vary, about four times the answer in #4.
6. The surface area of a tetrahedron is equal to the area of the original envelope.
7. Answers will vary.
8. Should be about the same
9. The cut is made up of a preimage and its 180-degree rotated image. A rotated image is congruent to its preimage. The position of the cut and the way the envelope is folded aligns the image and preimage. They fit exactly because they are congruent.

NCTM STUDENT MATH NOTES is published by the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 20191-9988. The five issues a year appear on the NCTM Web site, www.nctm.org, in September, November, January, March, and May/June. Pages may be reproduced for classroom use without permission.